Lecture 4

## NOISE AND ITS ANALYSIS

Voltage and current in all components of electronic circuits are always accompanied by noise voltage and current, which are called just ***noise.*** The common feature of all noises (they should not be mixed up with quasi-noise signals) is, firstly, impossibility to predict the appearing of any specific value at any specific moment, i.e. its ***randomness*** and, secondly, ***its zero average value***.

### Main Definitions

Despite the fact that the average value of noise is equal to zero, it exists physically and the ***squared*** noise value is not equal to zero. The root-mean-square value of noise is determined in the following way:

$$V_{n(rms)} \equiv \left[ \frac{1}{T} \int_0^T v_n^2(t)dt \right]^{\frac{1}{2}} \quad \text{for noise voltage} \qquad (4.1)$$

$$I_{n(rms)} \equiv \left[ \frac{1}{T} \int_0^T i_n^2(t)dt \right]^{\frac{1}{2}} \quad \text{for noise current} \qquad (4.2)$$

here *T* is the averaging time interval. The bigger the time interval T is, the more accurate values $V_{n(rms)}$ and $I_{n(rms)}$ are. It is clear that the squared values in (4.1) and (4.2) are ***power*** being dissipated in 1 Ohm resistor if the root-mean-square voltage $V_{n(rms)}$, or constant voltage numerically equal to it, is applied to it, and also if the root-mean-square current $I_{n(rms)}$, or constant one numerically equal to it, flows through the 1 Ohm resistor.

***The signal-to-noise ratio*** is determined as follows:

$$SNR = 10lg\left[\frac{signalpower}{noisepower}\right](dB) = 10lg\left[\frac{V_x^2}{V_n^2}\right](dB) = 20lg\left[\frac{V_x}{V_n}\right](dB). \qquad (4.3)$$

Here $V_x^2$ is the signal power, $V_n^2$ is the noise power.

Although decibels ***(dB)*** refer to the ***ratio*** of two values according to their definition it appeared useful to introduce the power value definition in ***dB*** for ***absolute*** values of the signal. It is provisionally accepted that the power equal to 1 mW is called 1dBm. For example, the power of 1 μW is designated as - 30 dBm. In case when ***voltages*** are indicated in dBm, 1 dBm is defined as voltages (root-mean-square or constant) at a number of resistors (600 Ohm, 75 Ohm and 50 Ohm), at which the same powers of 1 mW are dissipated.

### Summation of noises

Let us consider the case of noise voltage sources connected in series and noise current sources connected in parallel (Fig. 2.4.1).

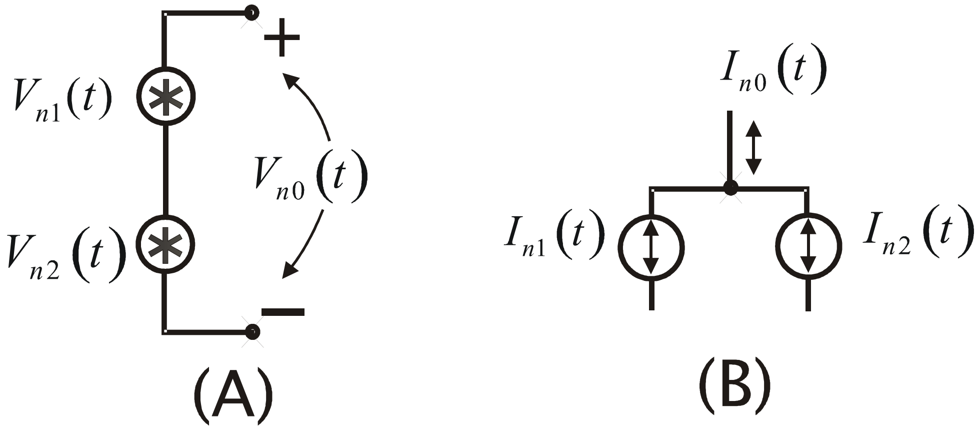


Fig.2.4.1. Combining two noise sources:
(a) of voltage and (b) of current.

