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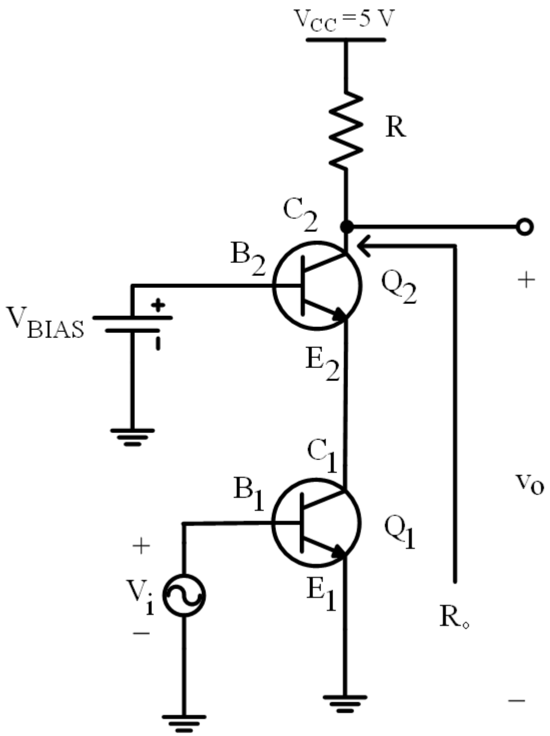
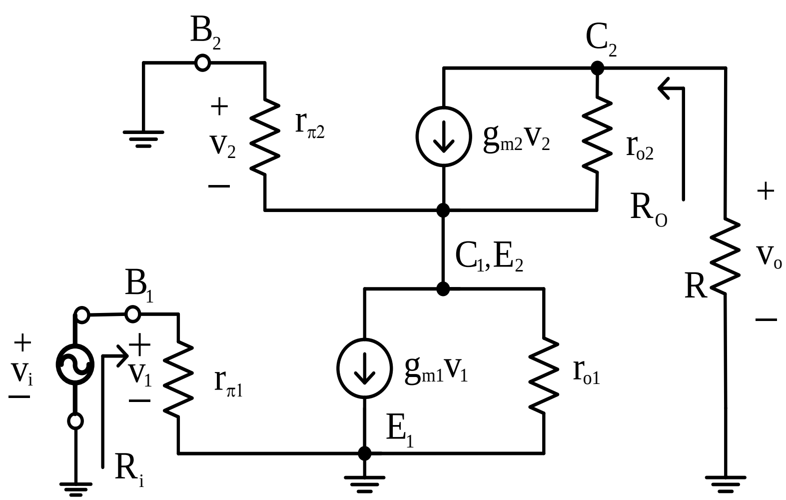
EEE 241
ANALOG ELECTRONICS
CLASS 15&16

DR NORLAILI MOHD NOH





G_M can also be obtained by inspection of the circuit. Since for Q_2, collector current is the output current and emitter current is the input current (CB configuration), then current gain of Q_2 is ≈ 1.

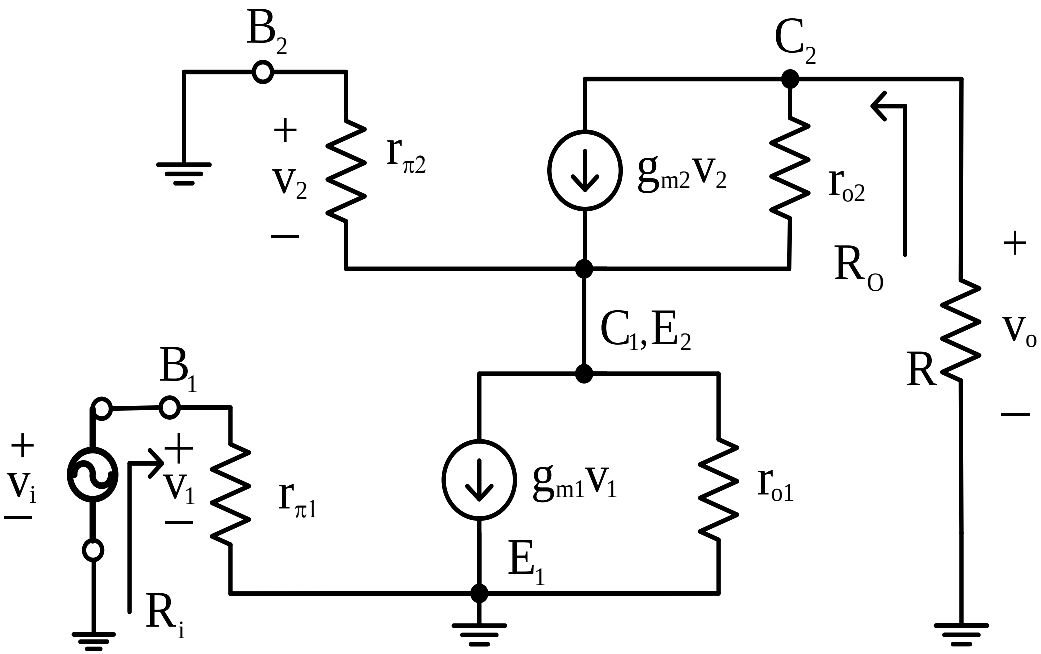
Hence, current gain of the cascode is equal to the current gain of the CE Q_1.

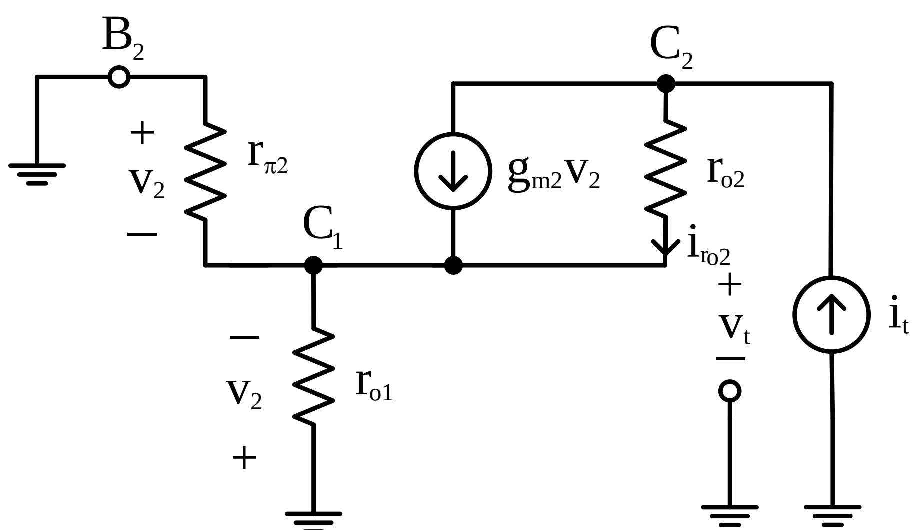
For the CE, a_{is} = β.

From the previous slide, we know that $R_i = r_{\pi 1}$

We know that $a_{is} = \frac{i_o}{i_i} = \frac{v_i}{i_i} \times \frac{i_o}{v_i} = R_i G_M$

Hence, $G_M = \frac{a_{is}}{R_i} = \frac{\beta_1}{r_{\pi 1}} = g_{m1}$





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To determine R_O :

$$R_o = \left.\frac{v_t}{i_t}\right|_{v_i=0}$$

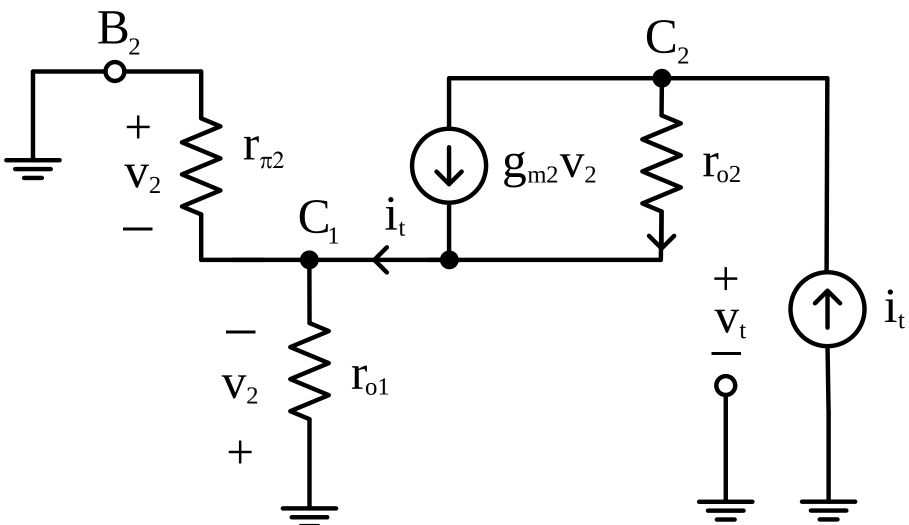
When v_i=0, then v_1=0.
Hence, $g_{m1}v_1$=0

