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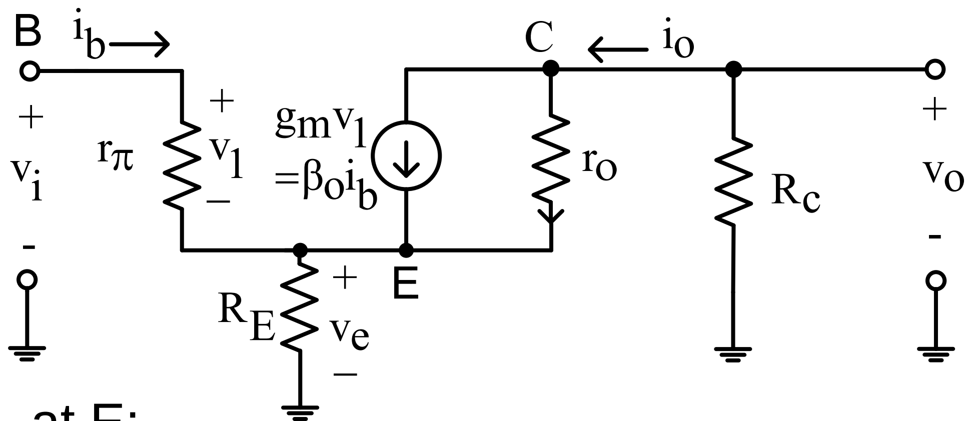
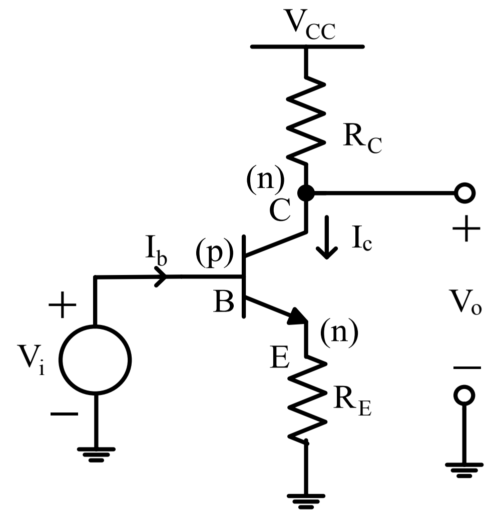
EEE 241
ANALOG ELECTRONICS 1
Lecture 10&11

DR NORLAILI MOHD NOH

CE with Emitter Degeneration

To determine R_i :





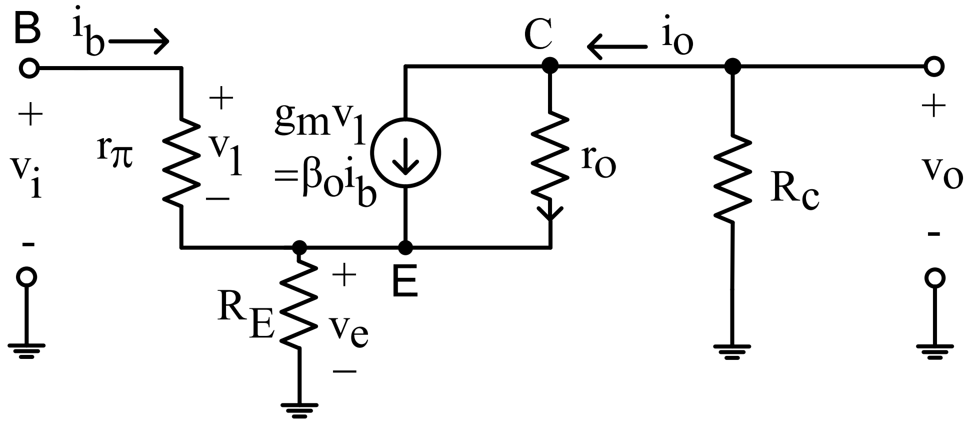
KCL at E:

$$i_b + \beta_o i_b + \frac{v_o - v_e}{r_o} = \frac{v_e}{R_E}$$

$$(1+\beta_o) i_b = \frac{v_e}{R_E} - \frac{-i_o R_C - v_e}{r_o} = \frac{v_e}{R_E} + \frac{i_o R_C + v_e}{r_o}$$

$$(1+\beta_o) i_b = \frac{v_e}{R_E} + \frac{v_e}{r_o} + \frac{i_o R_C}{r_o}$$

$$i_o = \left[(1+\beta_o) i_b - \left(\frac{1}{R_E} + \frac{1}{r_o} \right) v_e \right] \frac{r_o}{R_C}$$



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KCL at C:

$$i_o = \beta_o i_b + \frac{v_o - v_e}{r_o} = \beta_o i_b + \frac{-i_o R_C - v_e}{r_o}$$

Since KCL at E gives $i_o = \left[(1+\beta_o)i_b - \left(\frac{1}{R_E}+\frac{1}{r_o}\right)v_e\right]\frac{r_o}{R_C}$ then,

$$\left[(1+\beta_o)i_b - \left(\frac{1}{R_E}+\frac{1}{r_o}\right)v_e\right]\frac{r_o}{R_C} = \beta_o i_b + \frac{-\left[(1+\beta_o)i_b - \left(\frac{1}{R_E}+\frac{1}{r_o}\right)v_e\right]\frac{r_o}{R_C}R_C - v_e}{r_o}$$