Let us define $V_{n0}(t) as V_{n0}(t) = V_{n1}(t) + V_{n2}(t)$ (4.4)

Then

$$V_{no(rms)} = \frac{1}{T}\int_0^T \left[V_{n1}(t) + V_{n2}(t)\right]^2 dt = V_{n1(rms)}^2 + V_{n2(rms)}^2 + \frac{2}{T}\int_0^T \left[V_{n1}(t)V_{n2}(t)\right]dt \quad (4.5)$$

The first two terms in the right part (4.5) are noise powers of both sources. The last term expresses the ***correlation*** between the noise sources. The correlation C is commonly defined in the following way:

$$C \equiv \frac{\frac{1}{T}\int_0^T \left[V_{n1}(t)V_{n2}(t)\right]dt}{V_{n1(rms)}V_{n2(rms)}} \quad (4.6)$$

Taking into account this definition (4.5) we can write:

$$V_{n0(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2 + 2CV_{n1(rms)}V_{n2(rms)} \quad (4.7)$$

The correlation coefficient *C* satisfies the inequality $-1 \le C \le 1$. If $C = \pm 1$, the two signals are fully correlated, if $C = 0$, the signals are not correlated. Intermediate values of *C* mean partial correlation. Thus, in case of two non-correlated signals

$$V_{n0(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2, \quad (4.8)$$

in case of fully correlated signals (for example, two sinusoidal signals with the same frequencies and phases of 0 or 180 degrees)

$$V_{n0(rms)}^2 = \left[V_{n1(rms)} \pm V_{n2(rms)}\right]^2 \quad (4.9)$$

In amplifying electronic circuits, the specific noise value in most cases is much lower than the set or expected current or voltage values. Therefore noise voltages or currents can be considered not influencing characteristics of current and / or voltage dependent components (for example, the transistor transconductance), and noise voltages of two devices connected in series (or noise currents of two devices connected in parallel) do not correlate mutually.

## Noise analysis in the frequency domain

In view of the randomness and unpredictability of noise signal values its power $V_n^2(f)$ (or) $I_n^2(f)$) is continuously distributed in the frequency domain. Due to the ***continuous*** distribution the noise power in the infinitely small frequency band is equal to ***zero!*** When speaking about a specific spectral density of noise at some frequency, they mean the noise power in the 1 Hz frequency band by default, and the mentioned frequency is in the middle of this band. The total noise power can be obtained by integrating the noise density throughout the whole frequency spectrum:

$$V_{n(rms)}^2 = \int_0^\infty V_n^2(f)df \qquad (4.10)$$

Let us consider passage of the noise signal $V_{ni}(f)$ through the filter with the transfer function . The spectral density $V_{n0}(f)$ of the noise power is equal to:

$$V_{n0}^2(f) = |A(j2\pi f)|^2 V_{ni}^2(f) \qquad (4.11)$$

The total noise power $V_{n0}^2(rms)$ is equal to

$$V_{n0(rms)}^2 = \int_0^\infty |A(j2\pi f)|^2 V_{ni}^2(f)df \qquad (4.12)$$

The root-mean-square noise density value $V_{n0}(f)$ equals to

$$V_{n0}(f) = |A(j2\pi f)| V_{ni}(f) \qquad (4.13)$$

As the root-mean-square noise value is defined in the way usual for the linear filter, by means of the transfer function module, but not through its square it is more convenient to use the root-mean-square value but not the noise power.

Now, let us assume that the system noise $V_{n0}^2(f)$ is the sum $N$ of noise signals $V_{ni}^2(F)$, and each noise signal $V_{ni}^2(f)$ passes through the filter . Then

$$V_{n0}^2(f) = \sum_i^N |A_i(j2\pi f)|^2 V_{ni}^2(f) \qquad (4.14)$$

If the noise signals are not mutually correlated they do not correlate at the output either.

