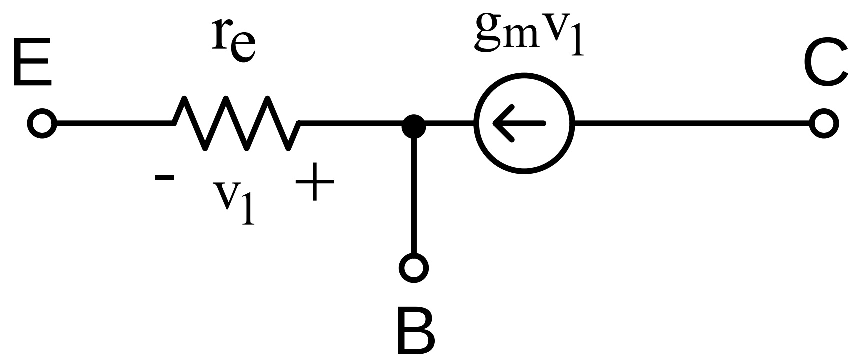
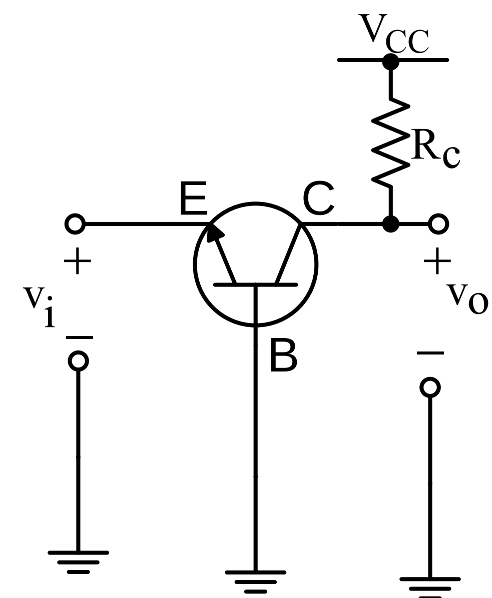
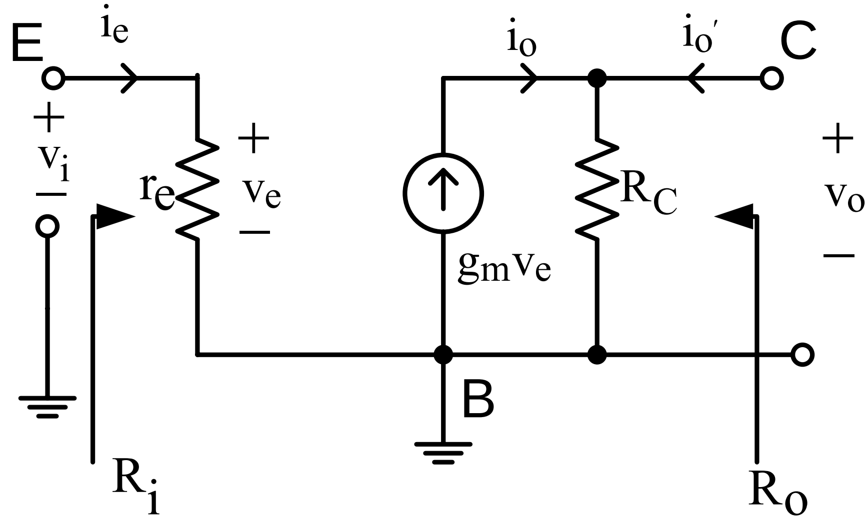
T-model:



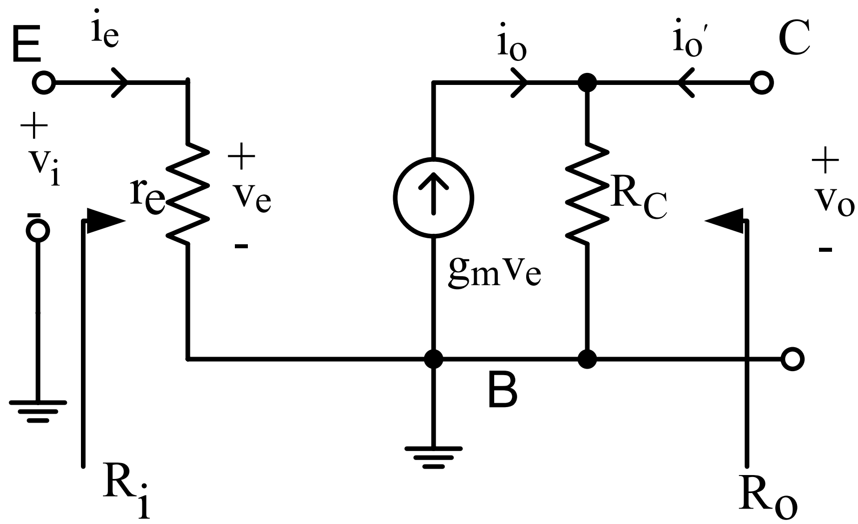
E
$r_e$
$g_m v_1$
C
-
$v_1$
+
B

$V_{CC}$
$R_c$
E
C
+
$v_i$
–
B
+
$v_o$
–

E
$i_e$
$i_o$
$i_o'$
C
+
$v_i$
–
$r_e$
+
$v_e$
–
$R_C$
+
$v_o$
–
$g_m v_e$
B
$R_i$
$R_o$




The s/c transconductance:

$$G_m = \left.\frac{i_o}{v_i}\right|_{v_o=0} = \frac{g_m v_e}{v_e} = g_m$$

The input resistance:

$$R_i = \frac{v_i}{i_e} = \frac{v_e}{i_e} = r_e$$

The output resistance:

$$R_o = \left.\frac{v_o}{i_o'}\right|_{v_i=0} = R_C$$

o/c or unloaded voltage gain,

$$a_v = \left.\frac{v_o}{v_i}\right|_{i_o'=0} = \frac{g_m v_e R_C}{v_e} = g_m R_C$$

$$a_i = \left.\frac{i_o}{i_i}\right|_{v_o=0} = \frac{g_m v_e}{v_e / r_e} = g_m r_e$$

$$a_i = \left.\frac{i_o}{i_i}\right|_{v_o=0} = \frac{g_m v_e}{v_e / r_e} = g_m r_e$$

Since, $r_e = \frac{\alpha_o}{g_m}$ , then $a_i = g_m \frac{\alpha_o}{g_m} \approx 1$

For the CE configuration: $R_i = r_\pi = \frac{\beta_o}{g_m}$

For the CB configuration: $R_i = r_e = \frac{\alpha_o}{g_m}$

$R_{i\_CE} > R_{i\_CB}$

$$R_{i\_CB} = \frac{\alpha_o}{\beta_o} R_{i\_CE} = \frac{1}{1+\beta_o} R_{i\_CE}$$

