### 1.3 Large Signal Operation of BJT (pg. 8)

In this book, the current direction and voltage polarity for the npn and pnp are as shown below:


Biasing current entering the device is considered positive for both npn and pnp.

## Symbol convention

| Condition | Examples |
| :--- | :--- |
| Bias or DC quantities are <br> represented by uppercase <br> symbols with uppercase <br> subscripts. | $\mathrm{I}_{\mathrm{B}}$ and $\mathrm{V}_{\mathrm{BE}}$ |
| Small-signal quantities are <br> represented by lowercase <br> symbols with lowercase <br> subscripts. |  |
| Elements in small-signal <br> circuit are represented by <br> lowercase symbols with <br> lowercase subscripts. | $\mathrm{i}_{\mathrm{b}}$ and $\mathrm{V}_{\mathrm{be}}$ |
| Total quantities, i.e. sum of <br> bias and signal, are <br> represented by uppercase <br> symbols and lowercase <br> subsripts. |  |

### 1.3.1. $\quad$ Large-signal model in the forward active region (pg. 9)




Carrier concentration along the cross-section $\mathrm{AA}^{\prime}$.
Uniform doping densities are assumed. Under this condition, the transistor is called uniform-base transistor.
$\mathrm{n}_{\mathrm{n}}$ and $\mathrm{n}_{\mathrm{p}}=$ electron concentration in the n -type and p -type region, respectively
$\mathrm{p}_{\mathrm{p}}$ and $\mathrm{p}_{\mathrm{n}}=$ hole concentration in the p -type and n -type region, respectively

In the forward active region, BE junction is fb and BC junction is rb.


Minority-carrier concentrations in the B:

$$
\begin{array}{ll}
\text { at } \mathrm{x}=0 ; & \mathrm{n}_{\mathrm{p}}(0)=\mathrm{n}_{\mathrm{p} 0} \exp \left(\mathrm{~V}_{\mathrm{BE}} / \mathrm{V}_{\mathrm{T}}\right) \\
\text { at } \mathrm{x}=\mathrm{W}_{\mathrm{B}} ; & \mathrm{n}_{\mathrm{p}}\left(\mathrm{~W}_{\mathrm{B}}\right)=\mathrm{n}_{\mathrm{p} 0} \exp \left(\mathrm{~V}_{\mathrm{BC}} / \mathrm{V}_{\mathrm{T}}\right) \approx 0 \tag{1.28}
\end{array}
$$

$\mathrm{n}_{\mathrm{p} 0}=$ equilibrium concentration of electrons in the B.

$$
\begin{aligned}
\mathrm{V}_{\mathrm{T}} & =\text { thermal voltage }=\mathrm{kT} / \mathrm{e} \\
& =26 \mathrm{mV} \text { at } 300 \mathrm{~K} .
\end{aligned}
$$

$\mathrm{k}=$ Boltzmann constant $=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$
$\mathrm{T}=$ temperature in Kelvin (K)
$\mathrm{e}=$ electronic charge $=1.6 \times 10^{-9}$ Coulomb $(\mathrm{C})$
Since $B C$ is $r b, V_{B C}$ is negative. In most cases, $V_{B C} \gg V_{T}$. This is the reason why $\mathrm{n}_{\mathrm{p}}\left(\mathrm{W}_{\mathrm{B}}\right) \approx 0$. Equations (1.27) and (1.28) are for low-level injection condition whereby minority-carrier concentrations are always assumed to be << majority-carrier concentrations.


If recombination of holes and electrons in B is small, minority-carrier concentration in $\mathrm{B}, \mathrm{n}_{\mathrm{p}}(\mathrm{x})$, varies linearly with distance.

For charge neutrality in B,
$\mathrm{N}_{\mathrm{A}}+\mathrm{n}_{\mathrm{p}}(\mathrm{x})=\mathrm{p}_{\mathrm{p}}(\mathrm{x})$
$\mathrm{p}_{\mathrm{p}}(\mathrm{x})-\mathrm{n}_{\mathrm{p}}(\mathrm{x})=\mathrm{N}_{\mathrm{A}}$
$\mathrm{p}_{\mathrm{p}}(\mathrm{x})=$ hole concentration in B
$\mathrm{N}_{\mathrm{A}}=\mathrm{B}$ doping density that is assumed constant
(1.30) shows that the hole and electron concentrations are separated by a constant. Hence, if $\mathrm{n}_{\mathrm{p}}(\mathrm{x})$ varies linearly with distance, $\mathrm{p}_{\mathrm{p}}(\mathrm{x})$ will also behave the same way.


Collector current is produced by minority-carriers (electrons) in B diffusing in the direction of the concentration gradient and being swept across the CB depletion region by the field existing there.