**White noise**

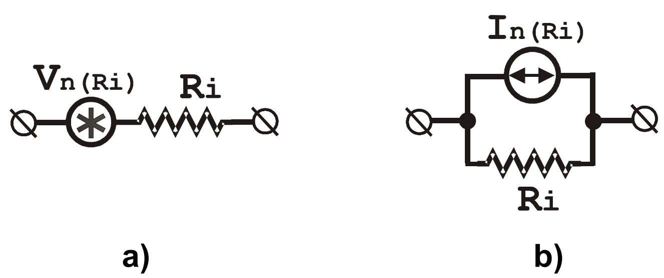


Fig. 2.4.2. Equivalent circuit of the noisy resistor:
a) is the noise voltage source connected in series;
b) is the noise current source connected in parallel

***White noise*** is defined as noise with a constant spectral noise density independent of the frequency. Resistor is the best known source of white noise. The resistor noise is simulated as a voltage source with the spectral density $V_R^2(f)$ connected in series with the resistor:

$$V_R^2(f) = 4kTR \qquad (4.15a)$$

Here $k \cong 1.38 \times 10^{-23}\left(\frac{Joule}{Kelvin}\right)$.

It should be noted that the symbol of the noise source itself in no way can be considered as an independent voltage source in the common representation of the noisy resistor Ri with the noise voltage source Vn(Ri) connected in series. Thereupon in Fig. 2.4.2, for example, ***there is no node between symbols of the voltage source and the resistor,*** as they must be considered as a single unit. This should be taken into account when analyzing and setting up Kirchhoff's equations.

Along with model (4.15) the resistor noise model is used equally as a current source with the spectral density $I_R^2(f)$ connected in parallel with the resistor:

$$I_R^2 = \frac{V_R^2(f)}{R^2} = \frac{4kT}{R} \quad . \qquad (4.15b)$$

**Noise Band**

As is well known, the ideal filter is a filter with a square amplitude-frequency characteristic within which the transfer coefficient module (gain) is equal to one and in the stopband it is equal to zero. Actual filters have a non-uniform amplitude-frequency characteristic in the pass band, a finite slope in the transition band, and different-from-zero transmission in the stopband.

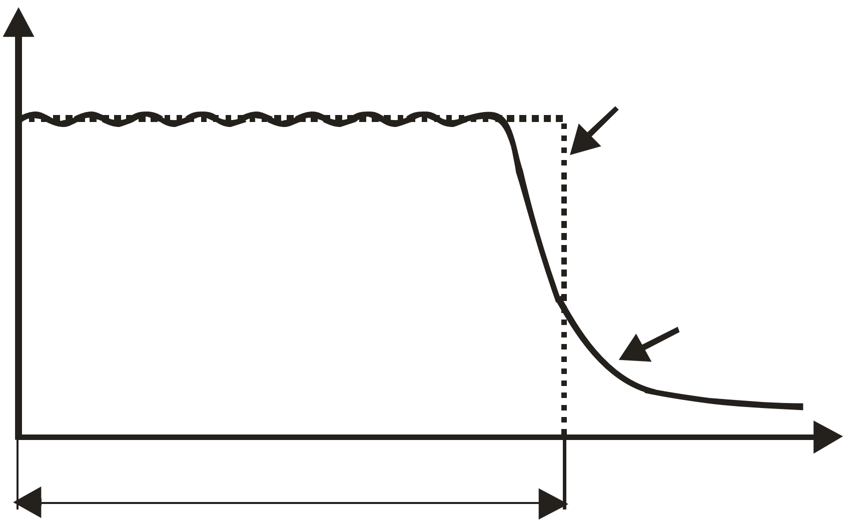

Fig.2.4.3. Actual and ideal filters with identical powers of output noise with the same spectral densities of white noise at the inputs.

By the noise bandwidth they mean the pass band of the ideal square low-pass filter where the noise at the output is the same as at the considered actual filter output provided that identical sources of white noise are connected to the inputs of both filters.

