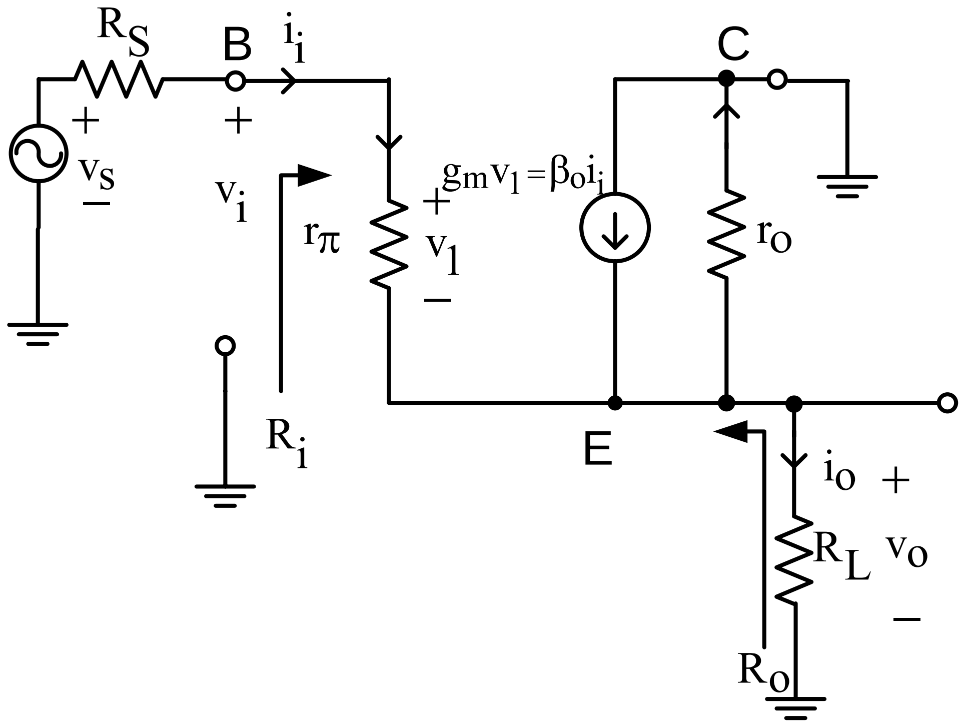
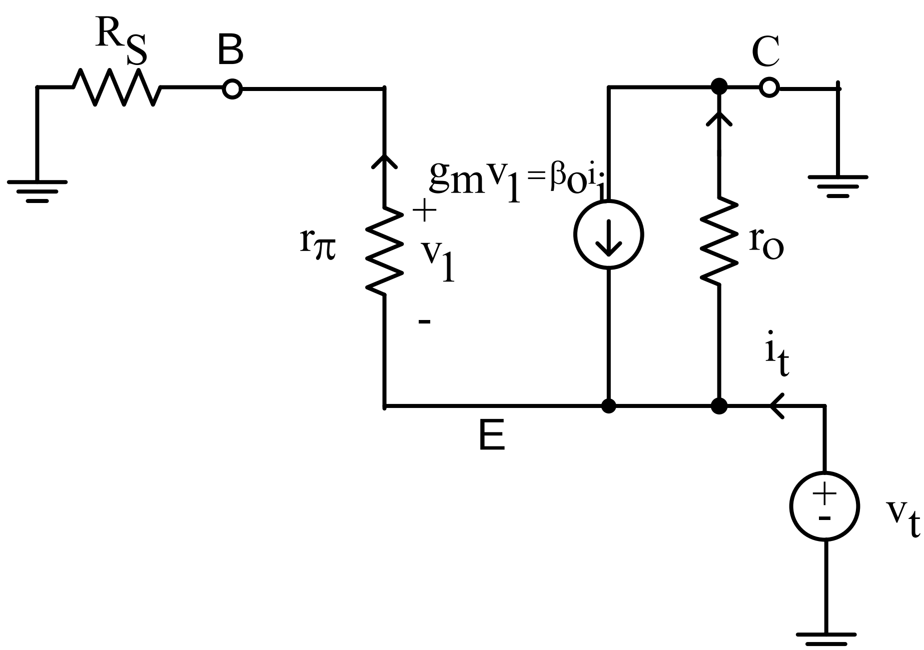
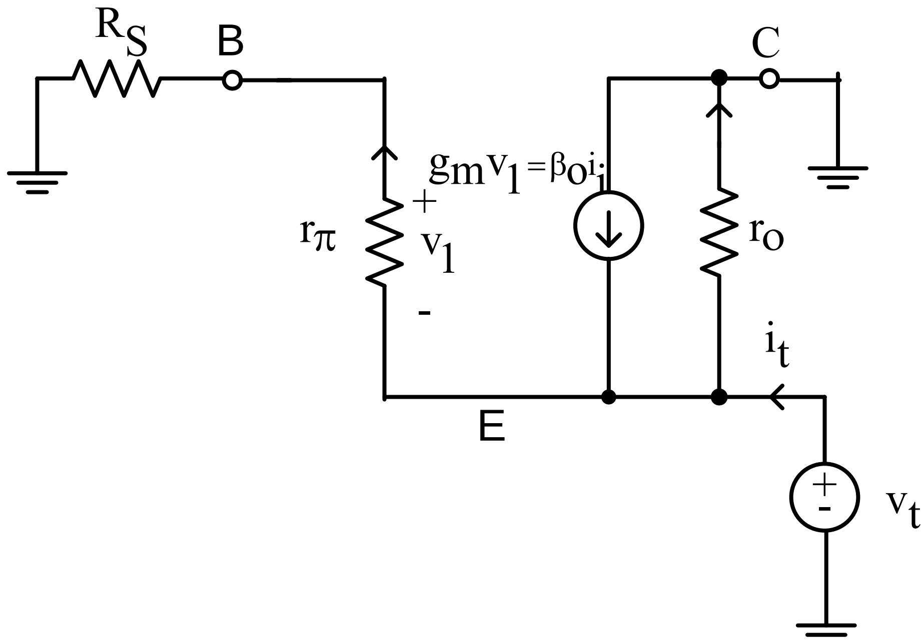