Diffusion current density due to electrons in B :
$\mathrm{J}_{\mathrm{n}}=\mathrm{q} \mathrm{D}_{\mathrm{n}}\left[\mathrm{dn}_{\mathrm{p}}(\mathrm{x}) / \mathrm{dx}\right]$
$\mathrm{D}_{\mathrm{n}}=$ diffusion constant for electrons
$\mathrm{J}_{\mathrm{n}}=\mathrm{q} \mathrm{D}_{\mathrm{n}}\left[\mathrm{n}_{\mathrm{p}}\left(\mathrm{W}_{\mathrm{B}}\right)-\mathrm{n}_{\mathrm{p}}(0)\right] /\left[\mathrm{W}_{\mathrm{B}}-0\right]$
$\mathrm{J}_{\mathrm{n}}=-\mathrm{q} \mathrm{D}_{\mathrm{n}} \mathrm{n}_{\mathrm{p}}(0) / \mathrm{W}_{\mathrm{B}} \quad$ as $\quad \mathrm{n}_{\mathrm{p}}\left(\mathrm{W}_{\mathrm{B}}\right) \approx 0$
Since $J_{n}=I_{C} / A$ where $A=$ area, then
$\mathrm{I}_{\mathrm{C}}=\mathrm{q} \mathrm{AD}_{\mathrm{n}} \mathrm{n}_{\mathrm{p}}(0) / \mathrm{W}_{\mathrm{B}}$
$\mathrm{I}_{\mathrm{C}}$ is positive as the collector current flows into the collector.


From equation (1.27),

$$
\begin{aligned}
& \mathrm{n}_{\mathrm{p}}(0)=\mathrm{n}_{\mathrm{p} 0} \exp \left(\mathrm{~V}_{\mathrm{BE}} / \mathrm{V}_{\mathrm{T}}\right) \\
& \mathrm{I}_{\mathrm{C}}=\mathrm{qADD} \mathrm{D}_{\mathrm{n}}(0) / \mathrm{W}_{\mathrm{B}} \\
& \mathrm{I}_{\mathrm{C}}=\mathrm{qADD} \mathrm{D}_{\mathrm{n} 0} \exp \left(\mathrm{~V}_{\mathrm{BE}} / \mathrm{V}_{\mathrm{T}}\right) / \mathrm{W}_{\mathrm{B}} \\
& \mathrm{I}_{\mathrm{C}}=\mathrm{I}_{\mathrm{S}} \exp \left(\mathrm{~V}_{\mathrm{BE}} / \mathrm{V}_{\mathrm{T}}\right) \\
& \mathrm{I}_{\mathrm{S}}=\mathrm{qAD} \mathrm{D}_{\mathrm{n}} \mathrm{n}_{\mathrm{p} 0} / \mathrm{W}_{\mathrm{B}}
\end{aligned}
$$

$\mathrm{I}_{\mathrm{S}}$ is a constant used to describe the transfer characteristic of the transistor in the forward active region.

$$
\begin{aligned}
& \mathrm{n}_{\mathrm{p} 0}=\mathrm{n}_{\mathrm{i}}^{2} / \mathrm{N}_{\mathrm{A}} \\
& \mathrm{n}_{\mathrm{i}}=\text { carrier concentration in intrinsic } \mathrm{Si} \\
& \quad \approx 1.5 \times 10^{16} \text { carriers } / \mathrm{m}^{3} \\
& \mathrm{I}_{\mathrm{S}}=\mathrm{q} \mathrm{~A} \mathrm{D}_{\mathrm{n}} \mathrm{n}_{\mathrm{i}}^{2} /\left(\mathrm{N}_{\mathrm{A}} \mathrm{~W}_{\mathrm{B}}\right) \text { typically } 10^{-14} \text { to } 10^{-16} \mathrm{~A} .
\end{aligned}
$$



Base current, $\mathrm{I}_{\mathrm{B}}$, consists of two major components, $\mathrm{I}_{\mathrm{B} 1}$ and $\mathrm{I}_{\mathrm{B} 2}$.
$\mathrm{I}_{\mathrm{B} 1}=$ recombination of holes and electrons in B.
$\mathrm{I}_{\mathrm{B} 1} \propto$ minority-carrier charge, $\mathrm{Q}_{\mathrm{e}}$, in B.
$\mathrm{Q}_{\mathrm{e}}=(1 / 2) \mathrm{n}_{\mathrm{p}}(0) \mathrm{W}_{\mathrm{B}} \mathrm{qA}$
$\mathrm{I}_{\mathrm{B} 1}=\mathrm{Q}_{\mathrm{e}} / \tau_{\mathrm{b}}=(1 / 2) \mathrm{n}_{\mathrm{p}}(0) \mathrm{W}_{\mathrm{B}} \mathrm{qA} / \tau_{\mathrm{b}}$
$\tau_{\mathrm{b}}=$ minority-carrier lifetime in B
$\mathrm{I}_{\mathrm{B} 1}$ represents a flow of majority holes from the B lead into the B region.