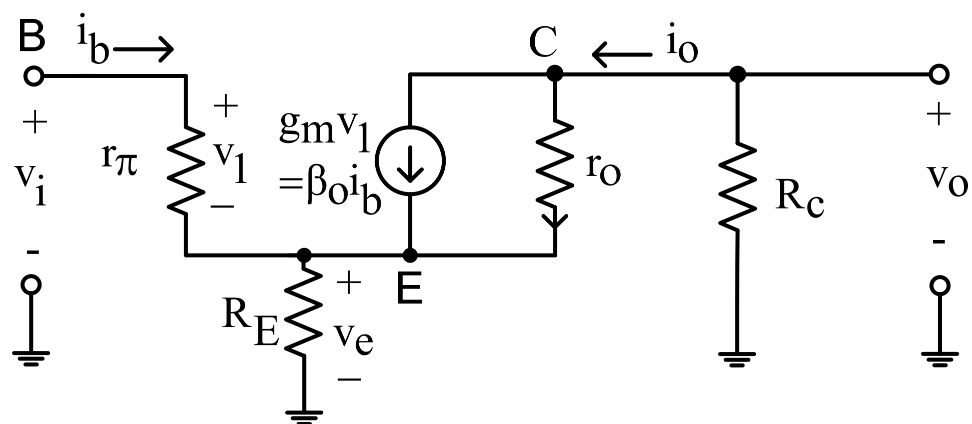
$$\left[(1+\beta_o)\frac{r_o}{R_C} - \beta_o + (1+\beta_o)\frac{r_o}{r_o R_C}R_C\right]i_b = \left\{\left(\frac{1}{R_E}+\frac{1}{r_o}\right)\frac{r_o}{R_C} + \left(\frac{1}{R_E}+\frac{1}{r_o}\right)\frac{r_o}{r_o R_C}R_C - \frac{1}{r_o}\right\}v_e$$

$$\left[(1+\beta_o)\frac{r_o}{R_C} - \beta_o + (1+\beta_o)\right]i_b = \left\{\left(\frac{1}{R_E}+\frac{1}{r_o}\right)\frac{r_o}{R_C} + \left(\frac{1}{R_E}+\frac{1}{r_o}\right) - \frac{1}{r_o}\right\}v_e$$

$$\left[(1+\beta_o)\frac{r_o}{R_C}-\beta_o+(1+\beta_o)\right]i_b=\left\{\left(\frac{1}{R_E}+\frac{1}{r_o}\right)\frac{r_o}{R_C}+\left(\frac{1}{R_E}+\frac{1}{r_o}\right)-\frac{1}{r_o}\right\}v_e$$

$$v_e=\left[\frac{(1+\beta_o)\frac{r_o}{R_C}+1}{\frac{r_o}{R_E R_C}+\frac{1}{R_C}+\frac{1}{R_E}}\right]i_b$$

KVL of the input loop:
$-v_i + v_1 + v_e = 0$.
Thus, $-v_i + i_b r_\pi + v_e = 0$



$$v_i=r_\pi i_b+\left[\frac{(1+\beta_o)\frac{r_o}{R_C}+1}{\frac{r_o}{R_E R_C}+\frac{1}{R_C}+\frac{1}{R_E}}\right]i_b$$

$$R_i=\frac{v_i}{i_b}=r_\pi+\left[\frac{(1+\beta_o)\frac{r_o}{R_C}+1}{\frac{r_o}{R_E R_C}+\frac{1}{R_C}+\frac{1}{R_E}}\right]\leftarrow \text{enough}$$

$$R_i = r_\pi + \left[\frac{(1+\beta_o)\frac{r_o}{R_C}+1}{\frac{r_o}{R_E R_C}+\frac{1}{R_C}+\frac{1}{R_E}}\right]$$

$$= r_\pi + \frac{R_E R_C\left[1+(1+\beta_o)\frac{r_o}{R_C}\right]}{r_o+R_C+R_E}$$

$$= r_\pi + \frac{(1+\beta_o)R_E\left[\frac{R_C}{(1+\beta_o)}+r_o\right]}{r_o+R_C+R_E}$$

$$R_i = r_\pi + \frac{(1+\beta_o)R_E\left[\frac{R_C}{(1+\beta_o)} + r_o\right]}{r_o + R_C + R_E}$$

If $r_o >> R_C$ and $r_o >> R_E$ (in the case of $r_o \rightarrow \infty$), then

$$R_i = r_\pi + \frac{(1+\beta_o)R_E[r_o]}{r_o} = r_\pi + (1+\beta_o)R_E$$

For a finite r_o, $\frac{\frac{R_C}{(1+\beta_o)} + r_o}{r_o + R_C + R_E} < 1$.