At node C_2 :

$$i_t = g_{m2}v_2 + i_{ro2}$$

KVL at the output loop,

$$-v_t + i_{ro2}(r_{o2}) - v_2 = 0$$
$$-v_t + (i_t - g_{m2}v_2)(r_{o2}) - v_2 = 0$$
$$-v_t + i_t r_{o2} - g_{m2}v_2 r_{o2} - v_2 = 0$$
$$v_t = i_t r_{o2} - (g_{m2}r_{o2} + 1)v_2$$



$$v_t = i_t r_{o2} - \left(g_{m2} r_{o2} + 1\right) v_2$$

$$i_t = \frac{-v_2}{r_{o1} // r_{\pi 2}}$$

$$v_t = i_t r_{o2} - \left(g_{m2} r_{o2} + 1\right)\left(-i_t\right)\left(r_{o1} // r_{\pi 2}\right)$$

$$R_O = \left(\frac{v_t}{i_t}\right) = r_{o2} + \frac{\left(g_{m2} r_{o2} + 1\right)}{\frac{1}{r_{o1}} + \frac{1}{r_{\pi 2}}}$$ ← enough

where $r_\pi = \frac{\beta_o}{g_m}$

$$R_O = r_{o2} + \frac{\left(g_{m2} r_{o2} + 1\right)}{\frac{1}{r_{o1}} + \frac{g_{m2}}{\beta_{o2}}}$$

$$R_O = r_{o2}\left[1 + \frac{g_{m2} + \frac{1}{r_{o2}}}{\frac{1}{r_{o1}} + \frac{g_{m2}}{\beta_{o2}}}\right]$$

$$R_O = r_{o2}\left[1 + \frac{g_{m2} + \frac{1}{r_{o2}}}{\frac{1}{r_{o1}}\left(1 + \frac{g_{m2} r_{o1}}{\beta_{o2}}\right)}\right]$$

$$R_O = r_{o2}\left[1+\frac{g_{m2}+\frac{1}{r_{o2}}}{\frac{1}{r_{o1}}\left(1+\frac{g_{m2}r_{o1}}{\beta_{o2}}\right)}\right] = r_{o2}\left[1+\frac{r_{o1}g_{m2}+\frac{r_{o1}}{r_{o2}}}{\left(1+\frac{g_{m2}r_{o1}}{\beta_{o2}}\right)}\right]$$

If $r_{o1} = r_{o2}$,

$$R_O = r_{o2}\left[1+\frac{g_{m2}r_{o1}+1}{1+\frac{g_{m2}r_{o1}}{\beta_{o2}}}\right]$$

If $g_{m2}\ r_{o1} >> \beta_{o2}$,

$$R_O = r_{o2}\left[1+\frac{g_{m2}r_{o1}+1}{\frac{g_{m2}r_{o1}}{\beta_{o2}}}\right]$$

$$R_O = r_{o2}\left[1+\frac{\beta_{o2}\left(g_{m2}r_{o1}+1\right)}{g_{m2}r_{o1}}\right]$$

If $\beta_{o2} >>1$, then $g_{m2}\ r_{o1} >> 1$,

$$R_O = r_{o2}\left[1+\frac{\beta_{o2}g_{m2}r_{o1}}{g_{m2}r_{o1}}\right]$$

$$R_O = r_{o2}\left(1+\beta_{o2}\right)$$

$$R_O \approx r_{o2}\beta_{o2}$$

$R_O \approx r_{o2}\beta_{o2}$

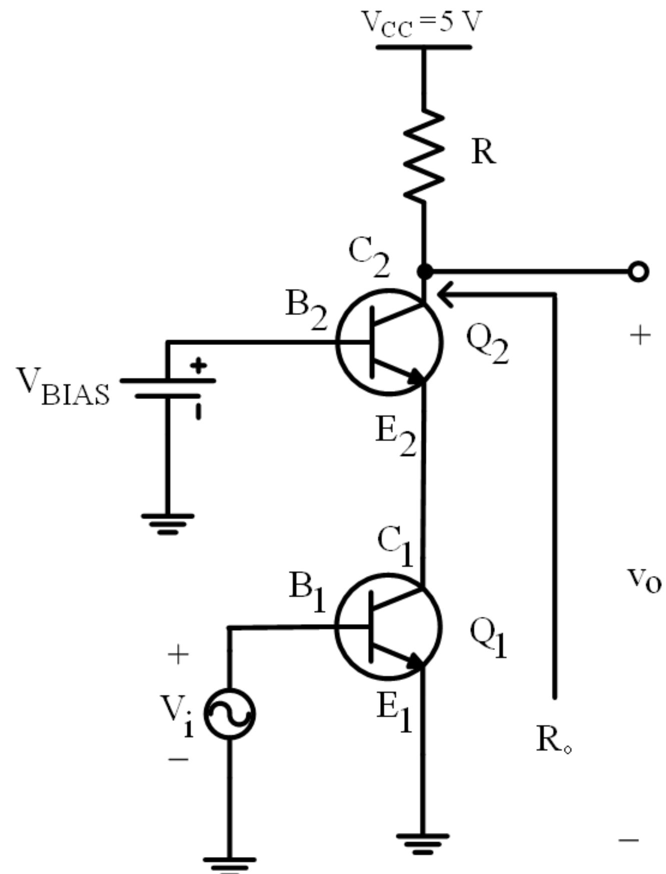
Hence, the CE-CB connection displays an output resistance that is larger by a factor of about β_{o2} then the CE stage alone (R_O of CE is r_o if R_C is neglected)

Open circuit voltage gain :

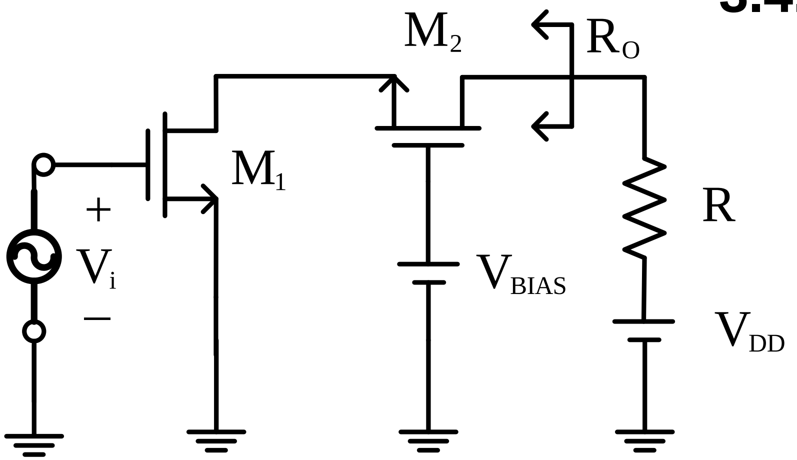
$$a_{vo} = \frac{v_o}{v_i} = \frac{v_o}{i_o} \times \frac{i_o}{v_i} = R_O \times G_M$$

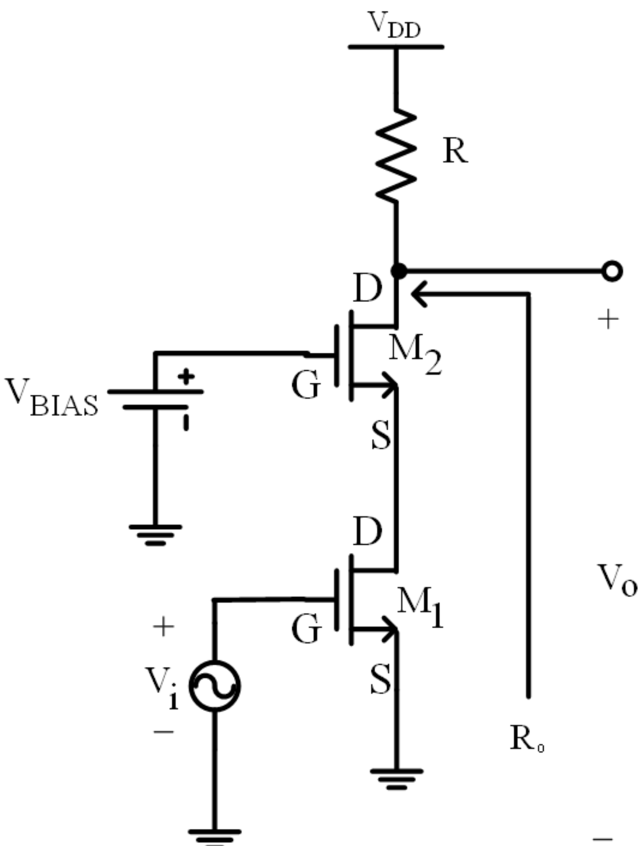
Hence, $a_{vo} = -r_{o2}\beta_{o2}g_{m1}$