To determine R_o :

$$R_o = \left.\frac{v_o}{i_o}\right|_{v_S=0}$$





$$v_1 = -\frac{r_\pi}{R_S + r_\pi}(v_t)$$

At node E,

$$i_t + g_m v_1 = \frac{v_t}{r_o} + \frac{v_t}{R_S + r_\pi}$$

$$i_t = \frac{v_t}{r_o} + \frac{v_t}{R_S + r_\pi} + \frac{g_m r_\pi v_t}{R_S + r_\pi}$$

$$R_o = \frac{v_t}{i_t} = \frac{1}{\frac{1}{r_o} + \frac{1+\beta_o}{R_S + r_\pi}} \quad \leftarrow \text{enough}$$

$$= \left(\frac{R_S + r_\pi}{1+\beta_o}\right) \Big\| r_o$$

$$R_o = \left(\frac{R_S + r_\pi}{1+\beta_o}\right) \| r_o$$

$$= \left(\frac{R_S}{1+\beta_o} + \frac{r_\pi}{1+\beta_o}\right) \| r_o$$

If $\beta_o >> 1$, then $R_o = \left(\frac{R_S}{1+\beta_o} + \frac{r_\pi}{\beta_o}\right) \| r_o$.

Since $g_m r_\pi = \beta_o$, then $R_o = \left(\frac{R_S}{1+\beta_o} + \frac{1}{g_m}\right) \| r_o$.

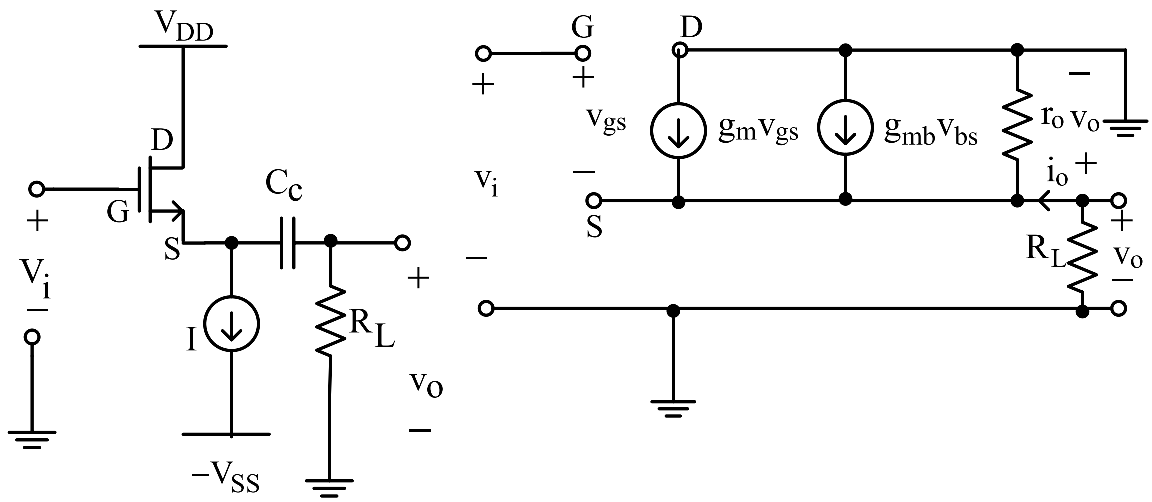
If $r_o \gg \frac{R_S}{1+\beta_o} + \frac{1}{g_m}$, then $R_o = \frac{R_S}{1+\beta_o} + \frac{1}{g_m}$.