The passive RC filter of the first order is an obvious example filter. The following expression is the module of its transfer function (transfer coefficient versus frequency):

$$|H(j\omega)| = \sqrt{\frac{1}{1+(\omega RC)^2}} = \sqrt{\frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^2}} = \sqrt{\frac{1}{1+\left(\frac{f}{f_0}\right)^2}}, \qquad (4.16)$$

where $\omega_0 = 2\pi f_0 = \frac{1}{RC}$ is the frequency of the passive RC-filter real pole. This is a representative example as, in fact, this filter exists in each system node in the system with real poles. Here the node capacitor, which is an attribute of any node of any system, is charged through the real resistor, which is called the output resistance in the node.

Let us connect to the filter input a white-noise source with the frequency-independent spectral density: $V_{ni}(f) = V_{nw} = const$ (4.17)

The total power $V^2_{no(rms)}$ of the noise, which has passed through the filter, equals to:

$$V^2_{no(rms)} = \int_0^\infty V^2_{nw}|H(j2\pi f)|^2 df = \int_0^\infty \frac{V^2_{nw}}{\left(1+\left(\frac{f}{f_0}\right)^2\right)} df = V^2_{nw} f_0 arcrtg\left(\frac{f}{f_0}\right)\Bigg|_0^\infty = \frac{V^2_{nw}\pi f_0}{2} = \frac{V^2_{nw}\omega_0}{4} \qquad (4.18)$$

The noise which is the same in value but has passed through the ideal square filter equals to:

$$V^2_{square(rms)} = \int_0^{f_x} V^2_{nw} df = V^2_{nw} f_x \qquad (4.19)$$

Comparing (4.18) and (4.19) we shall get:

$$f_x = \frac{\pi f_0}{2} = \frac{\omega_0}{4} \qquad (4.20)$$

The calculation given above is aimed also at obtaining the fundamental expression below, which is used in analyzing analog circuits on switching capacitors. Let us calculate the expression for the total power of the voltage noise on the passive RC-filter capacitor with a noisy resistor:

$$V^2_{no(rms)} = 4kTR\frac{\omega_0}{4} = 4kTR\frac{1}{4RC} = \frac{kT}{C} \qquad (4.21)$$

**The Elementary Sample–and-Hold Circuit Noise**

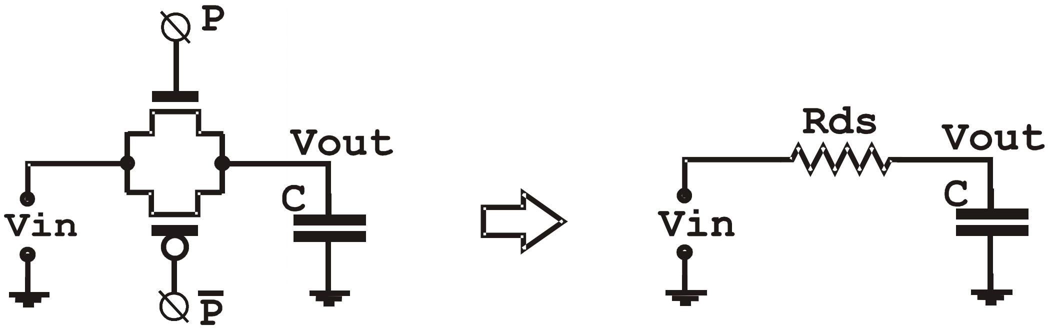


Fig.2.4.4. Elementary sample-and-hold circuit with the finite resistance Rds of the switch channel

Let the switch channel resistance make noise as a resistor of the Rds value. Then the root-mean-square noise $V_{nS\&H(rms)}$ on the capacitor C at the sampling moment equals to the fundamental value $\sqrt{\frac{kT}{C}}$ and does not depend on the sampling number and rate.