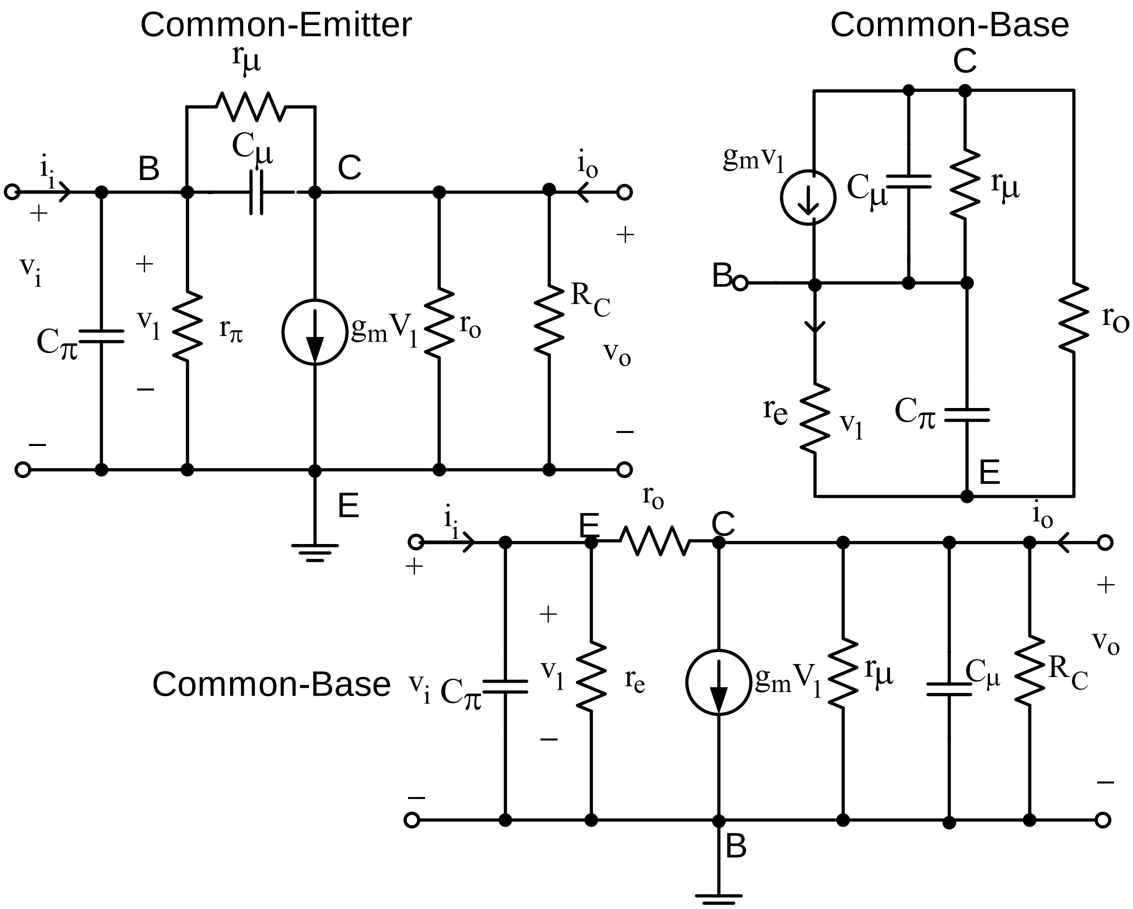
$a_{i\_CB} \approx \alpha_o$

$a_{i\_CE} \approx \beta_o$

$a_{i\_CB} < a_{i\_CE}$

$$a_{i\_CB} = \frac{\alpha_o}{\beta_o} a_{i\_CE} = \frac{1}{1+\beta_o} a_{i\_CE}$$

In terms of i/p resistance and current gain, the CE amplifier performs better than CB.

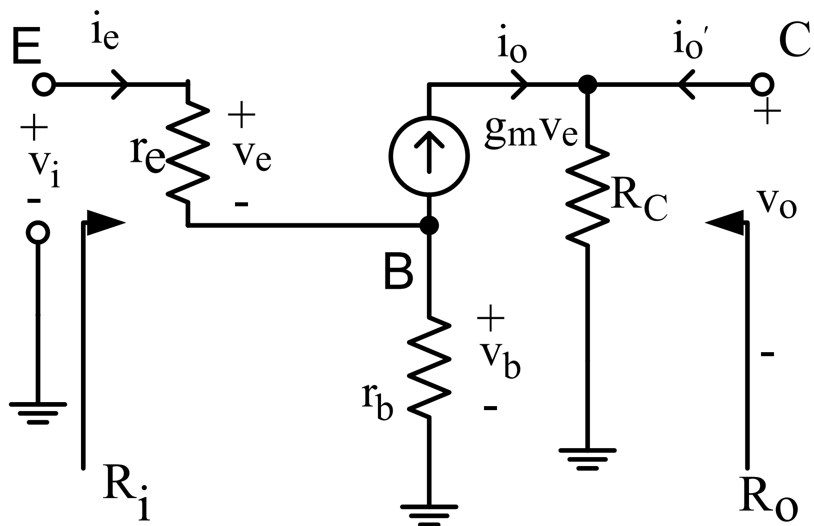



$C_\mu$ is between B-C. At high freqs., capacitive components are dominant.

For CE, $C_\mu$ is between i/p and o/p. Hence, at high-freqs., there will be a feedback from o/p to i/p.

For CB, i/p is at E and o/p is at C. Therefore, $C_\mu$ will not cause a feedback at high-freqs. CB circuits are used for high-freq. application.

Until now we have assumed that $r_b$ is negligible. In practice, $r_b$ has a significant effect on $G_m$ and $R_i$ when the CB stage is operated at sufficiently high current levels.




The s/c transconductance:

$$G_m = \left.\frac{i_o}{v_i}\right|_{v_o=0} = \frac{g_m v_e}{v_e + v_b}$$

KCL at B,

$i_e = i_{rb} + g_m v_e$

$$\frac{v_e}{r_e} = \frac{v_b}{r_b} + g_m v_e$$

$$v_b = \left(\frac{v_e}{r_e} - g_m v_e\right) r_b$$

$$G_m = \frac{g_m}{1 + \left(\frac{1}{r_e} - g_m\right) r_b}$$

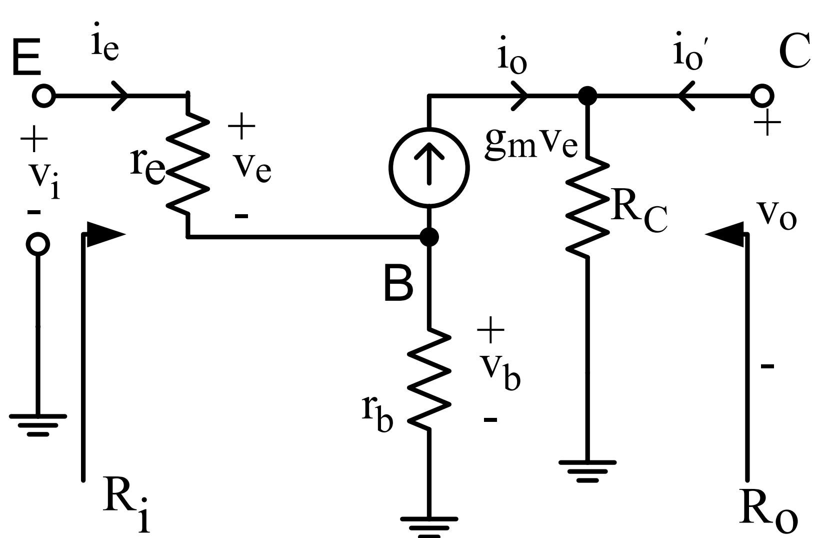
From Equation (3.29), $r_e = \dfrac{1}{g_m + \dfrac{1}{r_\pi}}$

$$\frac{1}{r_e} = g_m + \frac{1}{r_\pi}$$

$$G_m = \frac{g_m}{1 + \left(g_m + \dfrac{1}{r_\pi} - g_m\right) r_b}$$

$$= \frac{g_m}{1 + \dfrac{r_b}{r_\pi}}$$

Comparing this with the transconductance of the CB circuit if $r_b$ is neglected (i.e. $G_m = g_m$) shows that $G_m$ becomes lower when $r_b$ is included.




The input resistance:

$$R_i = \frac{v_i}{i_e} = \frac{v_b + v_e}{i_e} = \frac{\left(\dfrac{v_e}{r_e} - g_m v_e\right) r_b + v_e}{i_e} = \frac{\left(\dfrac{1}{r_e} - g_m\right) r_b + 1}{\dfrac{1}{r_e}}$$

$$R_i = \frac{v_i}{i_e} = \frac{v_b + v_e}{i_e} = \frac{\left(\frac{v_e}{r_e} - g_m v_e\right) r_b + v_e}{i_e} = \frac{\left(\frac{1}{r_e} - g_m\right) r_b + 1}{\frac{1}{r_e}}$$

$$R_i = \left[\left(\frac{1}{r_e} - g_m\right) r_b + 1\right] r_e$$

Since $\frac{1}{r_e} = g_m + \frac{1}{r_\pi}$ and $r_e = \frac{\alpha_o}{g_m}$, then

$$R_i = \left[\left(g_m + \frac{1}{r_\pi} - g_m\right) r_b + 1\right] \frac{\alpha_o}{g_m} = \left[\frac{r_b}{r_\pi} + 1\right] \frac{\alpha_o}{g_m}$$

Comparing this with the input resistance of the CB circuit if $r_b$ is neglected (i.e. $R_i = \frac{\alpha_o}{g_m}$) shows that $R_i$ becomes larger when $r_b$ is included.

At high current level, i.e. $I_C\uparrow$, $r_\pi \downarrow$ as $r_\pi = \frac{\beta_o}{g_m} = \frac{\beta_o V_T}{I_C}$ . So, if $r_\pi$ is small enough that it is comparable with $r_b$, then $r_b$ should be included in the analysis.

Example: If $r_b$ = 100 Ω, $\beta_o$ = 100 and $I_C$ = 26 mA, then $r_\pi = \frac{\beta_o}{g_m} = \frac{100 \times 26m}{26m} = 100\,\Omega$. Hence, $r_\pi$ = $r_b$ when $I_C$ is high.

In CB configuration, $R_o = R_C$

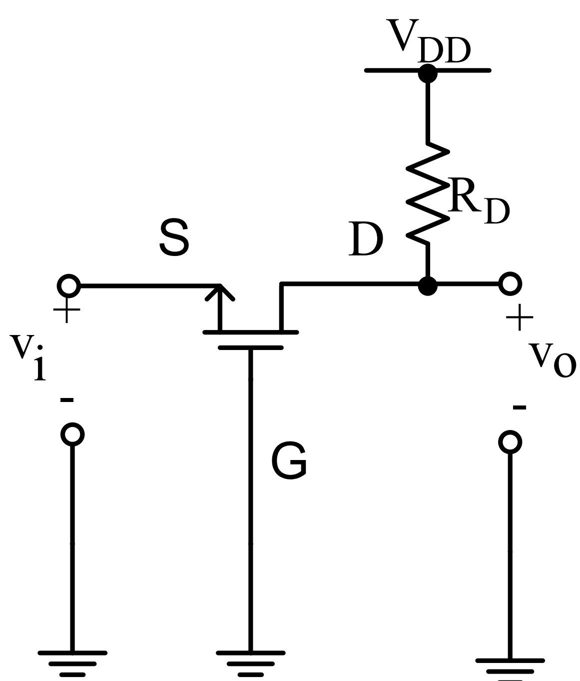
In CE configuration, $R_o = R_C \,//\, r_o$

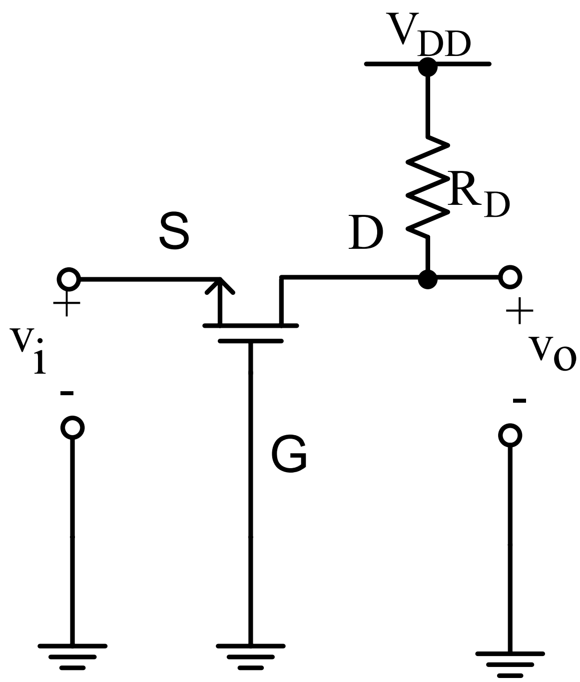
If $R_C \rightarrow \infty$, $R_{o\_CB} = \infty$ and $R_{o\_CE} = r_o$

Under this condition, $R_{o\_CB} > R_{o\_CE}$

Besides using the CB as high freq. amplifier, it can also be used as a current source whose current is nearly independent of the voltage across it (i.e. $i_o = g_m v_e$)

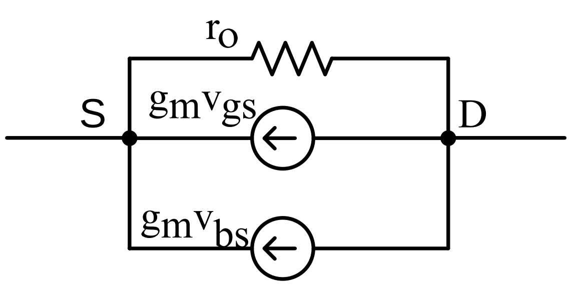
## 3.3.4 Common-gate (CG) configuration.