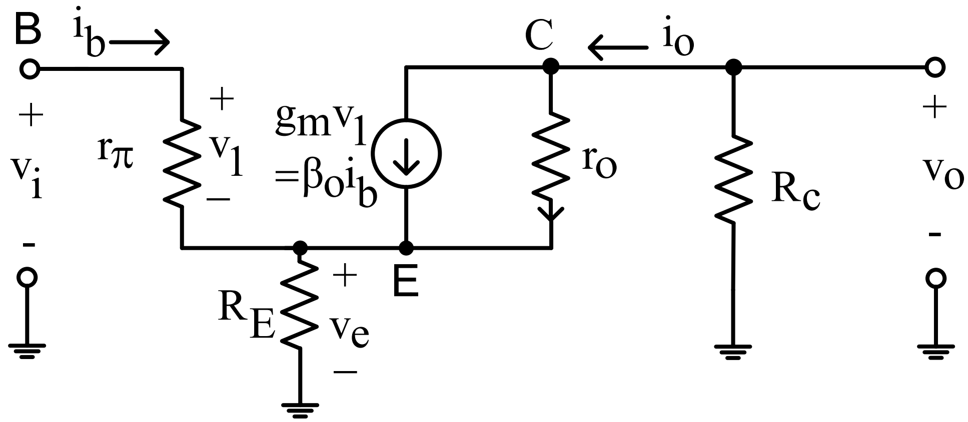
Hence, a finite r_o will reduce the R_i when compared to the R_i when $r_o \rightarrow \infty$.

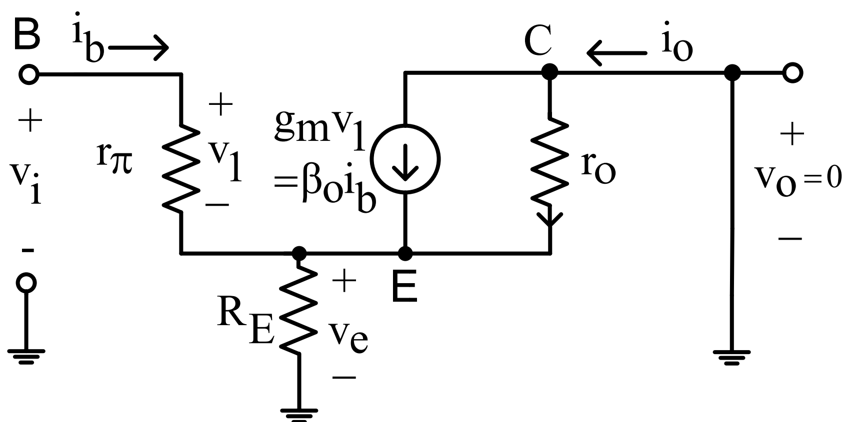
For a CE without degeneration, $R_i = \frac{v_i}{i_i} = r_\pi = \frac{\beta_o}{g_m}$.

Thus, a CE with degeneration will increase the R_i.

Normally, the R_i for a CE with emitter degeneration is taken as $r_\pi + (1+\beta_o)R_E$

To determine G_m :





Transconductance,

$$G_m = \left.\frac{i_o}{v_i}\right|_{v_o=0}$$

At node E,

$$i_b + \beta_o i_b = \frac{v_e}{R_E} + \frac{v_e}{r_o}$$

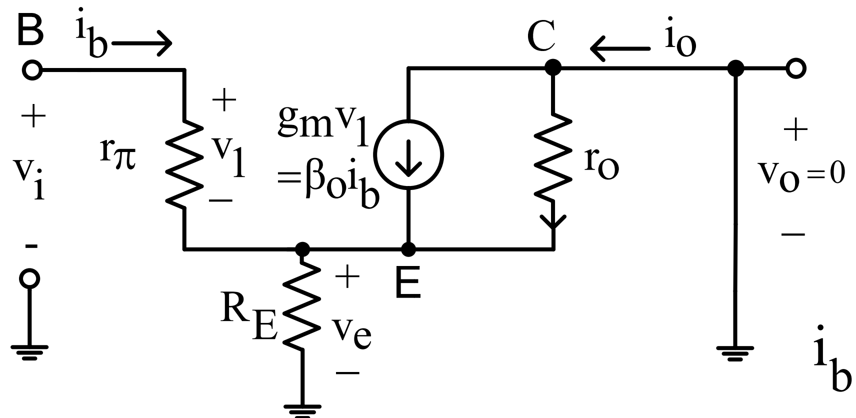
$$(1+\beta_o)\frac{v_i - v_e}{r_\pi} = \left(\frac{1}{R_E} + \frac{1}{r_o}\right) v_e$$

$$v_e = \frac{(1+\beta_o) v_i}{r_\pi\left[\frac{1}{R_E} + \frac{1}{r_o} + \frac{(1+\beta_o)}{r_\pi}\right]}$$