**Example calculation of noise for the ARC circuit based on the CMOS OA (1st order ARC filter)**

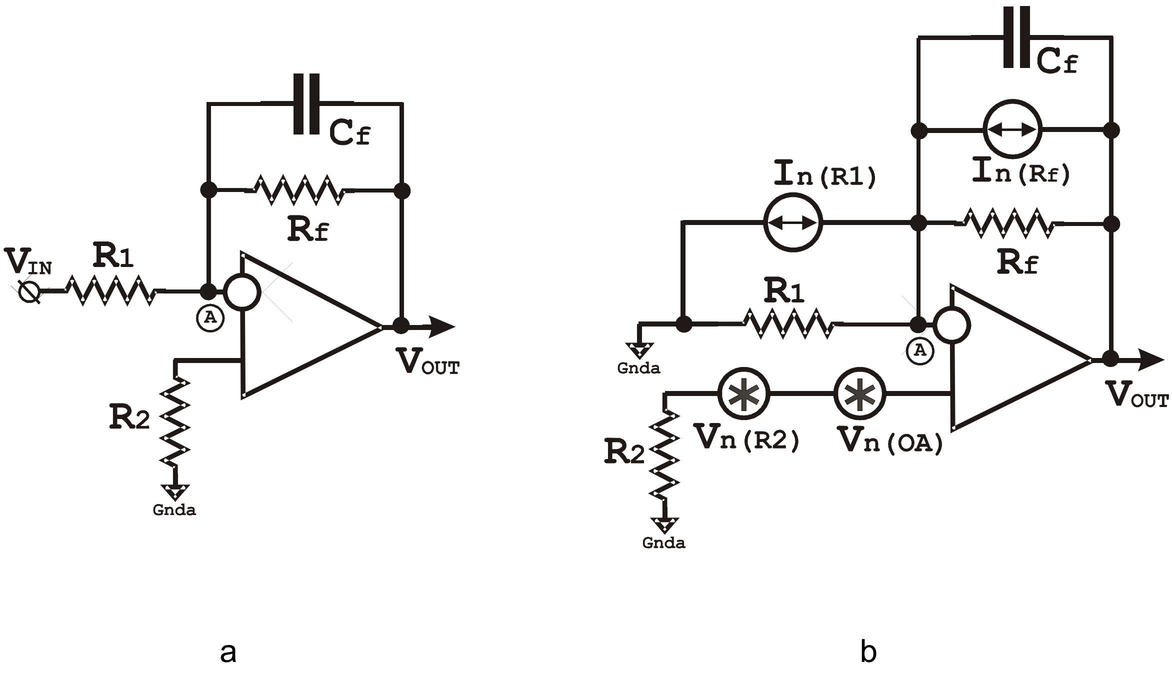


Fig. 2.4.5.a 1st order ARC filter
Fig. 2.4.5.b Equivalent circuit for the noise calculation

Let the OA have an infinite amplification for simplicity. Then the filter transfer function at applying the input signal as in Fig. 2.4.5a, i.e. to the inverting input circuit, will be:

$$H(s) = \frac{\frac{R_f}{R_1}}{\left(1 + sR_f C_f\right)} \qquad (4.22)$$

The filter transfer function at applying the input signal to the non- inverting input will be:

(4.23)

Now, some words about the relevancy of the chosen equivalent circuits of noisy resistors in Fig. 2.4.5b. As it can be seen from the figure, for R1 and Rf equivalent circuits with noise current sources connected in parallel are chosen, and for R2 and the OA a noise voltage source connected in series is chosen. The problem is that the voltage source connected in series at any place acts ***as if it divided*** the noisy resistor as the whole system into parts. This may result in the wrong analysis logic and errors in circuits with a through current. If we look forward to an obvious model we should represent a noise voltage source with an output resistance equal to the resistor rated value ***instead*** of the noisy resistor.

As to noise current sources, they do not disturb the obvious integrity and symmetry of the noisy resistor, when connected in parallel; thus using this very representation in any current circuits is preferable and convenient. However, in circuits with no constant current flowing, as at the non-inverting input of the CMOS OA with actually infinitely high resistance it is correct and convenient to use noise voltage sources.

First, we shall study the response of the ARC circuit shown in Fig. 2.4.5b to noise sources (current and voltage sources) separately (assuming that the others are equal to zero). Then we shall square the obtained output voltages, add them and get the total noise power at the output $V^2_{no(rms)}$.