$$R_i = r_\pi + (1+\beta_o)(R_L \| r_o)$$

$$a_v = \frac{1}{1 + \frac{R_S + r_\pi}{(1+\beta_o)(R_L \| r_o)}} \approx 1$$

The emitter follower has high i/p resistance, low o/p resistance and near-unity voltage gain. Therefore, it is widely used as an impedance transformer to reduce loading of a preceding signal source by the i/p impedance of a following stage.

Common-drain configuration (source follower)



I/p signal applied to G. O/p signal taken from S.

<u>To determine the open-circuit voltage gain, a_v</u> :

$$a_v = \left.\frac{v_o}{v_i}\right|_{R_L=\infty, i_o=0}$$

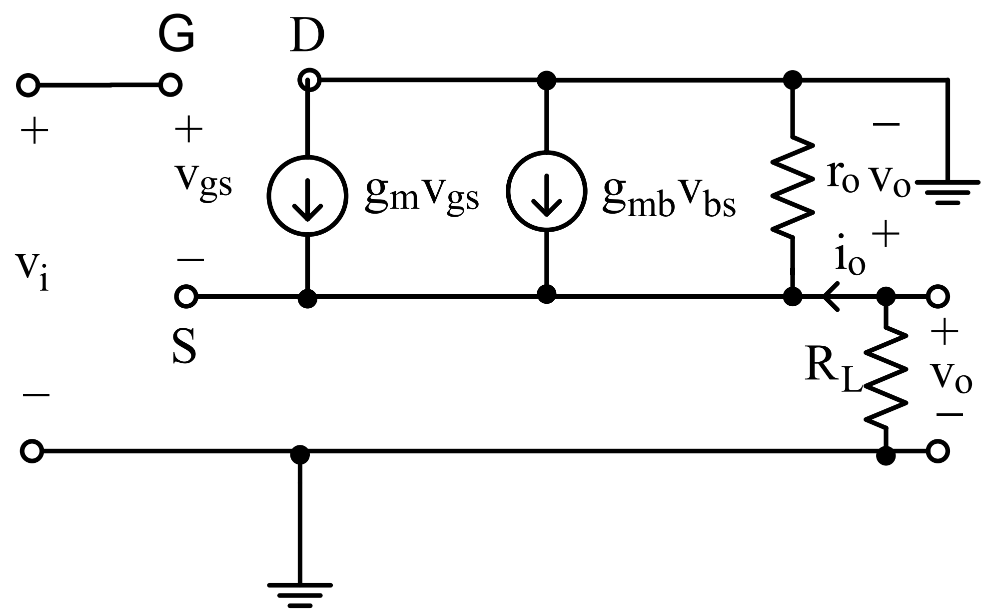
From KVL around the i/p loop,

$-v_i + v_{gs} + v_o = 0$

$v_i = v_{gs} + v_o$

KCL at the o/p node:

$g_m v_{gs} + g_{mb} v_{bs} = v_o / r_o$ for $R_L = \infty$



$$v_{bs} = v_b - v_s = -v_s = -v_o$$

$$g_m v_{gs} = v_o\left(\frac{1}{r_o} + g_{mb}\right)$$

$$g_m v_i = v_o\left(\frac{1}{r_o} + g_{mb} + g_m\right)$$

$$a_v = \frac{v_o}{v_i} = \frac{g_m}{\left(\frac{1}{r_o} + g_{mb} + g_m\right)} \quad \leftarrow \text{enough}$$

$$= \frac{g_m r_o}{1 + (g_{mb} + g_m) r_o}$$

If r_o is finite, this gain is < 1 even though the body effect is eliminated.