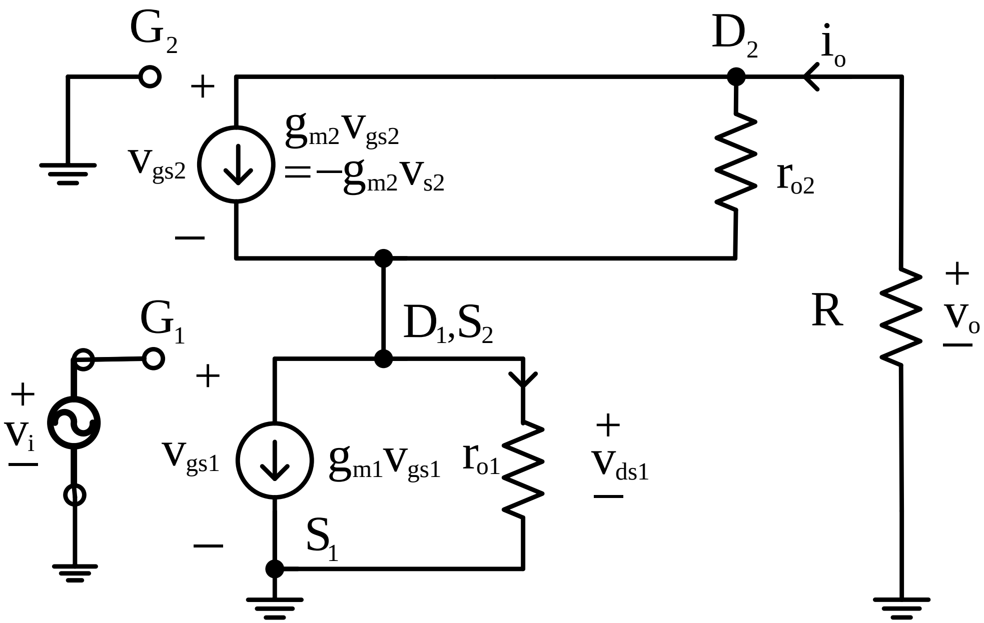
For a CE amplifier, $a_v = -g_m r_0$ if R_C is not included. Thus, the magnitude of the maximum available voltage gain is higher by a factor β_{o2} than for the case of a single transistor.



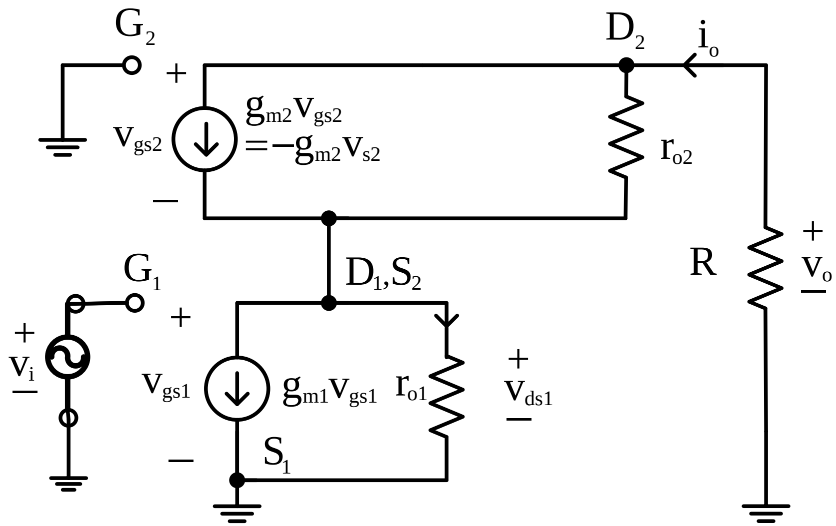
3.4.2.2 The MOS Cascode

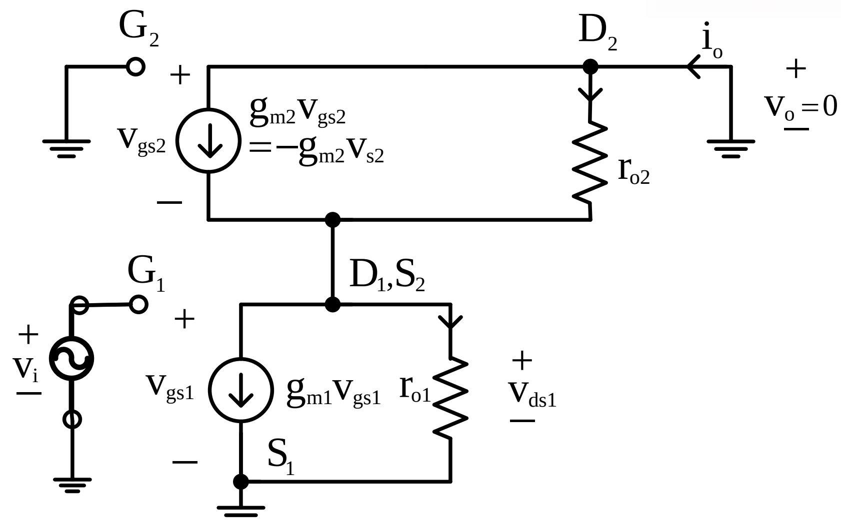






$$v_{s2} = v_{ds1}$$
$$v_{gs1} = v_i$$
$$v_{gs2} = -v_{s2}$$
$$R_i = \infty$$





$$G_M = \left.\frac{i_o}{v_i}\right|_{v_o=0}$$

KCL at node D_2,

$$i_o = -g_{m2}v_{ds1} - \frac{v_{ds1}}{r_{o2}}$$

$$i_o = v_{ds1}\left(-g_{m2} - \frac{1}{r_{o2}}\right)$$

KCL at node D_1,

$$i_o = g_{m1}v_i + \frac{v_{ds1}}{r_{o1}}$$

$$i_o = g_{m1}v_i + \frac{i_o}{r_{o1}}\left[\frac{1}{-g_{m2} - \frac{1}{r_{o2}}}\right]$$

$$i_o\left[1 + \frac{1}{r_{o1}\left(g_{m2} + \frac{1}{r_{o2}}\right)}\right] = g_{m1}v_i$$

$$G_M = \frac{i_o}{v_i} = \frac{g_{m1}}{1 + \frac{1}{r_{o1}\left(g_{m2} + \frac{1}{r_{o2}}\right)}} \leftarrow \text{enough}$$

$$G_M = \frac{i_o}{v_i} = \frac{g_{m1}}{1+\frac{1}{r_{o1}\left(g_{m2}+\frac{1}{r_{o2}}\right)}}$$

$$G_M = \frac{g_{m1} r_{o1}\left(g_{m2}+\frac{1}{r_{o2}}\right)}{r_{o1}\left(g_{m2}+\frac{1}{r_{o2}}\right)+1}$$

$$G_M = \frac{g_{m1}\left[g_{m2} r_{o1}+\frac{r_{o1}}{r_{o2}}+1\right]-g_{m1}}{\left[g_{m2} r_{o1}+\frac{r_{o1}}{r_{o2}}+1\right]}$$

$$G_M = g_{m1}\left[1-\frac{1}{g_{m2} r_{o1}+\frac{r_{o1}}{r_{o2}}+1}\right]$$

If $r_{o1} = r_{o2}$, then

$$G_M = g_{m1}\left[1-\frac{1}{g_{m2} r_{o1}+2}\right]$$

If $g_{m2} r_{o1} >> 1$, then $G_M \approx g_{m1}$

This equation shows that the transconductance of the cascode is always < g_{m1}. If $g_{m2} r_{o1} >> 1$, the difference is small. The main point here is that the cascode configuration has little effect on the transconductance since the overall G_M for the CS is g_m.

Another way of deriving to this conclusion is by inspecting the circuit.