At node C,

$$i_o + \frac{\dfrac{(1+\beta_o)v_i}{r_\pi\left[\dfrac{1}{R_E}+\dfrac{1}{r_o}+\dfrac{(1+\beta_o)}{r_\pi}\right]}}{r_o} = \beta_o i_b$$

$$v_e = \frac{(1+\beta_o)v_i}{r_\pi\left[\dfrac{1}{R_E}+\dfrac{1}{r_o}+\dfrac{(1+\beta_o)}{r_\pi}\right]}$$



$$i_b = \frac{v_i - v_e}{r_\pi}$$

$$i_o + \frac{\dfrac{(1+\beta_o)v_i}{r_\pi\left[\dfrac{1}{R_E}+\dfrac{1}{r_o}+\dfrac{(1+\beta_o)}{r_\pi}\right]}}{r_o}$$

$$= \beta_o \frac{\left\{v_i - \dfrac{(1+\beta_o)v_i}{r_\pi\left[\dfrac{1}{R_E}+\dfrac{1}{r_o}+\dfrac{(1+\beta_o)}{r_\pi}\right]}\right\}}{r_\pi} = \beta_o \frac{\left\{1 - \dfrac{(1+\beta_o)}{r_\pi\left[\dfrac{1}{R_E}+\dfrac{1}{r_o}+\dfrac{(1+\beta_o)}{r_\pi}\right]}\right\}v_i}{r_\pi}$$

$$i_o + \frac{\dfrac{(1+\beta_o)v_i}{r_\pi\left[\dfrac{1}{R_E}+\dfrac{1}{r_o}+\dfrac{(1+\beta_o)}{r_\pi}\right]}}{r_o} = \beta_o \frac{\left\{1-\dfrac{(1+\beta_o)}{r_\pi\left[\dfrac{1}{R_E}+\dfrac{1}{r_o}+\dfrac{(1+\beta_o)}{r_\pi}\right]}\right\}v_i}{r_\pi}$$

$$i_o = \beta_o \frac{\left\{1-\dfrac{(1+\beta_o)}{r_\pi\left[\dfrac{1}{R_E}+\dfrac{1}{r_o}+\dfrac{(1+\beta_o)}{r_\pi}\right]}\right\}v_i}{r_\pi} - \frac{\dfrac{(1+\beta_o)v_i}{r_\pi\left[\dfrac{1}{R_E}+\dfrac{1}{r_o}+\dfrac{(1+\beta_o)}{r_\pi}\right]}}{r_o}$$

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$$i_o = \left\{ \beta_o \frac{\left\{ 1 - \dfrac{(1+\beta_o)}{r_\pi \left[\dfrac{1}{R_E} + \dfrac{1}{r_o} + \dfrac{(1+\beta_o)}{r_\pi} \right]} \right\}}{r_\pi} - \frac{\dfrac{(1+\beta_o)}{r_\pi \left[\dfrac{1}{R_E} + \dfrac{1}{r_o} + \dfrac{(1+\beta_o)}{r_\pi} \right]}}{r_o} \right\} v_i$$

$$G_m = \frac{i_o}{v_i} = \beta_o \frac{\left\{ 1 - \dfrac{(1+\beta_o)}{r_\pi \left[\dfrac{1}{R_E} + \dfrac{1}{r_o} + \dfrac{(1+\beta_o)}{r_\pi} \right]} \right\}}{r_\pi} - \frac{\dfrac{(1+\beta_o)}{r_\pi \left[\dfrac{1}{R_E} + \dfrac{1}{r_o} + \dfrac{(1+\beta_o)}{r_\pi} \right]}}{r_o} \rightarrow \text{enough}$$

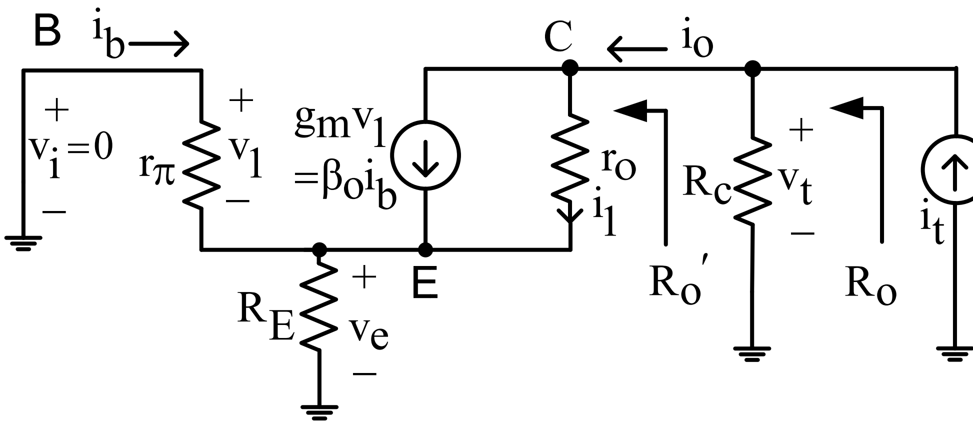
$$G_m = \frac{g_m\left[1-\frac{R_E}{r_o\beta_o}\right]}{1+g_mR_E\left(1+\frac{1}{g_mr_o}+\frac{1}{\beta_o}\right)}$$

Practically, $\beta_o >> 1$, $r_o >> R_E$ and $g_mr_o >> 1$

$$G_m = \frac{g_m}{1+g_mR_E}$$

This expression is often used to calculate the transconductance of the CE with emitter degeneration amplifier. Practically, $G_{m\_CE\_emitter\ degen} < G_{m\_CE}$ as $G_{m\_CE} = g_m$.