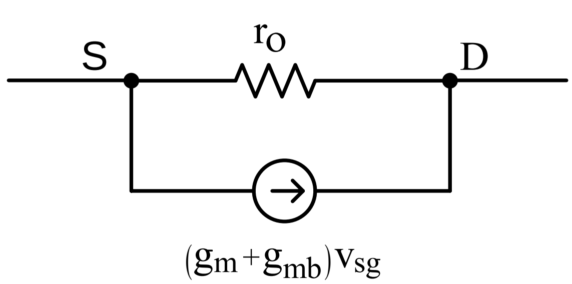




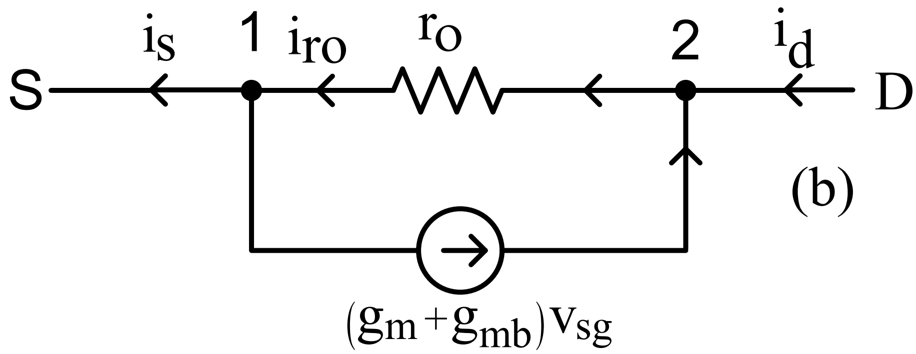

I/p signal is applied to the S. O/p is taken from the D. G is connected to the ac gnd.

The analysis of CG amplifiers can be simplified if the model is changed from a hybrid-π to a T-model.

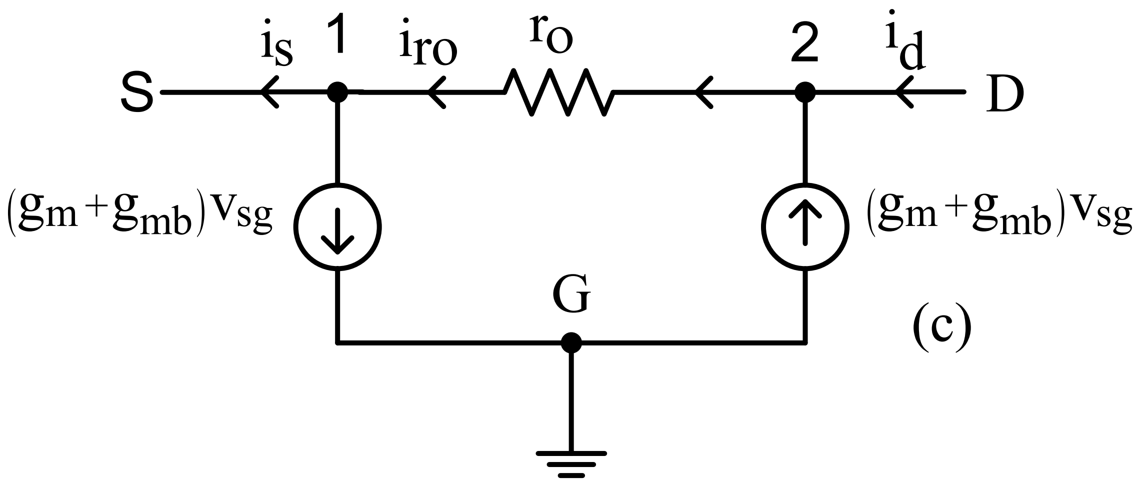



The body (B) is ac gnd, $v_{bs} = v_{gs}$ because G is also at gnd.




(b)




(c)

<u>Figure (b):</u>

Node 1: $i_{ro} = i_S + (g_m + g_{mb})v_{sg}$

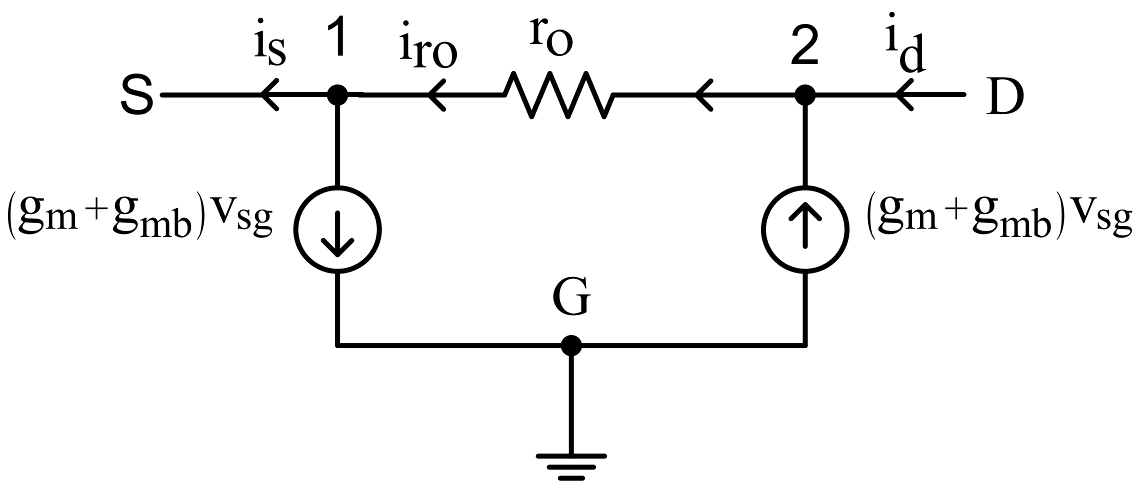
Node 2: $i_d + (g_m + g_{mb})v_{sg} = i_{ro}$