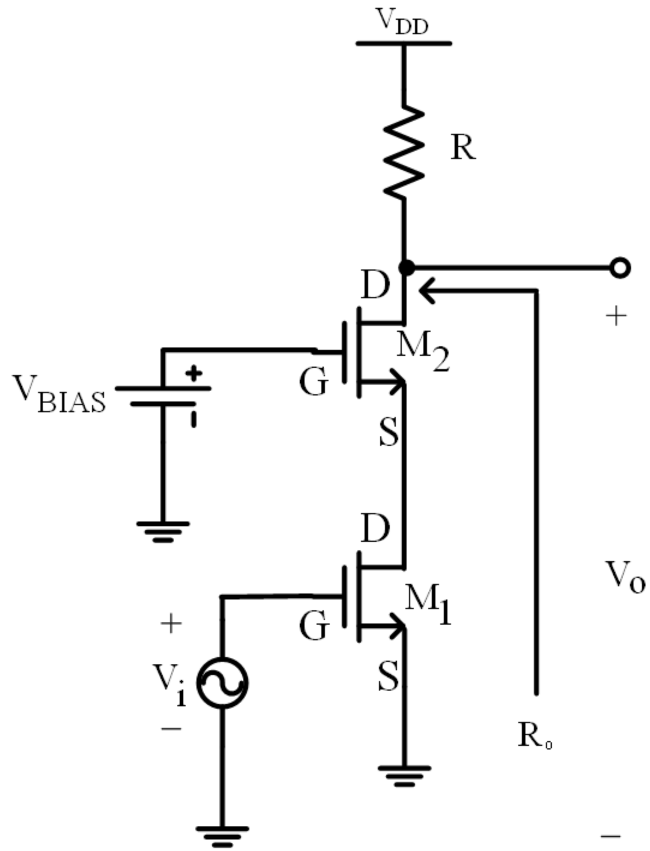
For CG,
$R_i = 1/g_m$.

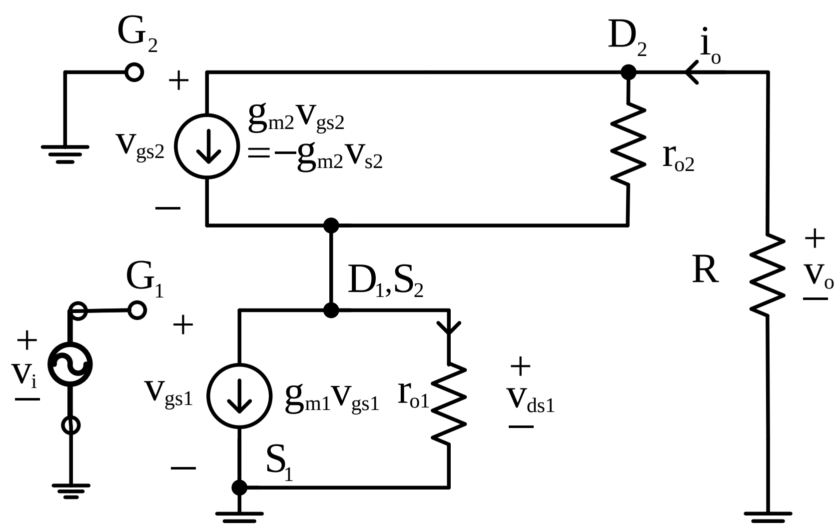
The current $g_{m1}v_i$ will mostly flow through this Ri rather than ro1 as
$1/\ g_m << r_{o1}$.

For a CG,
$a_i = 1$

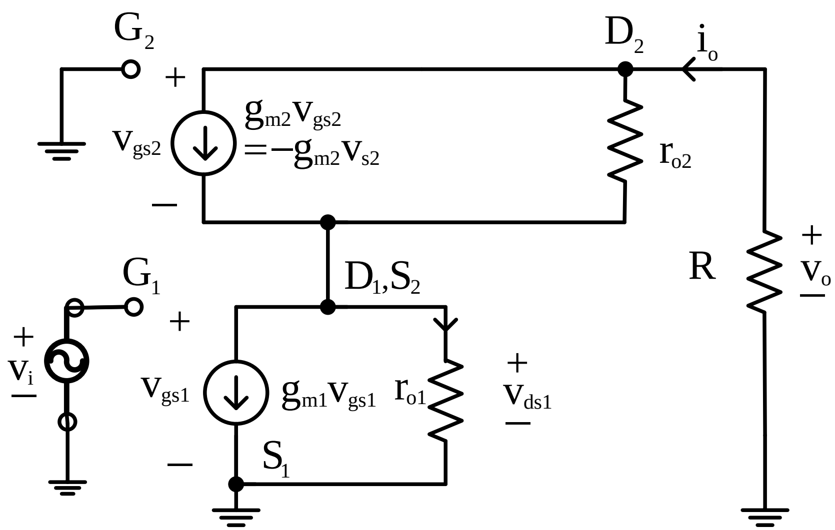
Hence, the output current will then be the same as the input current to the CG, which is $g_{m1}v_i$.

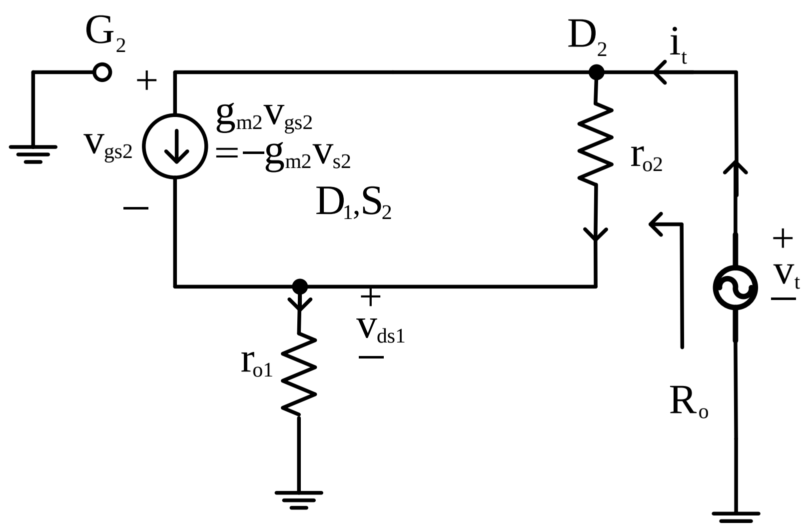
$G_M = i_o\ /v_i\ = g_{m1}v_i\ /v_i = g_m$.





Determining R_o





$$R_O = \left.\frac{v_t}{i_t}\right|_{v_i=0}$$

When v_i=0, $g_{m1}v_i$=0.