To determine R_o :



$$R_o = \left.\frac{v_t}{i_t}\right|_{v_i=0} = R_c // R_o'$$

$$v_1 = -v_e$$

$$v_1 = -i_o\left(r_\pi // R_E\right)$$

$$i_1 = i_o - g_m v_1 = i_o + g_m\left[i_o\left(r_\pi // R_E\right)\right]$$

$$-v_t + i_1 r_o + v_e = 0, \quad \therefore \quad -v_t + i_1 r_o - v_1 = 0$$

$$-v_t + \left[1 + g_m\left(r_\pi // R_E\right)\right] i_o r_o + i_o\left(r_\pi // R_E\right) = 0$$

$$R_o' = \frac{v_t}{i_o} = \left[1 + g_m\left(r_\pi // R_E\right)\right] r_o + \left(r_\pi // R_E\right)$$

$$\therefore \quad R_o = R_c // R_o' = R_c // \left\{\left[1 + g_m\left(r_\pi // R_E\right)\right] r_o + \left(r_\pi // R_E\right)\right\} \rightarrow \text{enough}$$

$$R_o' = \frac{v_t}{i_t} = \left[1 + g_m\left(r_\pi // R_E\right)\right] r_o + \left(r_\pi // R_E\right)$$

$$\left(r_\pi // R_E\right) << \left[1 + g_m\left(r_\pi // R_E\right)\right] r_o$$

$$R_o' = \left[1 + g_m\left(r_\pi // R_E\right)\right] r_o = \left[1 + g_m \frac{r_\pi R_E}{r_\pi + R_E}\right] r_o$$

$$R_o' = \left[1 + \frac{g_m R_E}{\frac{1}{r_\pi}\left(r_\pi + R_E\right)}\right] r_o = \left[1 + \frac{g_m R_E}{1 + \frac{R_E}{r_\pi}}\right] r_o = \left[1 + \frac{g_m R_E}{1 + \frac{g_m R_E}{\beta_o}}\right] r_o$$

$$\text{If } g_m R_E << \beta_o, R_o' = \left[1 + g_m R_E\right] r_o$$

$$R_o = R_c // \left[1 + g_m R_E\right] r_o$$

$$\text{If } g_m R_E >> \beta_o, R_o' = \left[1 + \frac{g_m R_E}{\frac{g_m R_E}{\beta_o}}\right] r_o = \left[1 + \beta_o\right] r_o$$

$$R_o' = \left[1 + \frac{g_m R_E}{\frac{g_m R_E}{\beta_o}}\right] r_o = \left[1 + \beta_o\right] r_o.$$

$$R_{o\_CE} = R_c // r_o. \quad R_{o\_CE\_degen} > R_{o\_CE}.$$

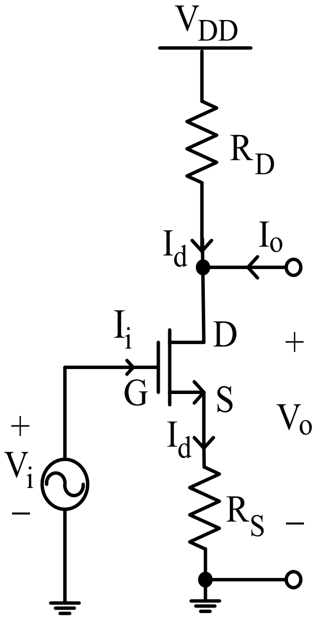
CS with Source Degeneration

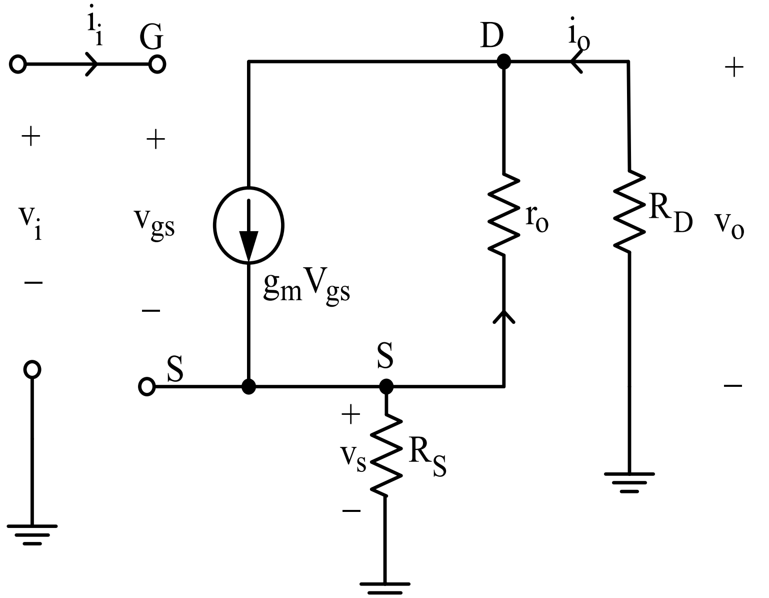
CS with S degeneration is not as typically implemented as the CE with E degeneration. The reasons are:

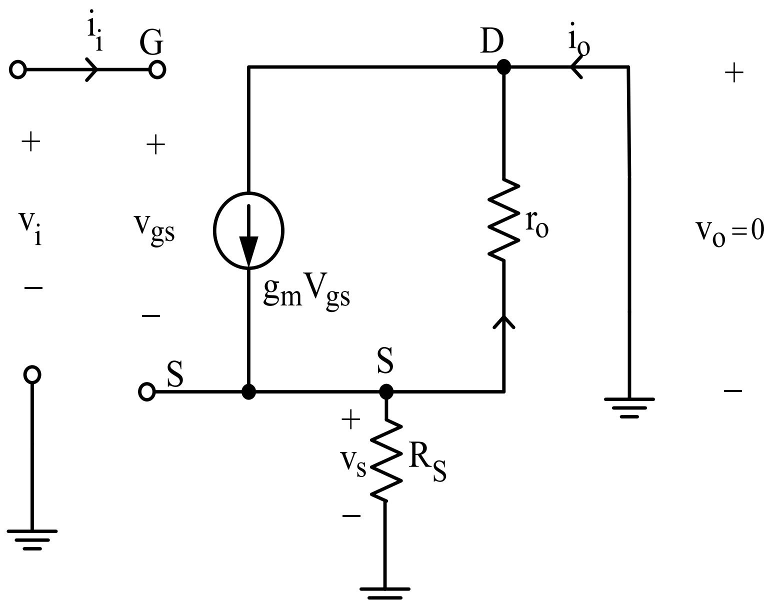
1. The transconductance of the MOS is far less than the transconductance of the BJT. When implementing S degeneration, the transconductance of the MOS will be reduced. This condition is undesired.
2. With E degeneration, R_i in CE is increased. However, $R_i \rightarrow \infty$ in the case of CS even without degeneration. Hence, the need for the degeneration circuit is not crucial.

   CS with S degeneration is sometimes implemented to increase the R_o of the MOS.

CS with Source Degeneration







$$R_i = \frac{v_i}{i_i} = \infty$$

$$G_m = \left.\frac{i_o}{v_i}\right|_{v_o = 0}$$

KCL at S when $v_o = 0$:

$$g_m v_{gs} = \frac{v_s}{r_o} + \frac{v_s}{R_s}$$

$$g_m (v_i - v_s) = \frac{v_s}{r_o} + \frac{v_s}{R_s}$$

$$g_m v_i = \frac{v_s}{r_o} + \frac{v_s}{R_s} + g_m v_s$$

$$g_m v_i = v_s \left(g_m + \frac{1}{r_o} + \frac{1}{R_s} \right)$$

$$g_m v_i = v_s\left(g_m + \frac{1}{r_o} + \frac{1}{R_s}\right)$$

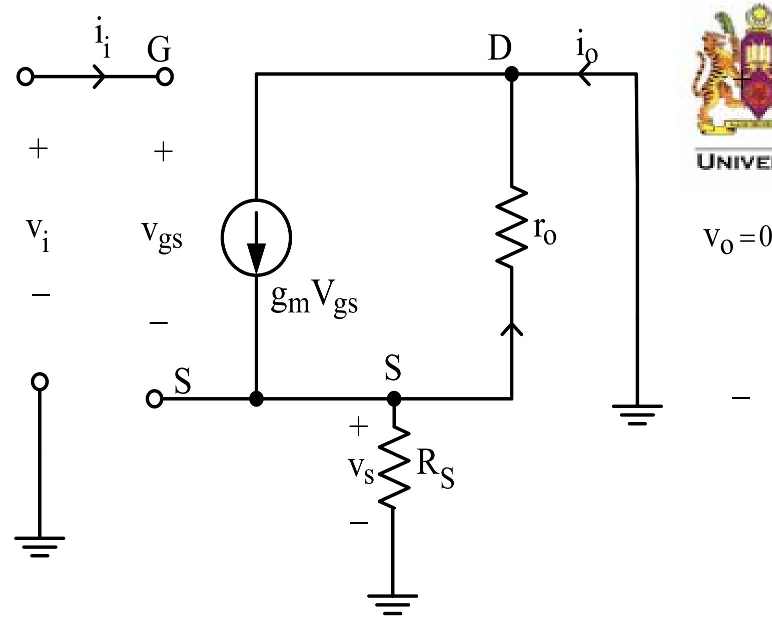
$$v_s = \frac{g_m v_i}{\left(g_m + \frac{1}{r_o} + \frac{1}{R_s}\right)}$$

$$G_m = \left.\frac{i_o}{v_i}\right|_{v_o=0}$$

KCL at D when $v_o = 0$:

$$g_m v_{gs} = \frac{v_s}{r_o} + i_o$$

$$v_{gs} = v_i - v_s$$



Therefore,

$$g_m\left\{v_i - \left[\frac{g_m v_i}{\left(g_m + \frac{1}{r_o} + \frac{1}{R_s}\right)}\right]\right\} = \frac{\left[\frac{g_m v_i}{\left(g_m + \frac{1}{r_o} + \frac{1}{R_s}\right)}\right]}{r_o} + i_o$$