$$a_v = \frac{g_m}{\left(\frac{1}{r_o} + g_{mb} + g_m\right)}$$

If $r_o \to \infty$, then $a_v = \frac{g_m}{(g_{mb} + g_m)} = \frac{1}{1+\chi}$

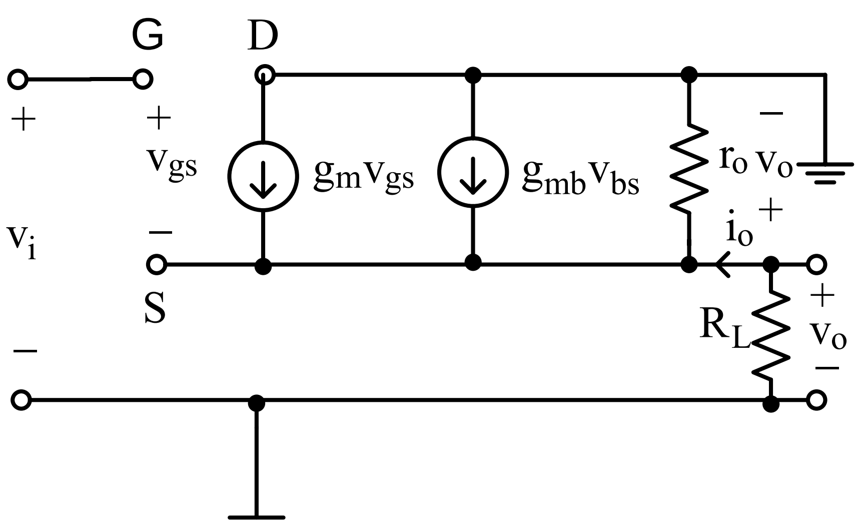
$\chi = g_{mb} / g_m = 0.1$ to 0.3.

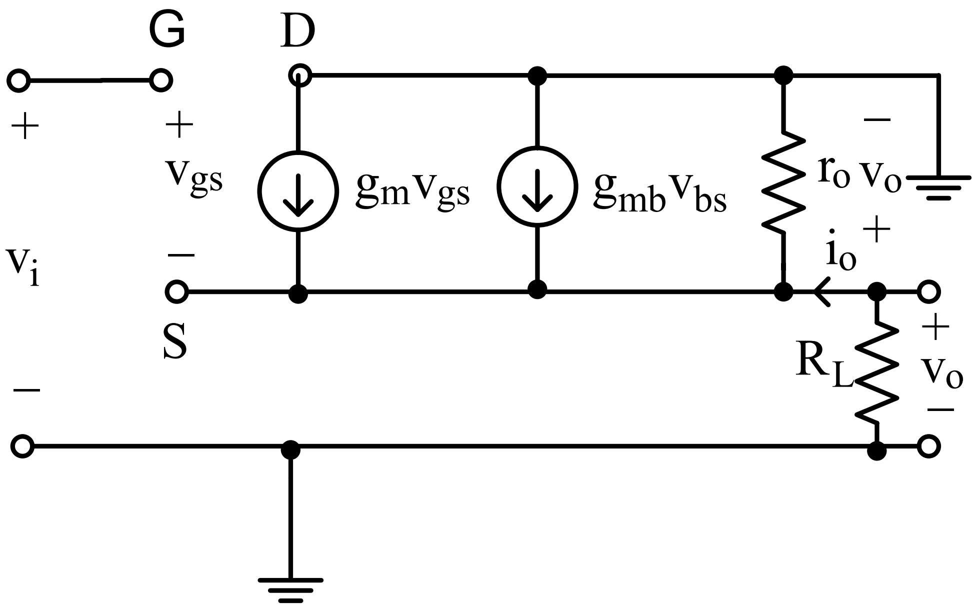
Comparing with CC (emitter follower), the open-circuit voltage gain for the CC is:

$$a_v = \left.\frac{v_o}{v_s}\right|_{R_L=\infty} = \frac{1}{\left(1 + \frac{(R_s + r_\pi)}{(1+\beta_o) r_o}\right)}$$

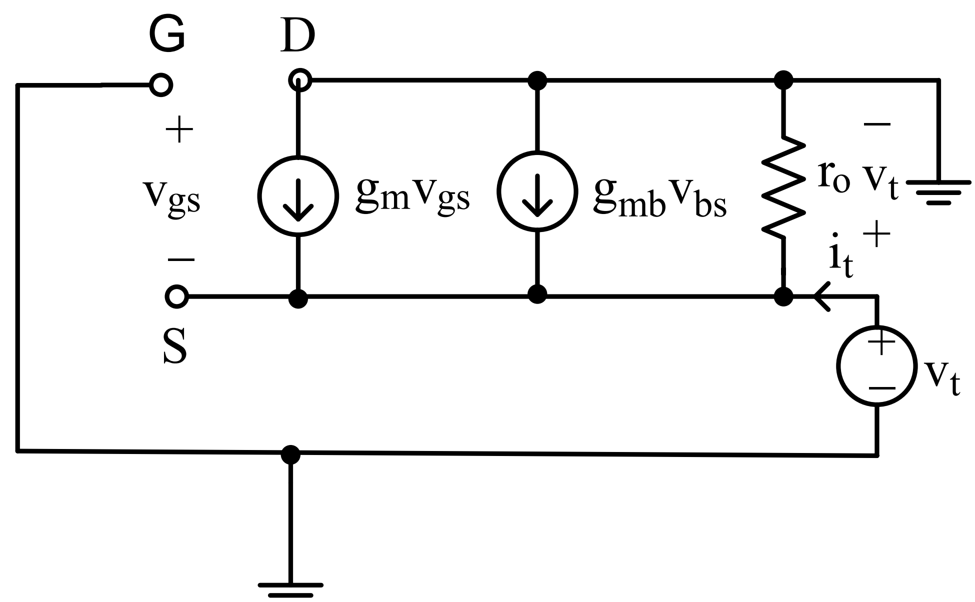
<u>To determine R_o</u> :