KCL at node D_2,

$$i_t = -g_{m2}v_{ds1} + \frac{v_{r_{o2}}}{r_{o2}}$$

$$v_{r_{o2}} = v_t - v_{ds1}$$

$$i_t = v_{ds1}\left(-g_{m2}\right) + \frac{v_t - v_{ds1}}{r_{o2}}$$

$$i_t = v_{ds1}\left(-g_{m2} - \frac{1}{r_{o2}}\right) + \frac{v_t}{r_{o2}}$$

KCL at node D_2,

$$i_t = v_{ds1}\left(-g_{m2} - \frac{1}{r_{o2}}\right) + \frac{v_t}{r_{o2}}$$

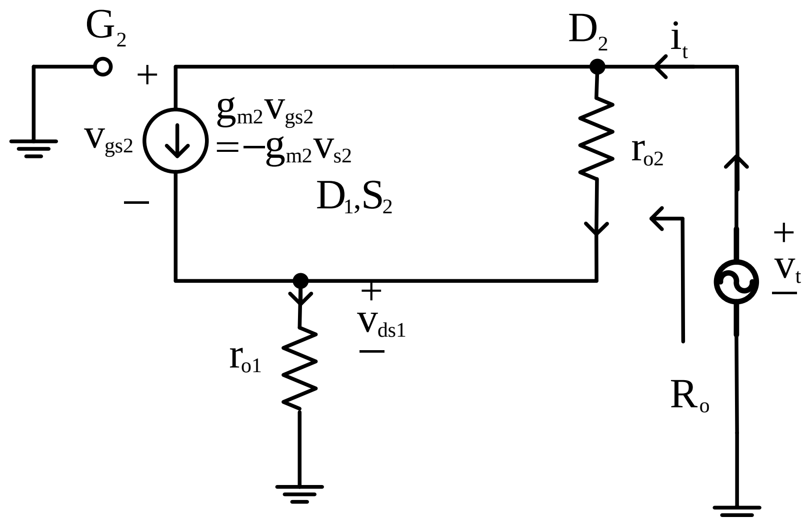
$$v_{ds1} = i_t r_{o1}$$

$$i_t = i_t r_{o1}\left(-g_{m2} - \frac{1}{r_{o2}}\right) + \frac{v_t}{r_{o2}}$$

$$i_t\left(1 + r_{o1} g_{m2} + \frac{r_{o1}}{r_{o2}}\right) r_{o2} = v_t$$

$$R_O = r_{o2} + r_{o1} r_{o2} g_{m2} + r_{o1}$$

$$R_O \approx g_{m2} r_{o1} r_{o2}$$



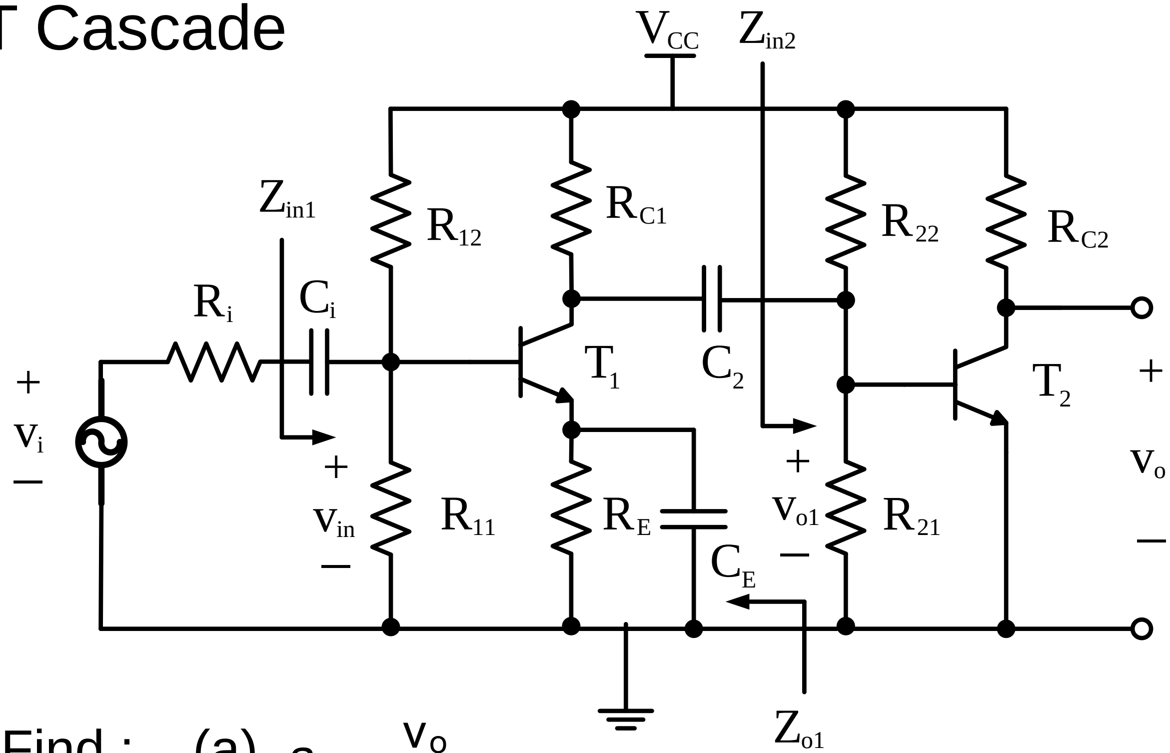
$$R_O \approx g_{m2} r_{o1} r_{o2}$$

This equation shows that the MOS cascode increases the output resistance by a factor of about $g_{m2} r_o$ as the R_O for CS is r_o if R_D is not included.

$$a_{vo} = -G_M R_O$$
$$a_{vo} = -g_{m1} g_{m2} r_{o1} r_{o2}$$
$$a_{vo} = -\left(g_m r_o\right)^2 \leftarrow \text{if } g_{m1} = g_{m2} \text{ and } r_{o1} = r_{o2}$$

The output resistance can further be increased by using more than one level of cascoding. However, number of levels of cascoding is limited by the power supply voltage and signal swing requirements.

BJT Cascade



Given :

$R_{B1} = R_{11}//R_{12} = 5k\Omega$

$R_{B2} = R_{21}//R_{22} = 5k\Omega$

$R_i = 1k\Omega$

$R_{C2} = 20k\Omega$

$R_{C1} = 10k\Omega$

$r_{\pi 1,2} = 1500\Omega$

$r_{o1,2} = 33.333\Omega$

$g_{m1,2} = 26.67mS$

Find :