***Response to the noise current source $I_{n(R1)}$.***

It should be noted that there are constant potentials equal to zero at both R1 outputs and the current flowing through R1 equals to zero.

The Kirchhoff's equation: $I_{n(R1)} = \left(0 - V_{out(R1)}\right)\left(\frac{1}{R_f} + sC_f\right)$ (4.24)

We shall obtain: $V_{out(R1)} = -I_{n(R1)} R_f \frac{1}{1 + sR_f C_f}$;

$$V^2_{out(R1)(rms)} = I^2_{n(R1)(rms)} R_f^2 \left|\frac{1}{1 + j\omega R_f C_f}\right|^2 \qquad (4.25)$$

***Response to the noise current source $I_{n(Rf)}$.***

We still suppose the current through R1 equal to zero.

The Kirchhoff's equation:

$$0 = I_{n(Rf)} + \left(0 - V_{out(Rf)}\right)\left(\frac{1}{R_f} + sC_f\right) \qquad (4.26)$$

We shall obtain: $V_{out(Rf)} = I_{n(Rf)} R_f \frac{1}{1 + sR_f C_f}$

$$V_{out(Rf)(rms)} = I^2_{n(Rf)(rms)} R_f^2 \left| \frac{1}{1 + j\omega R_f C_f} \right|^2 \quad (4.27)$$

As it can be seen from (4.25) and (4.27), the response to the noise current source in the OA inverting input circuit is equal to the voltage resulting from this current flowing in the resistor in the feedback circuit multiplied by the transfer coefficient of the ARC filter ***without amplification.***

***Response to the noise voltage source*** $V_{n(R2)}$ ***.***

The basic response comes to the fact that all the noise voltage $V_{n(R2)}$ is applied to the OA non-inverting input. As it is agreed that the OA has an infinite amplification the potential at the inverting input repeats completely the potential of the non-inverting input.

The Kirchhoff's equation:

$$\frac{0 - V_{n(R2)}}{R_1} = \left(V_{n(R2)} - V_{out(R2)}\right)\left(\frac{1}{R_f} + sC_f\right) \quad (4.28)$$

We shall obtain:

$$V_{out(R2)} = V_{n(R2)} \left( 1 + \frac{\frac{R_f}{R_1}}{1 + sR_f C_f} \right)$$

$$V^2_{out(R2)(rms)} = V^2_{n(R2)(rms)} \left( 1 + \frac{\frac{R_f}{R_1}}{1 + sR_f C_f} \right)^2 \quad (4.29)$$

***Response to the voltage source of the Operational Amplifier equivalent noise reduced to the input.***

It is most convenient to place this voltage source at the non-inverting input. The analysis is similar to the resistor R2 response analysis, thus, the result is also similar:

$$V^2_{out(OA)(rms)} = V^2_{n(OA)(rms)} \left( 1 + \frac{\frac{R_f}{R_1}}{1 + sR_f C_f} \right) \quad (4.30)$$