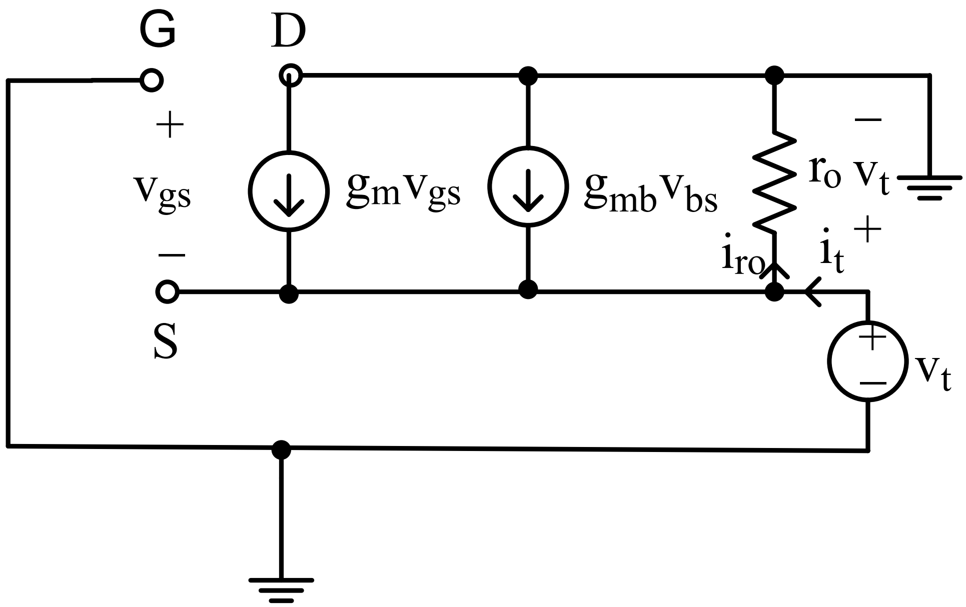
$$R_o = \left.\frac{v_o}{i_o}\right|_{v_i=0}$$





The output resistance can be calculated by setting $v_i = 0$ and driving the output with a voltage source v_t.





$$v_{gs} = -v_t$$
$$v_{gs} = v_g - v_s = -v_s$$
$$v_{bs} = v_b - v_s = -v_s = -v_t$$

At the S node,

$$g_m v_{gs} + g_{mb} v_{bs} + i_t = \frac{v_t}{r_o}$$
$$i_t = \left(\frac{1}{r_o} + g_m + g_{mb}\right) v_t$$
$$R_o = \frac{v_t}{i_t} = \frac{1}{\left(\frac{1}{r_o} + g_m + g_{mb}\right)}$$

If $r_o \to \infty$, $R_o = \frac{1}{(g_m + g_{mb})}$

Hence, body effect reduces R_o.

	CE	CB	CC
R_i	$r_\pi = \frac{\beta_o}{g_m}$	$r_e = \frac{1}{g_m + \frac{1}{r_\pi}} = \frac{1}{g_m\left(1+\frac{1}{\beta_o}\right)} = \frac{\alpha_o}{g_m}$	$r_\pi + (\beta_o+1)(R_L \Vert r_o)$
G_m	g_m	g_m	$\frac{1+\beta_o}{R_S+r_\pi}$
R_o	$R_C \Vert r_o$	R_C	$\frac{R_S+r_\pi}{1+\beta_o} \Big\Vert r_o$
a_v	$-g_m(R_C \Vert r_o)$	$g_m R_C$	$\frac{1}{1+\frac{R_S+r_\pi}{(1+\beta_o) r_o \Vert R_L}}$
a_i	β_o	$g_m r_e = \alpha_o$	$1+\beta_o$

	CE	CB	CC
R_i	medium	$\downarrow$	$\uparrow$
R_o	medium	$\uparrow$	$\downarrow$
a_v	$\uparrow$	$\uparrow$	≤ 1
a_i	$\uparrow$	≤ 1	$\uparrow$
$a_p = a_v a_i$	$\uparrow$	$\approx a_v$	$\approx a_i$
i/p–o/p phase shift	180° (voltage)	0°	0°