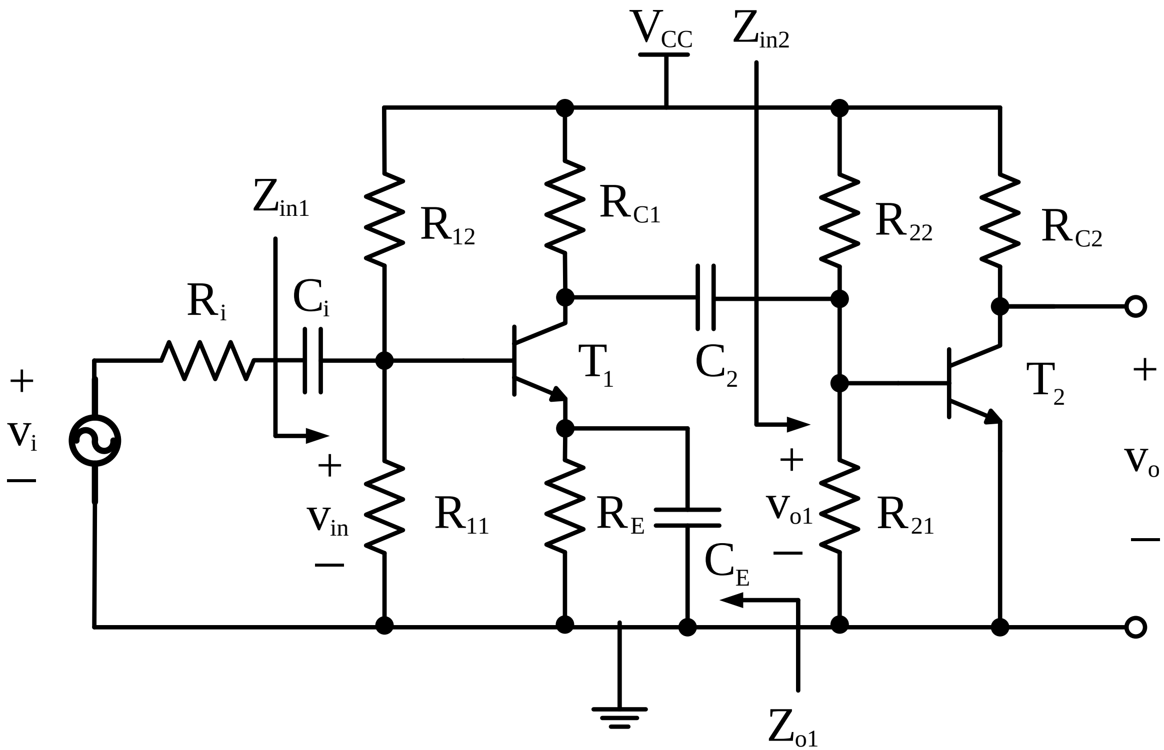
(a) $a_{v2} = \dfrac{v_o}{v_{o1}}$

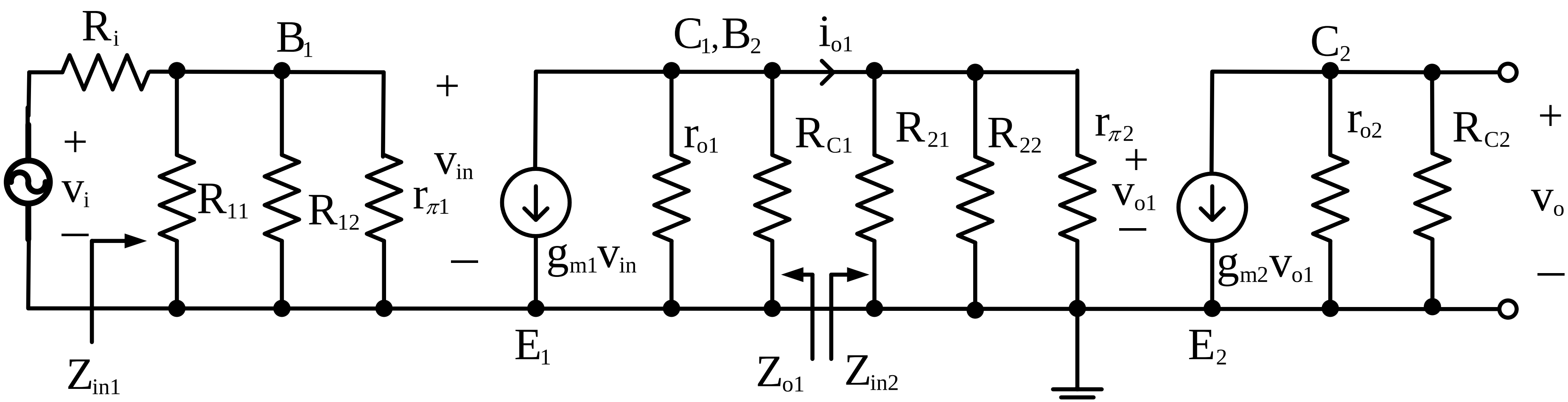
(b) Z_{in2}

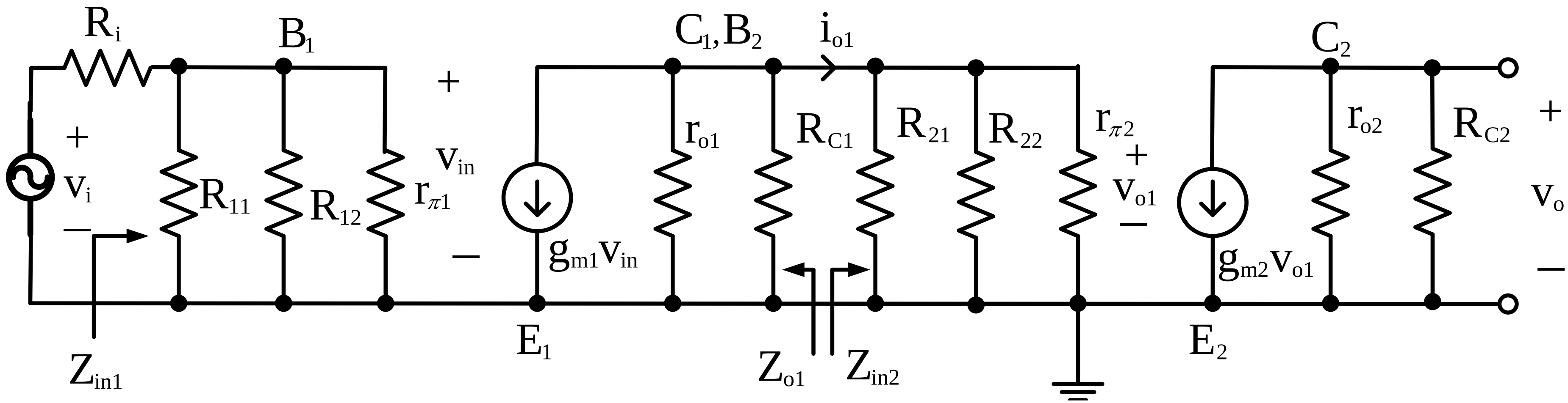
(c) $a_{v1} = \dfrac{v_{o1}}{v_{in}}$

(d) Z_{in1}

(e) $a_v = \dfrac{v_o}{v_i}$


VCC
Zin2
Zin1
R12
RC1
R22
RC2
Ri
Ci
T1
C2
T2
+
vi
–
+
vin
–
R11
RE
CE
+
vo1
–
R21
+
vo
–
Zo1


Ri
B1
C1,B2
io1
C2
+
vi
–
R11
R12
rπ1
+
vin
–
gm1vin
ro1
RC1
R21
R22
rπ2
+
vo1
–
gm2vo1
ro2
RC2
+
vo
–
E1
E2
Zin1
Zo1
Zin2



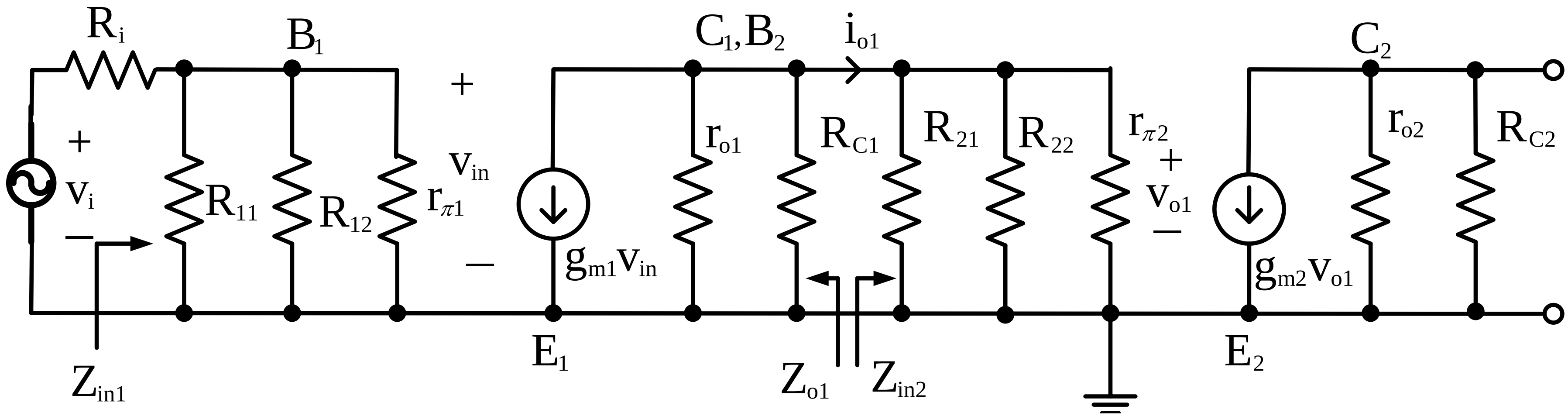
(a) Final stage voltage gain, a_{v2}

$$a_{v2} = \frac{v_o}{v_{o1}}$$

$$a_{v2} = \frac{-g_{m2}v_{o1}\left(r_{o2}//R_{C2}\right)}{v_{o1}}$$

$$a_{v2} = -26.67m\left(33.333k//20k\right)$$

$$a_{v2} = -333.37$$



(b) Final stage input impedance, Z_{in2}

$$Z_{in2} = \frac{v_{o1}}{i_{o1}}$$

$$Z_{in2} = R_{21} // R_{22} // r_{\pi 2}$$

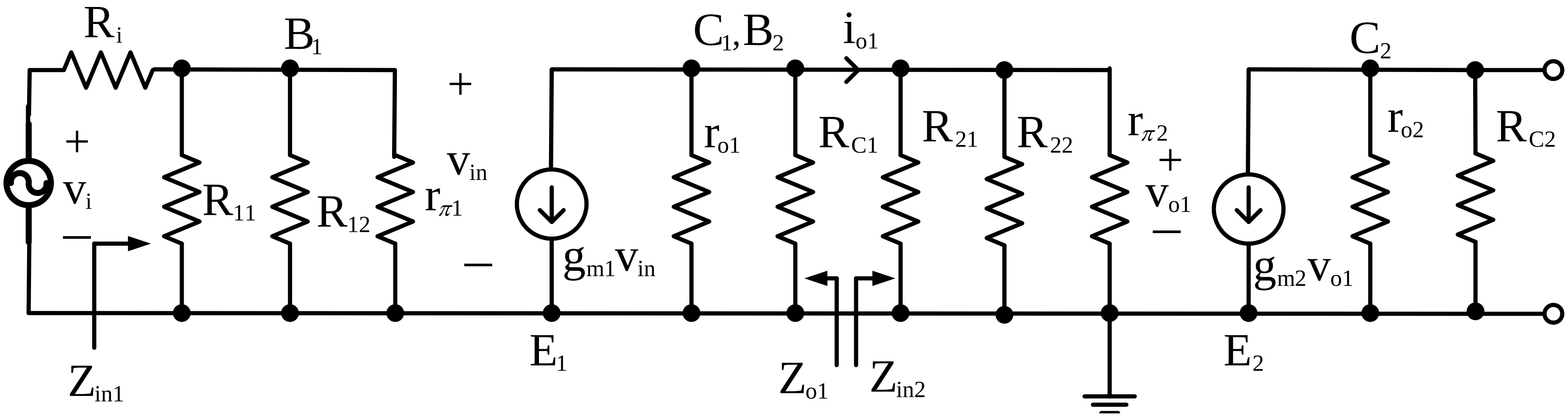
$$Z_{in2} = 5k // 1500 = 1.154k\Omega$$

(c) Initial stage voltage gain, a_{v1}

$$a_{v1} = \frac{v_{o1}}{v_{in}}$$

$$a_{v1} = \frac{-g_{m1} v_{in} \left(r_{o1} // R_{C1} // R_{21} // R_{22} // r_{\pi 2} \right)}{v_{in}}$$

$$a_{v1} = -26.76$$



(d) Amplifier input impedance, Z_{in1}

$$Z_{in1} = \frac{v_{in}}{i_i}$$

$$Z_{in1} = R_{11} // R_{12} // r_{\pi 1}$$

$$Z_{in1} = 5k//1500 = 1.154k\Omega$$

(e)

$$v_{in} = \frac{R_{11} // R_{12} // r_{\pi 1}}{R_{11} // R_{12} // r_{\pi 1} + R_i} v_i$$