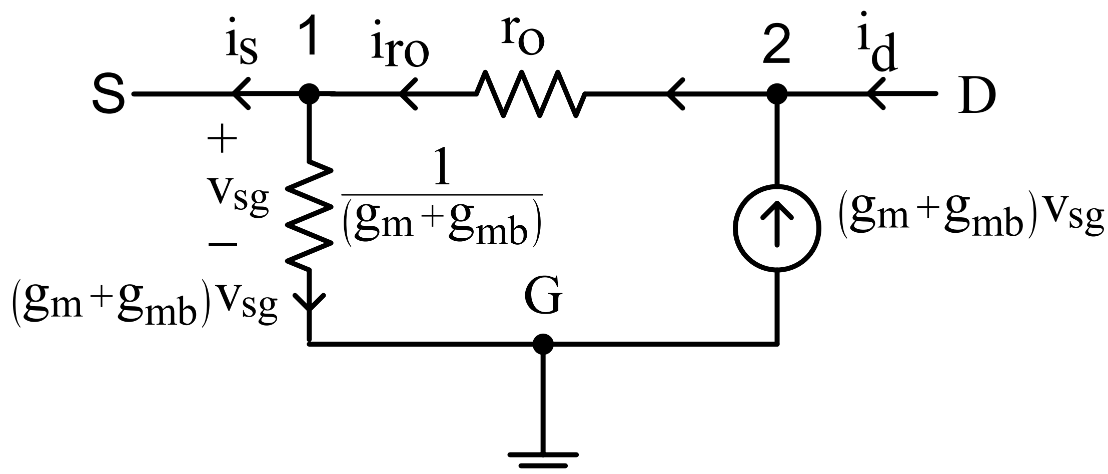
<u>Figure (c):</u>

Node 1: $i_{ro} = i_S + (g_m + g_{mb})v_{sg}$

Node 2: $i_d + (g_m + g_{mb})v_{sg} = i_{ro}$

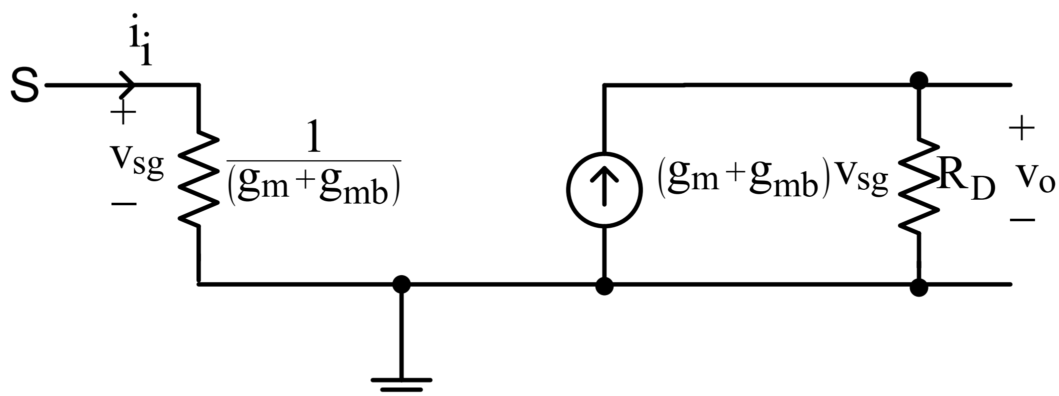
Equal currents are pushed into and pulled out of the G as the equations that describe the operation of the circuits are identical.

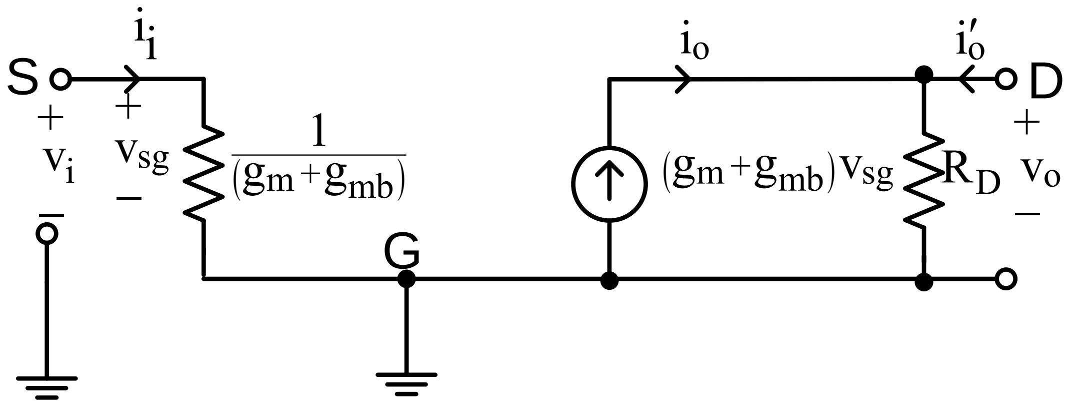



If $r_o$ is finite, the circuit is bilateral because of the feedback. If $r_o \to \infty$, the cct. is unilateral.

For $r_o \to \infty$ :






$$G_m = \left.\frac{i_o}{v_i}\right|_{v_o=0} = \frac{(g_m+g_{mb})v_{sg}}{v_{sg}} = g_m + g_{mb}$$

$$R_i = \frac{v_i}{i_i} = \frac{v_{sg}}{i_i} = \frac{1}{g_m+g_{mb}}$$

$$R_o = \left.\frac{v_o}{i_o'}\right|_{v_i=0} = R_D$$

$$a_v = \left.\frac{v_o}{v_i}\right|_{i_o'=0} = \frac{(g_m+g_{mb})v_{sg}R_d}{v_{sg}}$$