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$$g_m\left\{v_i-\left[\frac{g_m v_i}{\left(g_m+\frac{1}{r_o}+\frac{1}{R_s}\right)}\right]\right\}=\frac{\left[\frac{g_m v_i}{\left(g_m+\frac{1}{r_o}+\frac{1}{R_s}\right)}\right]}{r_o}+i_o$$

$$g_m\left\{v_i-\left[\frac{g_m v_i}{\left(g_m+\frac{1}{r_o}+\frac{1}{R_s}\right)}\right]\right\}-\frac{\left[\frac{g_m v_i}{\left(g_m+\frac{1}{r_o}+\frac{1}{R_s}\right)}\right]}{r_o}=i_o$$

$$i_o=v_i\left\{g_m\left\{1-\left[\frac{g_m}{\left(g_m+\frac{1}{r_o}+\frac{1}{R_s}\right)}\right]\right\}-\frac{g_m}{r_o\left(g_m+\frac{1}{r_o}+\frac{1}{R_s}\right)}\right\}$$

$$i_o = v_i\left\{g_m\left\{1-\left[\frac{g_m}{\left(g_m+\frac{1}{r_o}+\frac{1}{R_s}\right)}\right]\right\}-\frac{g_m}{r_o\left(g_m+\frac{1}{r_o}+\frac{1}{R_s}\right)}\right\}$$

$$G_m = \frac{i_o}{v_i} = \left\{\begin{array}{l} g_m\left\{\dfrac{g_m+\frac{1}{r_o}+\frac{1}{R_s}}{g_m+\frac{1}{r_o}+\frac{1}{R_s}}-\left[\dfrac{g_m}{\left(g_m+\frac{1}{r_o}+\frac{1}{R_s}\right)}\right]\right\} \\ -\dfrac{g_m}{r_o\left(g_m+\frac{1}{r_o}+\frac{1}{R_s}\right)} \end{array}\right\}$$

$$G_m = \left\{g_m\left\{\frac{g_m+\frac{1}{r_o}+\frac{1}{R_s}-g_m-\frac{1}{r_o}}{g_m+\frac{1}{r_o}+\frac{1}{R_s}}\right\}\right\}$$

$$G_m = \left\{g_m\left[\frac{\frac{1}{R_s}}{g_m+\frac{1}{r_o}+\frac{1}{R_s}}\right]\right\} = \frac{g_m}{R_s g_m+\frac{R_s}{r_o}+1}$$

$$G_m = \frac{g_m}{R_s g_m + \frac{R_s}{r_o} + 1}$$

If $r_o >> R_s$, $G_m = \frac{g_m}{R_s g_m + 1}$

If R_s is large, $G_m = \frac{g_m}{R_s g_m} = \frac{1}{R_s}$

Without S degeneration, $G_m = g_m$. This shows that with source degeneration, G_m has reduced. $G_{m\_CS\_degen} < G_{m\_CS}$.

$$G_{m\_CS\_degen} = \frac{1}{R_s} \text{ and } G_{m\_CE\_degen} = \frac{g_m\left[1 - \frac{R_E}{r_o\beta_o}\right]}{1 + g_m R_E\left(1 + \frac{1}{g_m r_o} + \frac{1}{\beta_o}\right)}$$

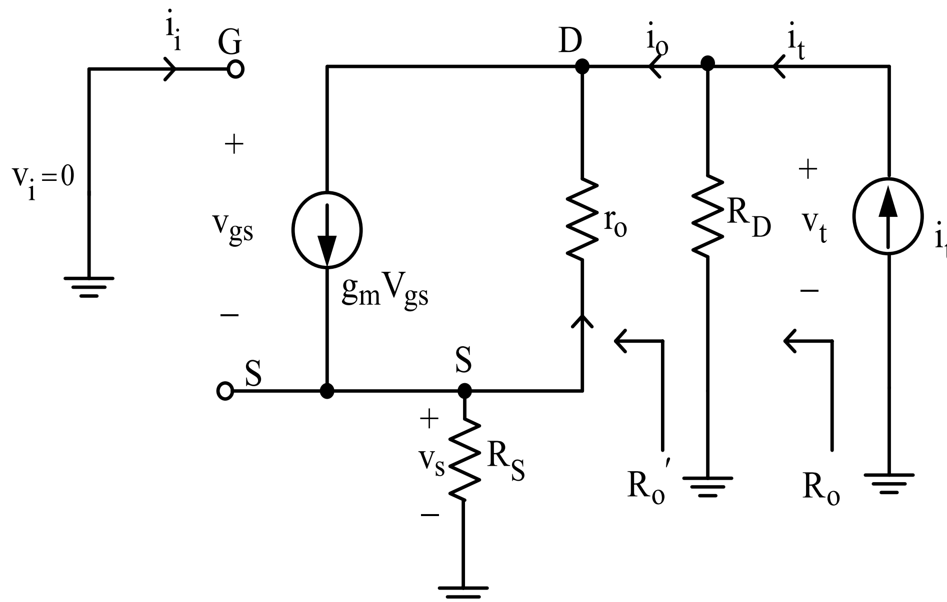
If $r_o >> R_E$, and $R_E >> 1$, then

$$G_{m\_CE\_degen} = \frac{g_m[1]}{g_m R_E\left(1 + \frac{1}{\beta_o}\right)} = \frac{1}{R_E\left(1 + \frac{1}{\beta_o}\right)} = \frac{\beta_o}{R_E(1+\beta_o)}$$