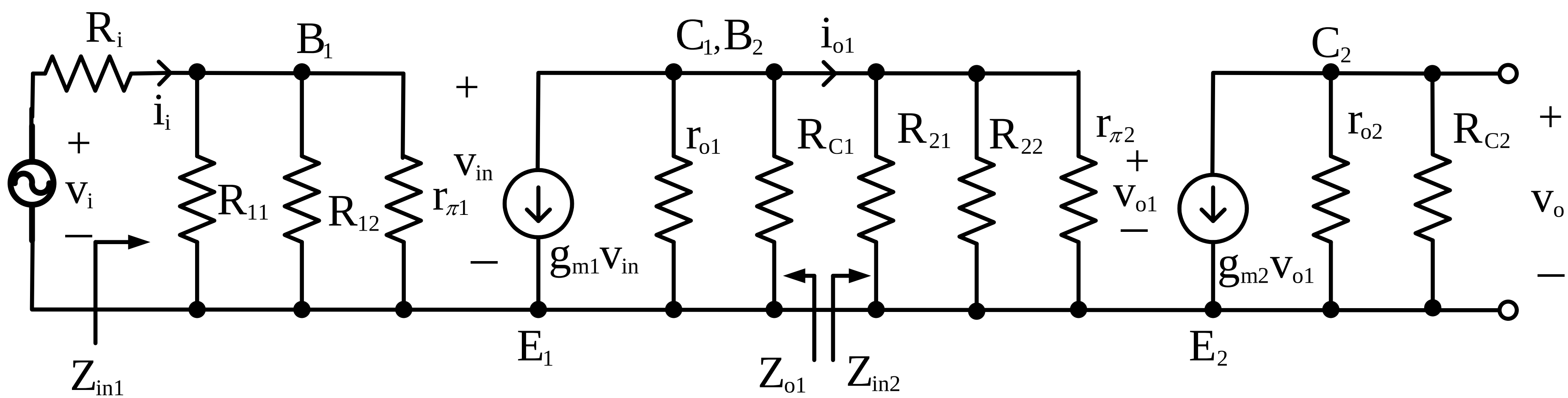
$$\frac{v_{in}}{v_i} = \frac{5k//1500}{5k//1500 + 1k} = 0.5357$$

Overall voltage gain of the cascaded amplifier, a_v

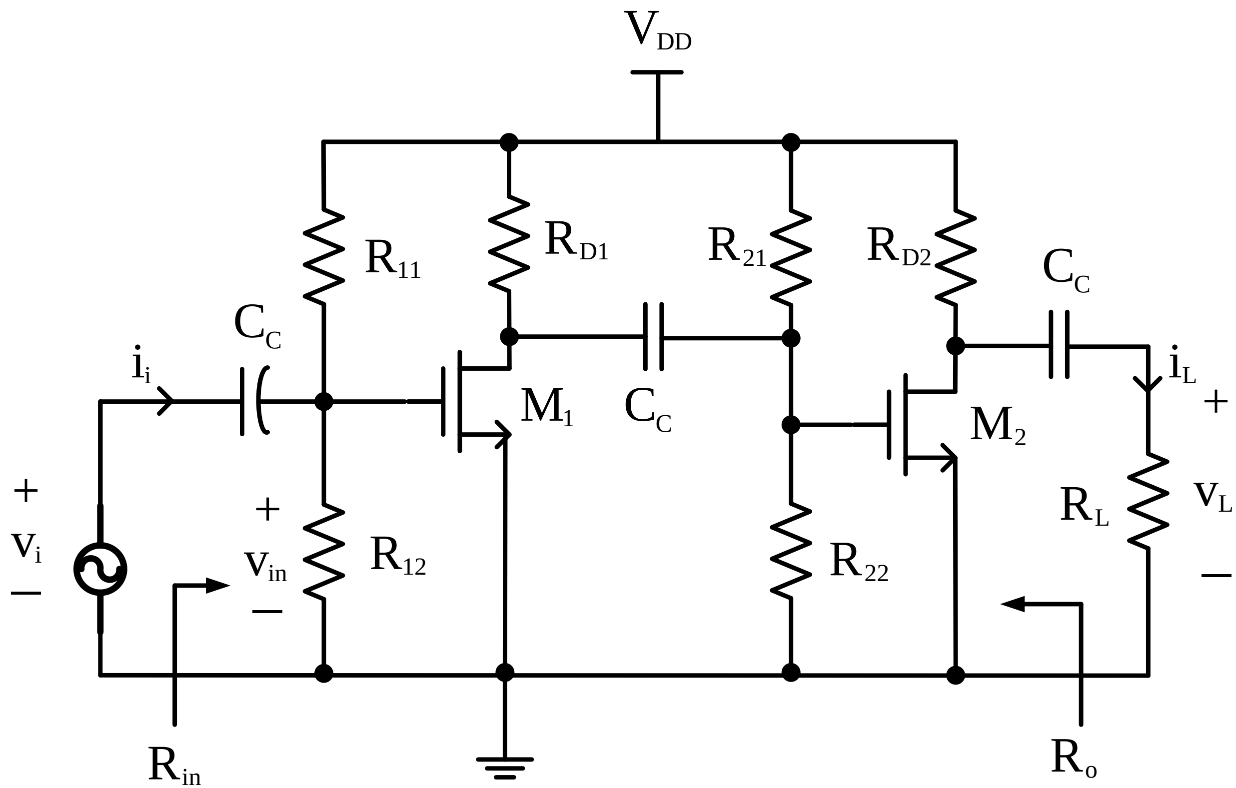
$$a_v = \frac{v_o}{v_i} = \frac{v_{in}}{v_i} \times \frac{v_{o1}}{v_{in}} \times \frac{v_o}{v_{o1}}$$

$$a_v = 0.5357 \times a_{v1} \times a_{v2} = 4778.97$$

The total gain of the cascaded amplifier is the multiplication of the gain at each stage of the cascaded network.