**Example calculation of the input referred noise of the CMOS differential stage with an active load**

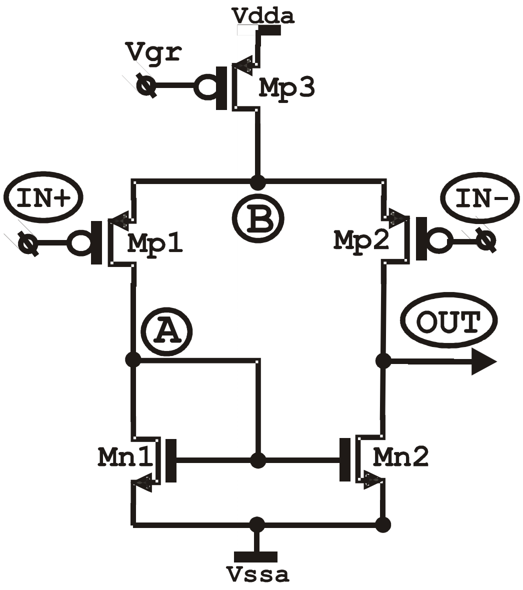


Fig.2.4.6. CMOS differential stage with an active IOA

Let the operation current source on Mp3 have a large output resistance. It gives an opportunity to consider that, for example, a current increase in Mp1 by some value results in current decrease in Mp2 by the same value (this behavior is caused by the potential change on node B, which is a source both for Mp1 and Mp2). Let the mentioned current increase be due to the internal noise in Mp1. Thus, the total noise current of the transistor Mp1 is halved between Mp1 and Mp2. The same with the noise current of Mp2, i.e. half of it goes to Mp2 and the other half goes to Mp1.

Let us consider transistors Mn1 and Mn2. Source potentials are invariable for Mn1, as well as for Mn2, so intrinsic noise currents flow fully in them. However, it should be noted that Mn1 is connected as a diode, thus, the current change in Mn1, due to the noise, results in excess over the threshold $(V_{GS} - V_T)_{Mn1}$ and, therefore, the potential of node A. Additionally, in transistor Mn1 there is also a current, noise current included, of transistor Mp1 and, consequently, of halves of noise currents of transistors Mp1 and Mp2. All the above mentioned noise currents (Mn1 noise current and halves of Mp1 and Mp2 noise currents) modulate the potential of node A. However, node A is connected with the gate of transistor Mn2, and transistor Mn2 becomes a source of the currents listed above, in brackets, **BUT OF THE OPPOSITE SIGN.** This implies that the halves of Mp1 and Mp2 noise currents, “reflected” in Mn2 flowing in Mn1, are in phase with those halves of Mp1 and Mp2 noise currents which flowed in Mp2 “from the very beginning”. Mn1 noise current is fully “reflected” in Mn2, and Mn2 “intrinsic” noise current always flows in it.

Thus, in Mn2, and, therefore, in Mp2, and in the differential stage output circuit total noise currents of four transistors, Mp1, Mp2, Mn1, Mn2 flow, and their squares are added arithmetically. It is clear, that phase coincidence of halves of Mp1 and Mp2 noise currents which “initially” were in Mp2 and “reflected” halves of the same currents is possible in LOW frequency area only, when the inevitable phase delay due to parasitic capacitances in nodes A and B can be neglected.

Now let us study the noise caused by transistor Mp3 in the output circuit. The total current $I_{Mp3}$, generated by it is equal to $I_0 + I_{n(Mp3)}$, where $I_0$ is an operation current of the differential stage, $I_{n(Mp3)}$ is a noise current of transistor Mp3. In the diode on Mn1 there is the current $\frac{I_0 + I_{n(Mp3)}}{2}$ from Mp3, and the voltage $(V_{GS} - V_T)_{Mn1}$ on the diode is equal to:

$$(V_{GS} - V_T)_{Mn1} = \frac{I_0 + I_{n(Mp3)}}{\beta_{Mn1}(V_{GS} - V_T)_{Mn1}} = \frac{I_0 + I_{n(Mp3)}}{g_{Mn1}} \qquad (4.31)$$

Here: $\beta_{Mn1} = C_0 \mu_n \frac{W_E}{L_E}$, where $C_0$ is the specific capacitance of the gate dielectric per unit area, $\mu_n$ is electron mobility, $W_E$ and $L_E$ are effective width and length of the Mn1 channel; $g_{Mn1}$ is the Mn1 transconductance.

The evident should be noted: the role of noise grows as the differential stage becomes closer to the symmetrical state. For the ideal differential stage (it is this case that is considered now) the symmetrical state means, in addition, the equality of constant components of A and OUT node potentials. The in-phase signal on node B, caused by Mp3 transistor noise, varies A and OUT node potentials IN THE IN-PHASE MANNER as well, with the root-mean-square value $V_{n(Mp3)(rms)} = \frac{I_{n(Mp3)(rms)}}{g_{Mn1}}$.