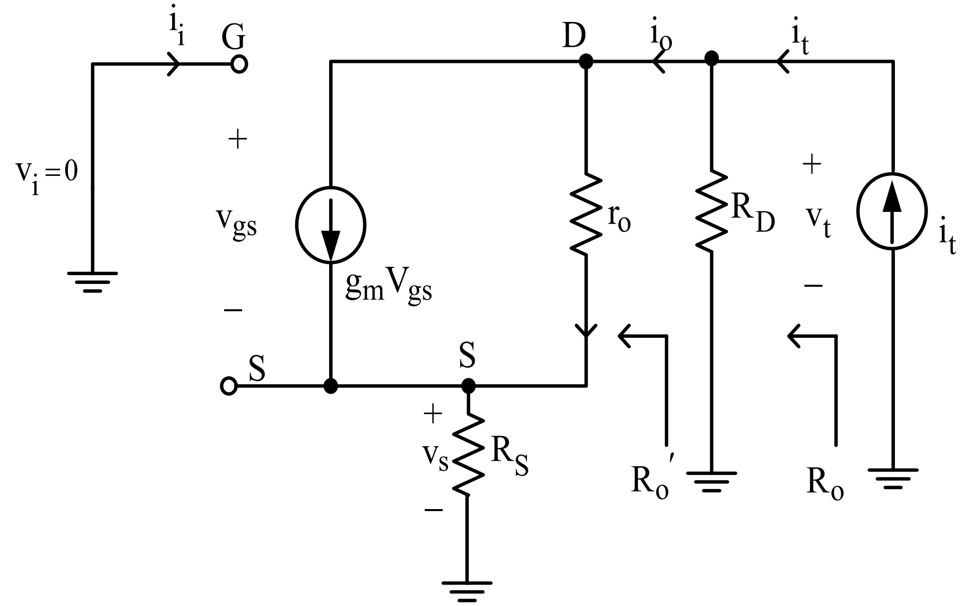
$$G_{m\_CS\_degen} = \frac{1}{R_S}$$

$$G_{m\_CE\_degen} = \frac{1}{R_E\left(1+\frac{1}{\beta_o}\right)} = \frac{\beta_o}{R_E(1+\beta_o)} \approx \frac{1}{R_E} \quad \text{if } \beta_o >> 1$$

$$R_o = \left.\frac{v_t}{i_t}\right|_{v_i=0}$$



$$R_o = \left.\frac{v_t}{i_t}\right|_{v_i=0}, R_o' = \left.\frac{v_t}{i_o}\right|_{v_i=0}$$



$$v_{gs} = -v_s$$

KCL at node D:

$$i_o = -g_m v_s + \frac{v_t - v_s}{r_o}$$

KCL at node S:

$$g_m v_{gs} + \frac{v_t - v_s}{r_o} = \frac{v_s}{R_s}$$

$$-g_m v_s + \frac{v_t - v_s}{r_o} = \frac{v_s}{R_s}$$

$$\frac{v_s}{R_s} + g_m v_s + \frac{v_s}{r_o} = \frac{v_t}{r_o}$$

$$v_s = \frac{v_t}{\left(\frac{1}{R_s} + g_m + \frac{1}{r_o}\right) r_o}$$

$$i_o = -g_m v_s - \frac{v_s}{r_o} + \frac{v_t}{r_o}$$

$$= \left[\frac{\left(-g_m - \frac{1}{r_o}\right)}{\left(\frac{1}{R_s} + g_m + \frac{1}{r_o}\right) r_o} + \frac{1}{r_o}\right] v_t$$

$$i_o = -g_m v_s - \frac{v_s}{r_o} + \frac{v_t}{r_o}$$

$$= \left[\frac{\left(-g_m - \frac{1}{r_o} \right)}{\left(\frac{1}{R_s} + g_m + \frac{1}{r_o} \right) r_o} + \frac{1}{r_o} \right] v_t$$

$$R_o' = \frac{v_t}{i_o} = \frac{1}{\left[\frac{\left(-g_m - \frac{1}{r_o} \right)}{\left(\frac{1}{R_s} + g_m + \frac{1}{r_o} \right) r_o} + \frac{1}{r_o} \right]} = \frac{1}{\left[\frac{\left(-g_m - \frac{1}{r_o} \right) + \left(\frac{1}{R_s} + g_m + \frac{1}{r_o} \right)}{\left(\frac{1}{R_s} + g_m + \frac{1}{r_o} \right) r_o} \right]}$$

$$R_o' = \frac{1}{\left[\frac{\frac{1}{R_s}}{\left(\frac{1}{R_s} + g_m + \frac{1}{r_o} \right) r_o} \right]} = \frac{1}{\left[\frac{1}{R_s \left(\frac{1}{R_s} + g_m + \frac{1}{r_o} \right) r_o} \right]} = r_o + r_o R_s g_m + R_s$$

$$R_o' = r_o (1 + R_s g_m) + R_s$$

$$R_o' = r_o(1 + R_s g_m) + R_s$$

If $R_s \uparrow$, R_o' will also $\uparrow$.

For the CE with E degeneration:

$$R_o' = [1 + \beta_o] r_o.$$

Even though $R_E \to \infty$, R_o' will be limited to its maximum value of $r_o(1 + \beta_o)$.