	CS	CG	CD
R_i	∞	$\frac{1}{g_m + g_{mb}}$	∞
G_m	g_m	$g_m + g_{mb}$	g_m
R_o	$R_D \| r_o$	R_D	$\frac{1}{g_m + g_{mb} + \frac{1}{r_o}}$
a_v	$-g_m(R_D \| r_o)$	$(g_m + g_{mb})R_D$	$\frac{g_m r_o}{1 + (g_m + g_{mb})r_o + \frac{r_o}{R_L}}$
a_i	∞	1	∞

	CS	CG	CD
R_i	∞	↓	∞
R_o	medium	↑	↓
a_v	↑	↑	< 1
a_i	∞	1	∞
$a_p = a_v\, a_i$	∞	$\approx a_v$	∞
i/p–o/p phase shift	180° (voltage)	0°	0°

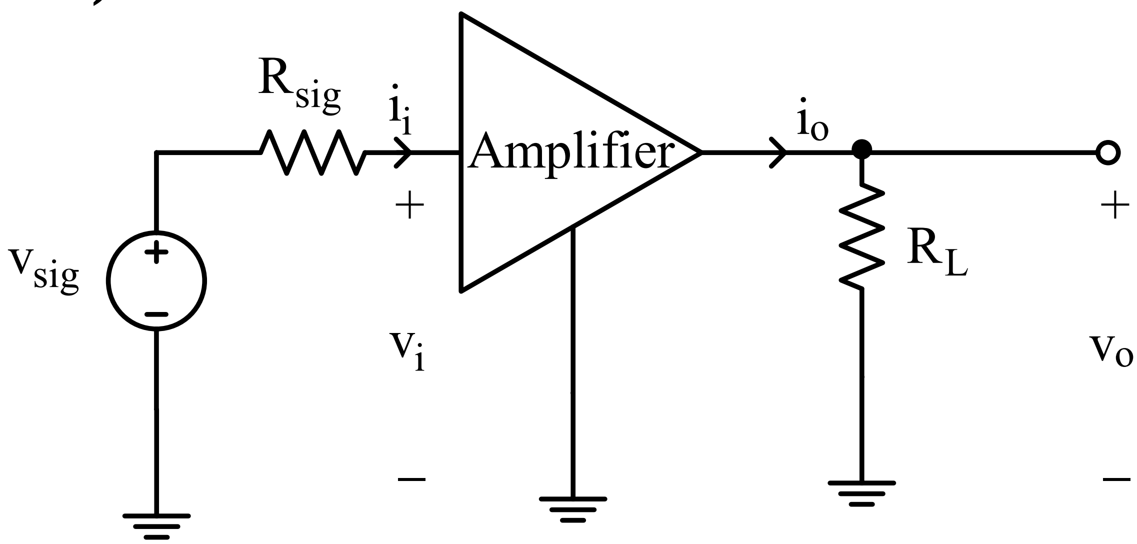
Open-circuit voltage gain ($R_L = \infty$),

$$a_{vo} = \frac{v_o}{v_i} = \frac{v_o}{i_o} \times \frac{i_o}{v_i} = R_o G_m$$

Short-circuit current gain ($R_L = 0$),

$$a_{is} = \frac{i_o}{i_i} = \frac{i_o}{v_i} \times \frac{v_i}{i_i} = R_i G_m$$

Characteristic Parameters of Amplifiers (Sedra & Smith, “Microelectronic Circuits”, Oxford 2004, Table 5.5, pg. 462)



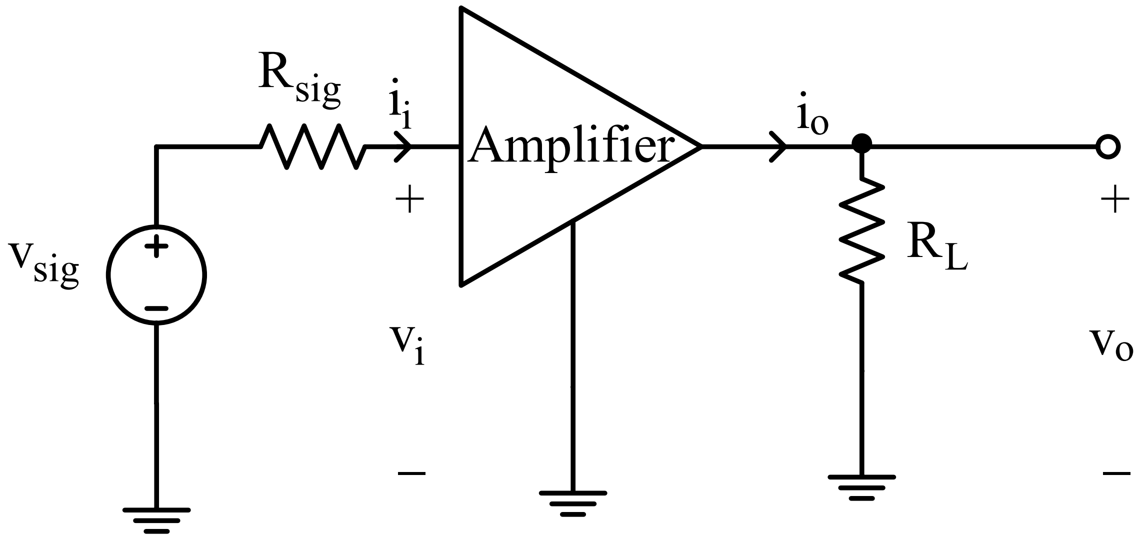
Definitions:

1. Input resistance with no load

$$R_i = \left.\frac{v_i}{i_i}\right|_{R_L=\infty}$$

2. Input resistance

$$R_i = \frac{v_i}{i_i}$$



3. Open-circuit voltage gain

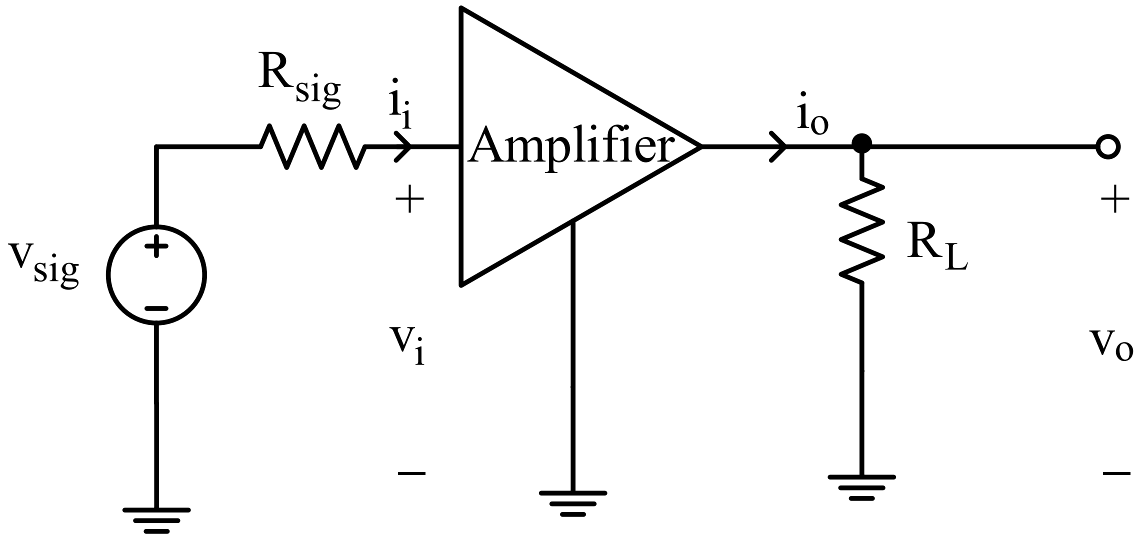
$$a_{vo} = \left.\frac{v_o}{v_i}\right|_{R_L=\infty}$$