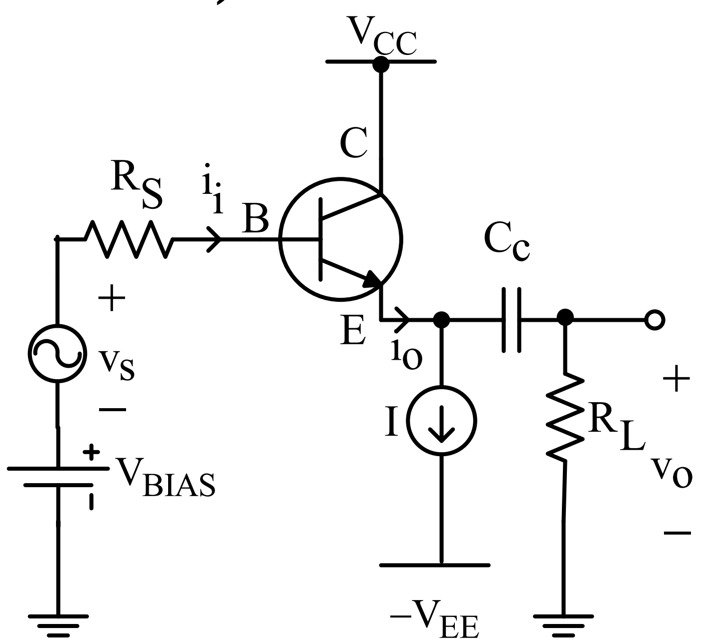
$$= (g_m+g_{mb})R_d$$

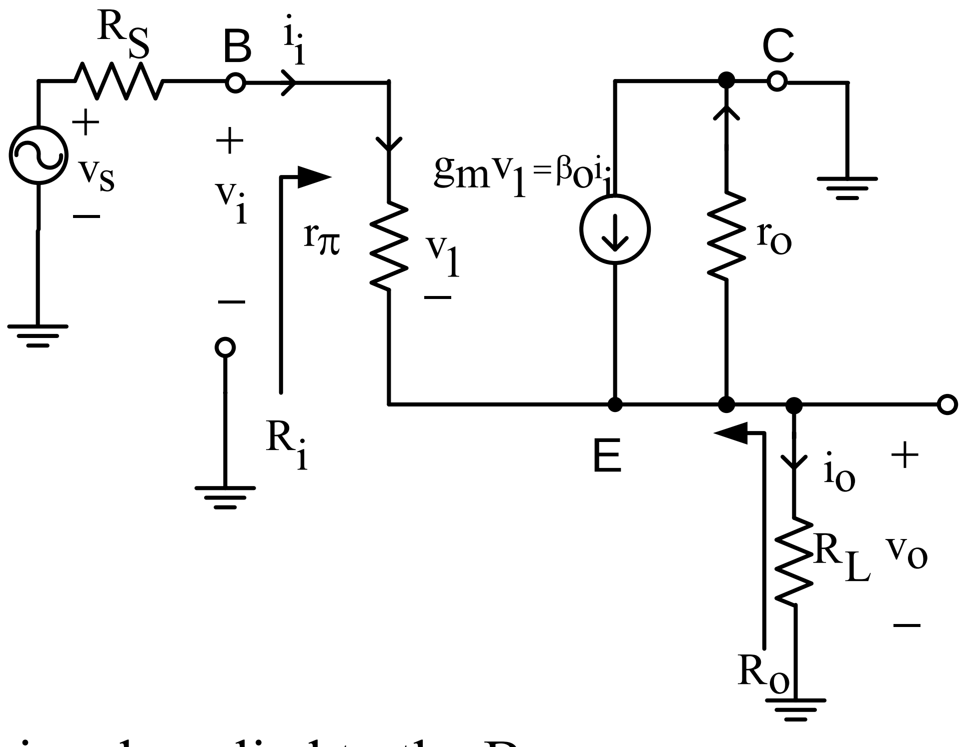
$$a_i = \left.\frac{i_o}{i_i}\right|_{v_o=0} = \frac{(g_m+g_{mb})v_{sg}}{\frac{v_{sg}}{\frac{1}{g_m+g_{mb}}}}$$

$$= (g_m+g_{mb})\left(\frac{1}{g_m+g_{mb}}\right)$$

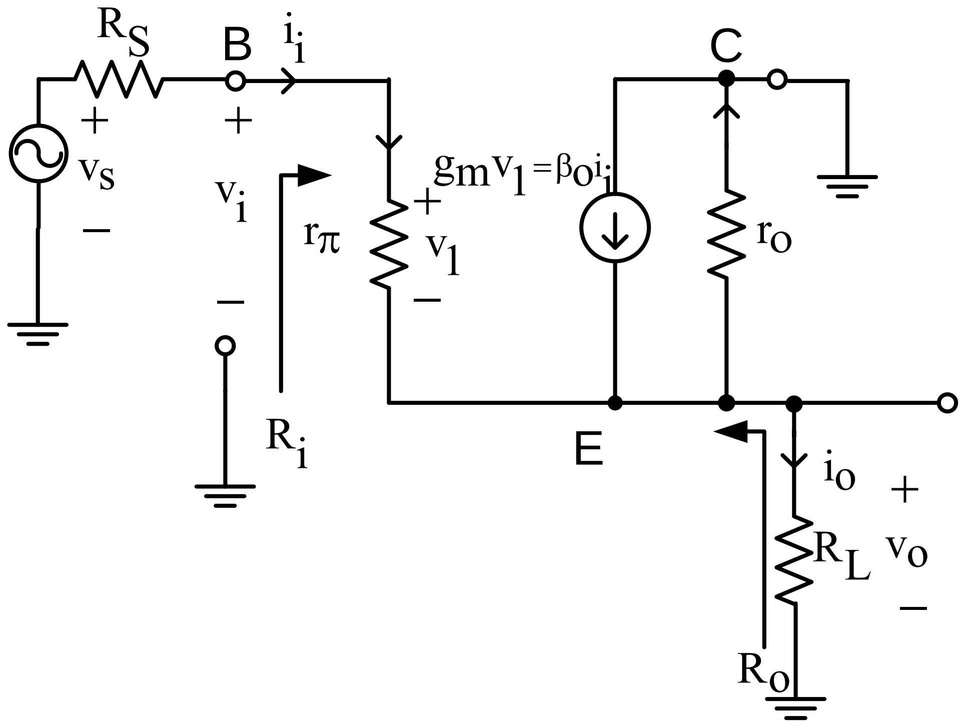
$$= 1$$

### 3.3.6 Common-collector (CC) configuration (Emitter follower)




I/p signal applied to the B.
O/p signal taken from the E.




$v_S = i_i\,(R_S + r_\pi) + v_o$

$v_i = i_i r_\pi + v_o$

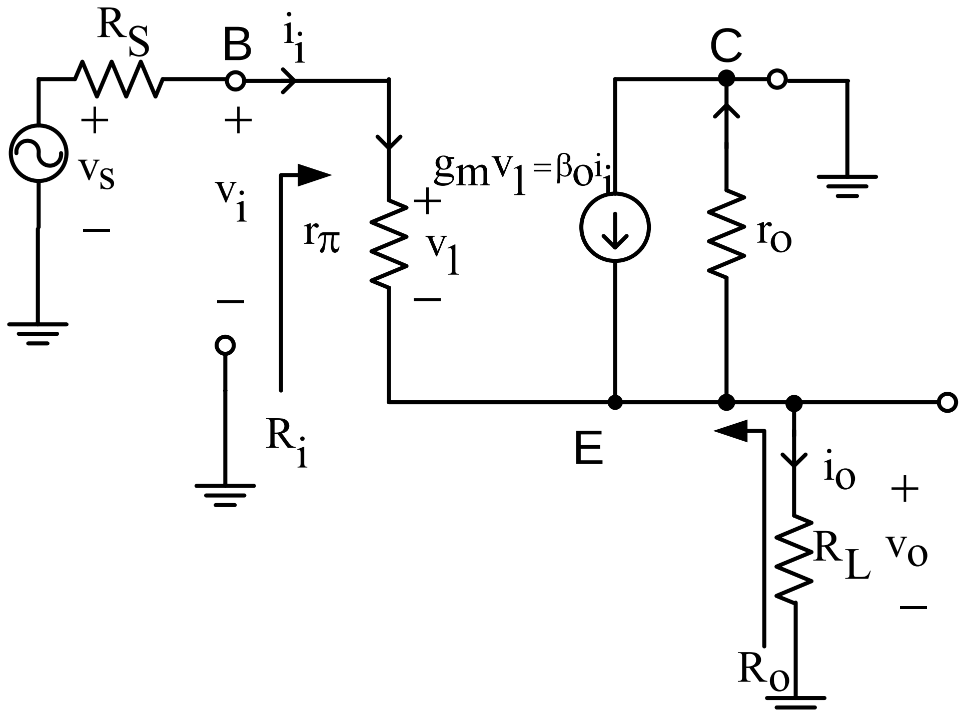
$R_i = v_i \,/\, i_i$

Hence, this circuit is not unilateral as the input resistance depends on the load resistor $R_L$ and the output resistance depends on the source resistance $R_S$.