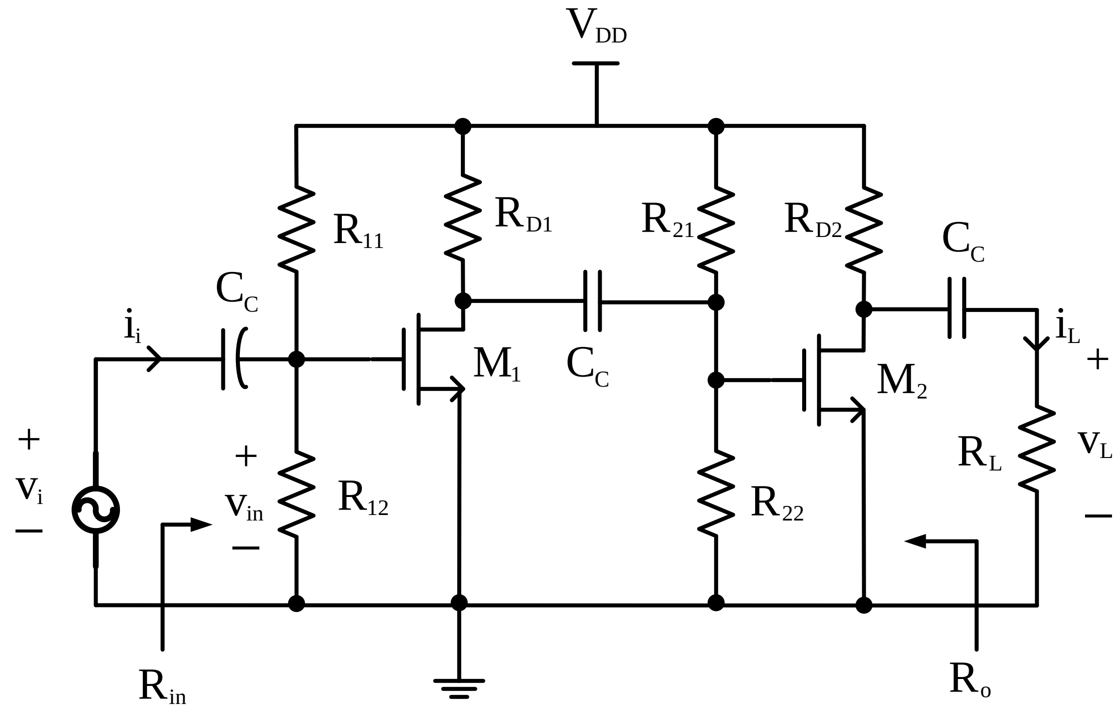


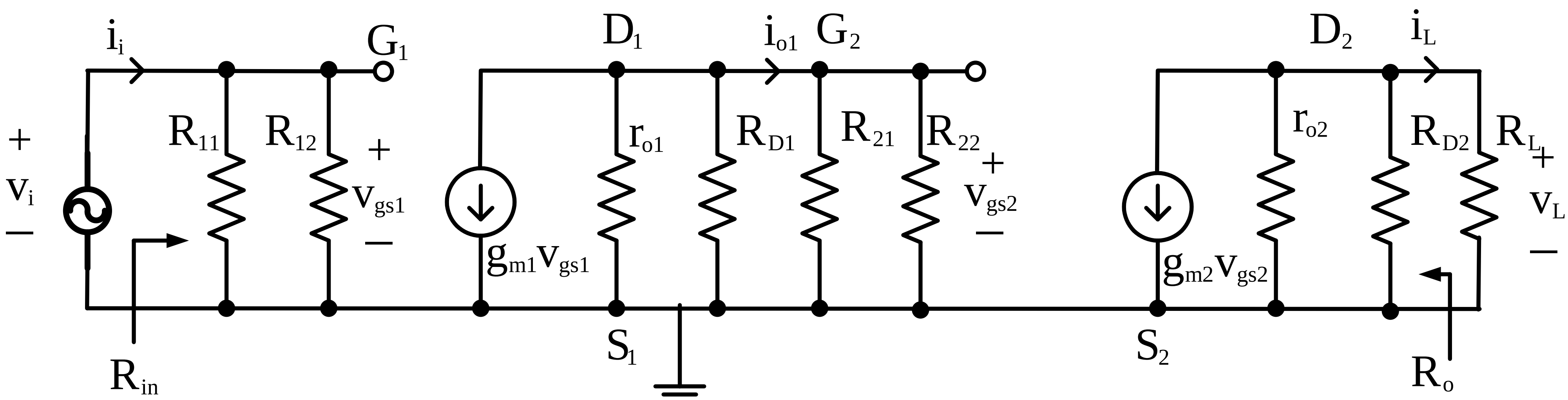
MOSFET Cascade

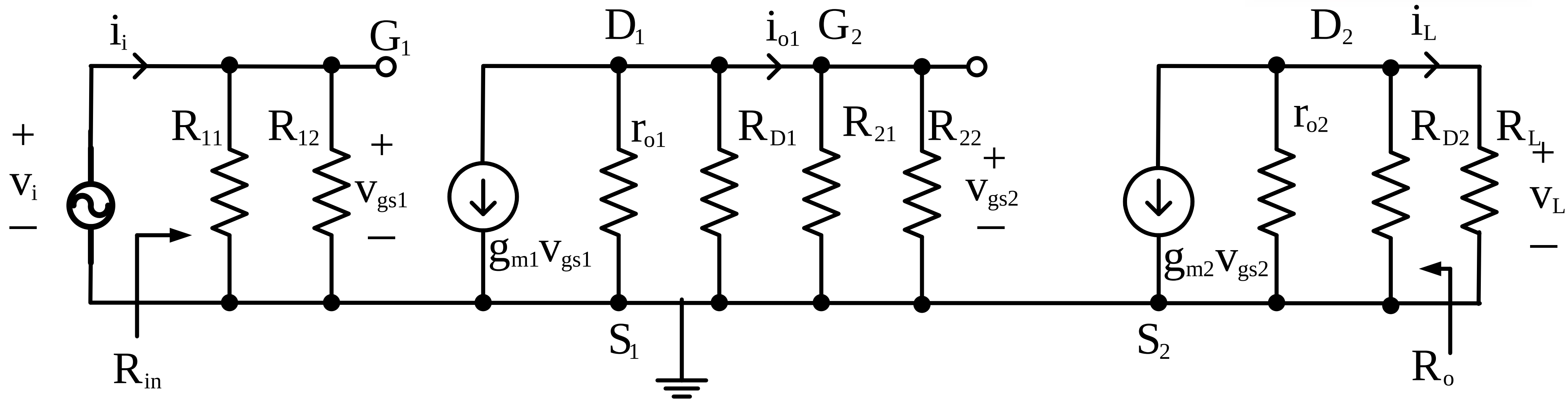


Find (a) Voltage gain ratio $a_v = \frac{v_L}{v_i}$

(b) The current gain ratio $a_i = \frac{i_L}{i_i}$


V_DD
R_11
R_D1
R_21
R_D2
C_C
i_i
M_1
C_C
M_2
i_L
+
+
v_i
–
+
v_in
–
R_12
R_22
R_L
v_L
–
R_in
R_o


i_i
G_1
D_1
i_o1
G_2
D_2
i_L
+
v_i
–
R_11
R_12
+
v_gs1
–
r_o1
R_D1
R_21
R_22
+
v_gs2
–
g_m1v_gs1
r_o2
R_D2
R_L
+
v_L
–
g_m2v_gs2
R_in
S_1
S_2
R_o



(a) Voltage gain ratio

$$a_{v2} = \frac{v_L}{v_{gs2}} = \frac{-g_{m2} v_{gs2} r_{o2} // R_{D2} // R_L}{v_{gs2}}$$

$$a_{v2} = -g_{m2} r_{o2} // R_{D2} // R_L$$

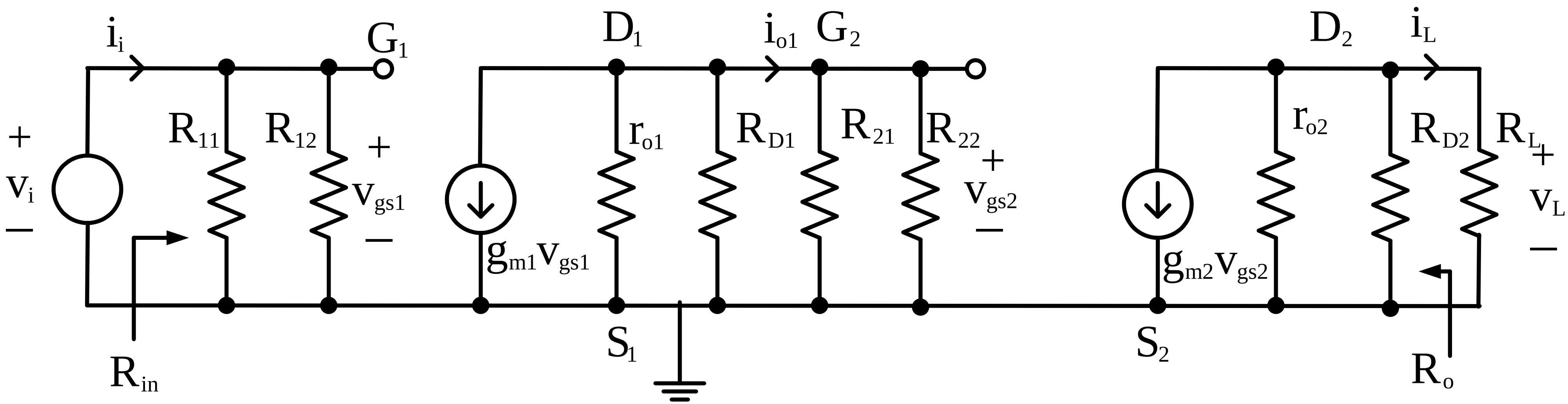
$$a_{v1} = \frac{v_{gs2}}{v_{gs1}} = \frac{-g_{m1} v_{gs1}\, r_{o1} // R_{D1} // R_{21} // R_{22}}{v_{gs1}}$$

$$a_{v1} = -g_{m1} r_{o1} // R_{D1} // R_{21} // R_{22}$$

$$v_{gs1} = v_i$$

$$a_v = \frac{v_L}{v_i} = \frac{v_{gs2}}{v_{gs1}} \times \frac{v_L}{v_{gs2}} \times \frac{v_{gs1}}{v_i}$$

$$a_v = \frac{g_{m1} g_{m2} r_{o1} r_{o2} \left(R_{D1} // R_{21} // R_{22}\right)\left(R_{D2} // R_L\right)}{\left[r_{o1} + \left(R_{D1} // R_{21} // R_{22}\right)\right]\left[r_{o2} + \left(R_{D2} // R_L\right)\right]}$$



(b) The current gain ratio,

$$a_i = \frac{i_L}{i_i} = \frac{v_L\left(R_{11}//R_{12}\right)}{R_L v_i} = \frac{a_v\left(R_{11}//R_{12}\right)}{R_L}$$

$$a_v = \frac{g_{m1}g_{m2}r_{o1}r_{o2}\left(R_{D1}//R_{21}//R_{22}\right)\left(R_{D2}//R_L\right)}{\left[r_{o1}+\left(R_{D1}//R_{21}//R_{22}\right)\right]\left[r_{o2}+\left(R_{D2}//R_L\right)\right]}$$

$$a_i = \frac{g_{m1}g_{m2}r_{o1}r_{o2}\left(R_{D1}//R_{21}//R_{22}\right)\left(R_{D2}//R_L\right)R_{11}//R_{12}}{\left[r_{o1}+\left(R_{D1}//R_{21}//R_{22}\right)\right]\left[r_{o2}+\left(R_{D2}//R_L\right)\right]\left[R_L\right]}$$