4. Voltage gain

$$a_v = \frac{v_o}{v_i}$$

5. Short-circuit current gain

$$a_{is} = \left.\frac{i_o}{i_i}\right|_{v_o=0, R_L=0}$$



6. Current gain

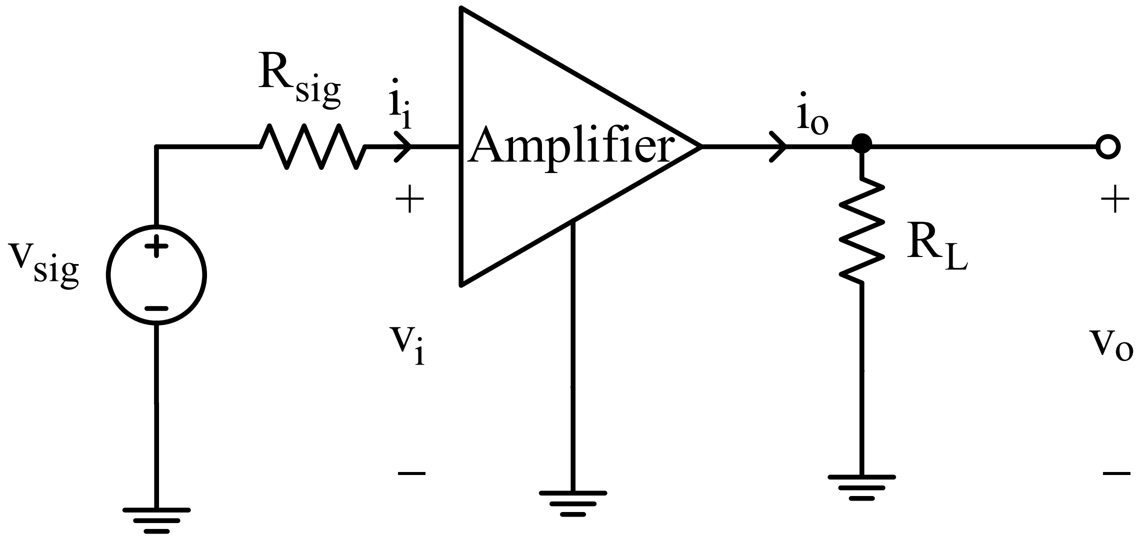
$$a_i = \frac{i_o}{i_i}$$

7. Short-circuit transconductance

$$G_m = \left.\frac{i_o}{v_i}\right|_{v_o=0, R_L=0}$$

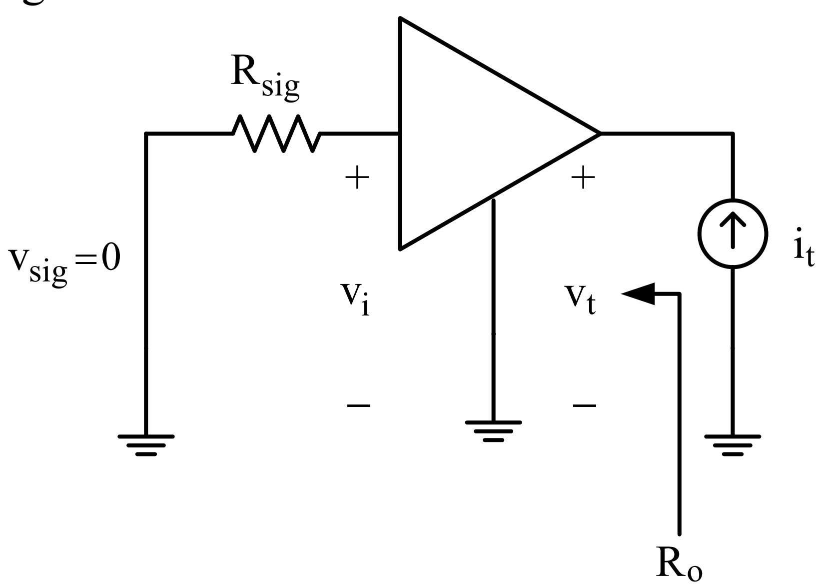
8. Open-circuit overall voltage gain

$$G_{vo} = \left.\frac{v_o}{v_{sig}}\right|_{R_L=\infty}$$



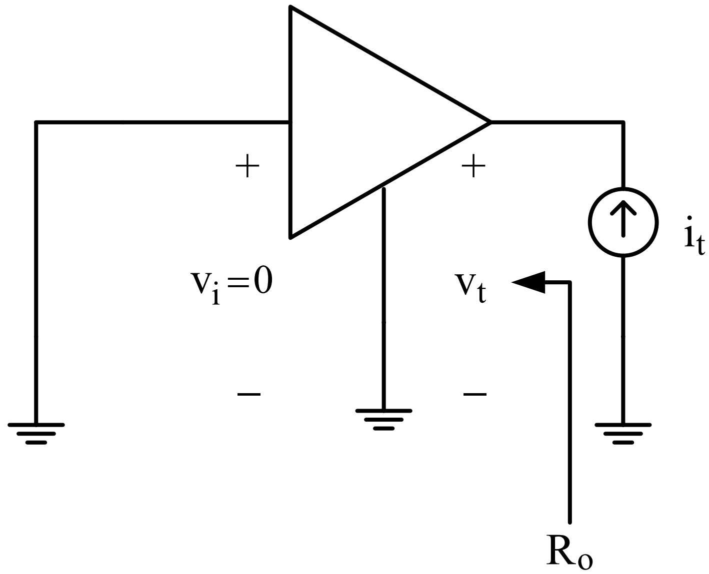
9. Overall voltage gain

$$G_V = \frac{v_o}{v_{sig}}$$



10. Output resistance

$$R_o = \left.\frac{v_t}{i_t}\right|_{v_{sig}=0}$$



11. Output resistance of the amplifier proper

$$R_o = \left.\frac{v_t}{i_t}\right|_{v_i=0}$$