Let us compare this value with the root-mean-square noise value from any of the other four transistors of the differential stage, for example, from Mp1. It was shown above that (in the low-frequency area) the total noise current $I_{n(Mp1)(rms)}$ flows in the differential stage output circuit. The corresponding root-mean-square voltage $V_{n(Mp1)(rms)}$ in OUT node: , where $R_{OUT}$ is the output resistance in OUT node. The $R_{OUT}$ value is defined by parallel connection of drain - source resistances of Mp2 and Mn2 transistors, so it exceeds the $\frac{1}{g_{Mn1}}$ value by the number of times equal in the order of magnitude to the intrinsic transistor gain, i.e. by a few tens of times. As SQUARED noise currents and voltages are added in the output circuit, it is necessary that $R^2_{OUT}$ and $\left(\frac{1}{g_{Mn1}}\right)^2$ values should be compared which differ by hundreds of times! ***It is obvious that the Mp3 transistor noise can be really neglected.***

Thus, we shall consider that the squared spectral density of the noise current in the output circuit is equal to the arithmetical sum of squared spectral densities of noise currents of FOUR transistors (the same is true for the squared total noise current):

$$I^2_{n(OUT)(rms)} = I^2_{n(Mp1)(rms)} + I^2_{n(Mp2)(rms)} + I^2_{n(Mn1)(rms)} + I^2_{n(Mn2)(rms)} \tag{4.32}$$

We shall use the known ratio: for the squared noise current $I^2_{n(i)(rms)}$ of the i-th transistor, where $g^2_{mi}$ is the squared transconductance of the i-th transistor, and $V^2_{n(i)(IN)(rms)}$ is the squared noise voltage reduced to the input, which represents the noise behavior of the i-th transistor. The latter is the sum of two squared noise voltages referred to the transistor input:

first, it is the resistive channel noise $V^2_{ni(CH)(rms)} = \frac{4kT}{\frac{3}{2}g_{mi}}$ (4.33)

and, second, flicker noise, or $\frac{1}{f}$ of the noise $V^2_{ni\left(\frac{1}{f}\right)(rms)} \approx \frac{K_i}{(WL)_i C_0 f}$ . (4.34)

Here $W$ and $L$ are effective channel width and length of the i-th transistor, $C_0$ is the specific capacitance of the gate dielectric, $f$ is a frequency, and $K_i$ is a constant dependent on the transistor type and, especially, on the technological process. (We shall remind that (4.33) and (4.34) are spectral densities of noises of different nature, and the squared voltage of the ***total*** noise within the frequency range is equal to the spectral density integral in this range).

The squared spectral density of the noise current is equal to (in view of the differential stage symmetry, i.e. identity of all transistor characteristics in pairs Mp1, Mp2, and Mn1, Mn2, there is no further name identification of transistor parameters):

$$I^2_{n(OUT)(rms)} = 2g^2_{m(PMOS)}\left(\frac{2}{3}\times\frac{4kT}{g_{m(PMOS)}}+\frac{K}{(WL)C_0 f}\right)_{PMOS} + 2g^2_{m(NMOS)}\left(\frac{2}{3}\times\frac{4kT}{g_{m(NMOS)}}+\frac{K}{(WL)C_0 f}\right)_{NMOS}$$

(4.35)

Then, the squared spectral density of the noise current, referred to the input, is equal to:

$$V^2_{n(IN)(rms)} = \frac{I^2_{n(OUT)(rms)}}{g^2_{m(PMOS)}} = 2\left(\frac{2}{3}\times\frac{4kT}{g_{m(PMOS)}}+\frac{K}{(WL)C_0 f}\right)_{PMOS} + 2\frac{g^2_{m(NMOS)}}{g^2_{m(PMOS)}}\left(\frac{2}{3}\times\frac{4kT}{g_{m(NMOS)}}+\frac{K}{(WL)C_0 f}\right)_{NMOS}$$

(4.36)

It is convenient to represent the transconductance $g_{mi}$ in the form of: $g_{mi} = \sqrt{C_0 \mu_i \frac{W}{L} I_0}$

(4.37)

The fact that half of operating current $I_0$ flows in each of the transistors is taken into account here.