<u>To determine $R_i$ :</u>

$R_i = v_i \,/\, i_i$

$$i_i + \beta_o i_i = \frac{v_o}{R_L} + \frac{v_o}{r_o}$$




$$i_i + \beta_o i_i = \frac{v_o}{R_L} + \frac{v_o}{r_o}$$

$$i_i(1+\beta_o) = (v_i - i_i r_\pi)\left(\frac{1}{R_L} + \frac{1}{r_o}\right)$$

$$i_i\left[1+\beta_o + r_\pi\left(\frac{1}{R_L} + \frac{1}{r_o}\right)\right] = v_i\left(\frac{1}{R_L} + \frac{1}{r_o}\right)$$

$$R_i = \frac{v_i}{i_i} = \frac{\left[1+\beta_o + r_\pi\left(\frac{1}{R_L} + \frac{1}{r_o}\right)\right]}{\left(\frac{1}{R_L} + \frac{1}{r_o}\right)} \qquad \leftarrow \text{enough.}$$

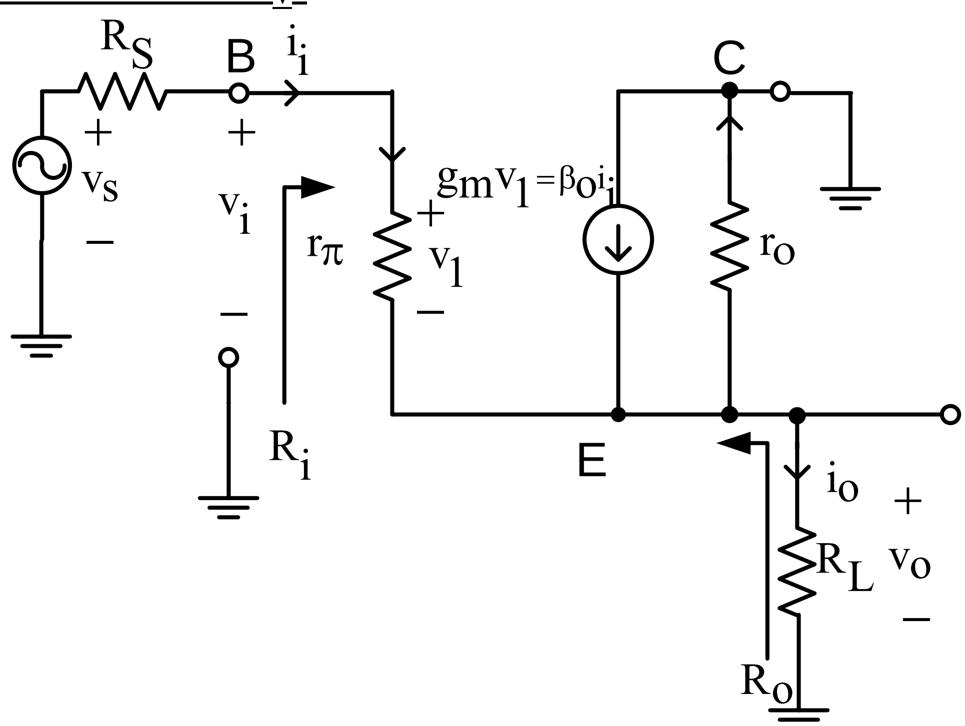
$$R_i = \frac{1+\beta_o + r_\pi\left(\frac{1}{R_L}+\frac{1}{r_o}\right)}{\left(\frac{1}{R_L}+\frac{1}{r_o}\right)}$$

$$= \frac{1+\beta_o}{\left(\frac{1}{R_L}+\frac{1}{r_o}\right)} + r_\pi$$

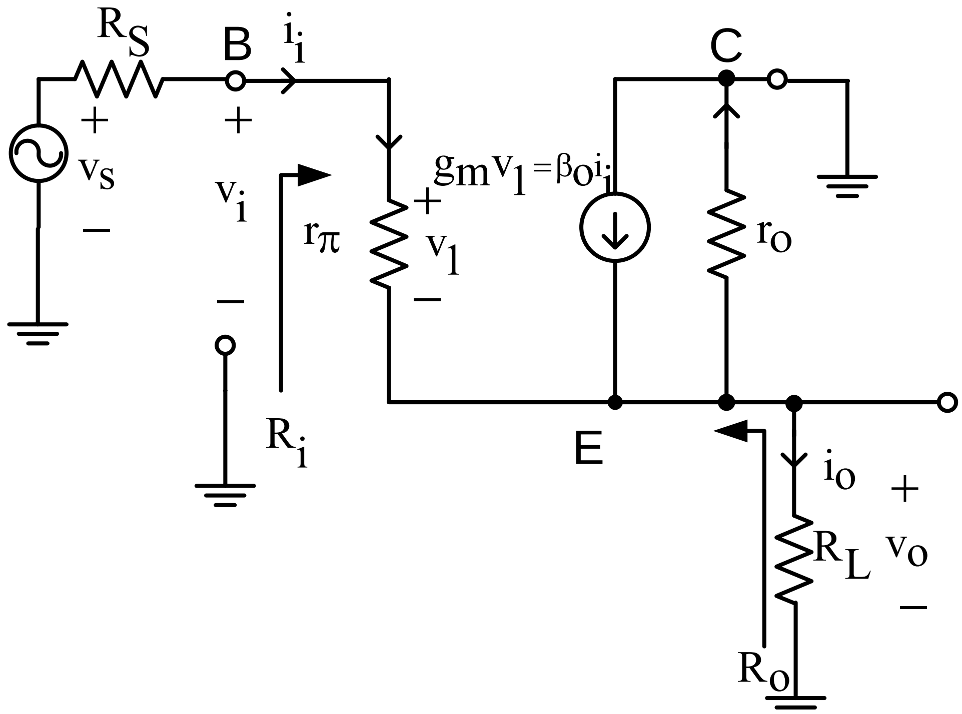
For $R_i$ with no load, i.e. $R_i = \left.\frac{v_i}{i_i}\right|_{R_L=\infty}$ ,

$$R_i = \left(1+\beta_o\right) r_o + r_\pi$$

<u>To determine $a_v$:</u>




Overall voltage gain: $a_v = v_o / v_s$




At node E,

$$\frac{v_s - v_o}{R_s + r_\pi} + g_m v_1 = v_o\left(\frac{1}{R_L} + \frac{1}{r_o}\right)$$

$$\frac{v_s}{R_s + r_\pi} + g_m \frac{r_\pi (v_s - v_o)}{R_s + r_\pi} = v_o\left(\frac{1}{R_L} + \frac{1}{r_o} + \frac{1}{R_s + r_\pi}\right)$$

Since $g_m r_\pi = \beta_o$,

$$\frac{v_s}{R_s + r_\pi} + \frac{\beta_o (v_s - v_o)}{R_s + r_\pi} = v_o\left(\frac{1}{R_L} + \frac{1}{r_o} + \frac{1}{R_s + r_\pi}\right)$$

$$v_s\left(\frac{1}{R_s + r_\pi} + \frac{\beta_o}{R_s + r_\pi}\right) = v_o\left(\frac{1}{R_L} + \frac{1}{r_o} + \frac{1}{R_s + r_\pi} + \frac{\beta_o}{R_s + r_\pi}\right)$$

$$a_v = \frac{v_o}{v_s} = \frac{(1+\beta_o)\left(\frac{1}{R_s + r_\pi}\right)}{\left(\frac{1}{R_L} + \frac{1}{r_o} + \frac{1+\beta_o}{R_s + r_\pi}\right)} \qquad \leftarrow \text{enough.}$$

$$= \frac{1}{\frac{1}{(1+\beta_o)\left(\frac{1}{R_s + r_\pi}\right)}\left(\frac{1}{R_L} + \frac{1}{r_o} + \frac{1+\beta_o}{R_s + r_\pi}\right)}$$

$$= \frac{1}{\left(\frac{R_s + r_\pi}{(1+\beta_o)R_L} + \frac{R_s + r_\pi}{(1+\beta_o)r_o} + 1\right)}$$

$$= \frac{1}{\left(\frac{r_o(R_s + r_\pi) + R_L(R_s + r_\pi)}{(1+\beta_o)R_L r_o} + 1\right)}$$

$$= \frac{1}{\left(\frac{(R_s + r_\pi)(r_o + R_L)}{(1+\beta_o)R_L r_o} + 1\right)}$$

$$= \frac{1}{\left(\frac{(R_s + r_\pi)}{(1+\beta_o)R_L // r_o} + 1\right)}$$

Open-circuit overall voltage gain,

$$a_v = \left.\frac{v_o}{v_s}\right|_{R_L = \infty} = \frac{1}{\left(\frac{(R_s + r_\pi)}{(1+\beta_o)r_o} + 1\right)}$$

$$a_V = \frac{1}{\left(\frac{(R_S + r_\pi)}{(1+\beta_o)R_L // r_o} + 1\right)}$$

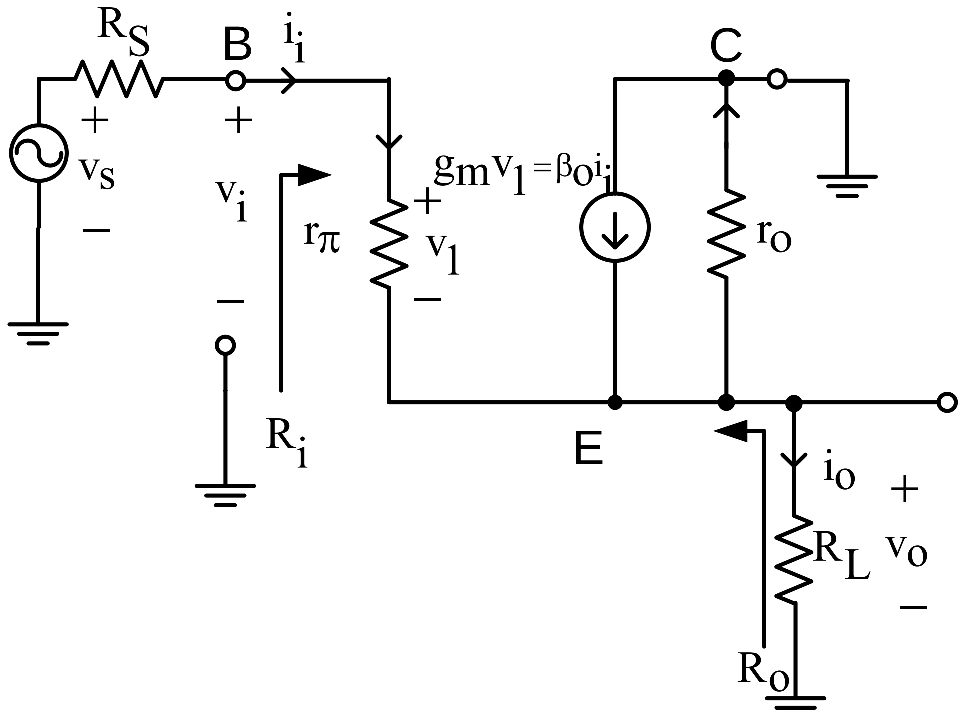
If the B resistance, $r_b$, is significant, it can be simply be added to $R_S$ in the expression. From the $a_v$ expression, $a_v$ will always be less than unity.

If $\beta_o(R_L // r_o) >> R_S + r_\pi$, then $a_v \approx 1$. This means that the output signal follows the input signal. Hence, this topology is also known as the emitter follower.

If $r_\pi >> R_S$, $\beta_o >> 1$ and $r_o >> R_L$, then

$$a_V = \frac{1}{\left(\frac{r_\pi}{(\beta_o)R_L} + 1\right)}$$

Since $g_m r_\pi = \beta_o$, then $a_V = \frac{1}{\left(\frac{1}{g_m R_L} + 1\right)} = \frac{g_m R_L}{(1 + g_m R_L)}$




<u>To determine the short circuit current gain, $a_i$ :</u>

$$a_i = \left.\frac{i_o}{i_i}\right|_{v_o=0}$$

$i_o = i_i + g_m v_1$

$v_1 = i_i r_\pi$

$i_o = i_i + g_m i_i r_\pi$

$$a_i = \frac{i_o}{i_i} = \frac{i_i\left(1+g_m r_\pi\right)}{i_i} = 1 + g_m r_\pi = 1 + \beta_o$$

<u>or</u>

$g_m v_1 = g_m i_i r_\pi = \beta_o i_i$

$i_o = i_i + \beta_o i_i = i_i\,(1 + \beta_o\,)$

$a_i = i_o \,/\, i_i = 1 + \beta_o$