Since $n_{p}(0)=n_{p 0} \exp \left(V_{B E} / V_{T}\right)$,
then $\mathrm{I}_{\mathrm{B} 1}=\frac{1}{2} \frac{\mathrm{~W}_{\mathrm{B}} \mathrm{qAn}_{\mathrm{p} 0}}{\tau_{\mathrm{b}}} \exp \frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{V}_{\mathrm{T}}}$.

The dominant current component of $\mathrm{I}_{\mathrm{B}}$ is $\mathrm{I}_{\mathrm{B} 2} . \mathrm{I}_{\mathrm{B} 2}$ is due to the injection of holes from $B$ into $E$. $I_{B 2}$ depends on the gradient of minority-carrier holes in E .

$\mathrm{I}_{\mathrm{B} 2}=\mathrm{qAD}_{\mathrm{p}} \mathrm{p}_{\mathrm{nE}}(0) / \mathrm{L}_{\mathrm{p}}$
$D_{p}=$ diffusion constant for holes
$L_{p}=$ diffusion length (assumed small) for holes in the $E$
$\mathrm{p}_{\mathrm{nE}}(0)=$ concentration of holes in the E at the edge of the BE depletion region.
$\mathrm{p}_{\mathrm{nE}}(0)=\mathrm{p}_{\mathrm{nE} 0} \exp \left(\mathrm{~V}_{\mathrm{BE}} / \mathrm{V}_{\mathrm{T}}\right)$
$\mathrm{p}_{\mathrm{nE} 0}=\mathrm{n}_{\mathrm{i}}^{2} / \mathrm{N}_{\mathrm{D}}$
$\mathrm{N}_{\mathrm{D}}=$ donor atom concentration in E.

E is deliberately doped much more heavily than the B , making $\mathrm{N}_{\mathrm{D}}$ large and $\mathrm{p}_{\mathrm{nE}}$ small, so that $\mathrm{I}_{\mathrm{B} 2}$ is minimized.

$$
\mathrm{I}_{\mathrm{B} 2}=\left(\mathrm{qAD}_{\mathrm{p}} / \mathrm{L}_{\mathrm{p}}\right)\left(\mathrm{n}_{\mathrm{i}}^{2} / \mathrm{N}_{\mathrm{D}}\right) \exp \left(\mathrm{V}_{\mathrm{BE}} / \mathrm{V}_{\mathrm{T}}\right)
$$

Total B current:
$I_{B}=I_{B 1}+I_{B 2}=\frac{1}{2} \frac{W_{B} q A n_{p 0}}{\tau_{b}} \exp \frac{V_{B E}}{V_{T}}+\frac{q A D_{p}}{L_{p}} n_{i}^{2}{ }_{N_{D}} \exp \frac{V_{B E}}{V_{T}}$

$$
\mathrm{I}_{\mathrm{B}}=\left(\frac{1}{2} \frac{\mathrm{~W}_{\mathrm{B}} q A n_{\mathrm{p} 0}}{\tau_{\mathrm{b}}}+\frac{q A D_{\mathrm{p}}}{L_{\mathrm{p}}} \frac{n_{i}^{2}}{\mathrm{~N}_{\mathrm{D}}}\right) \exp \frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{T}}}
$$

Forward current gain:

$$
\begin{aligned}
\beta_{\mathrm{F}}= & \frac{\mathrm{I}_{\mathrm{C}}}{\mathrm{I}_{\mathrm{B}}}
\end{aligned}=\frac{\frac{\mathrm{qAD}_{\mathrm{n}} \mathrm{n}_{\mathrm{p} 0}}{W_{\mathrm{B}}}}{\left(\frac{1}{2} \frac{\mathrm{~W}_{\mathrm{B}} \mathrm{AAn}_{\mathrm{p} 0}}{\tau_{\mathrm{b}}}+\frac{\mathrm{qAD}_{\mathrm{p}}}{\mathrm{~L}_{\mathrm{p}}} \frac{n_{\mathrm{i}}^{2}}{\mathrm{~N}_{\mathrm{D}}}\right)}
$$

$\beta_{\mathrm{F}}$ is maximized by minimizing base width $\mathrm{W}_{\mathrm{B}}$ and maximizing the ratio of E to B doping densities, $\mathrm{N}_{\mathrm{D}} / \mathrm{N}_{\mathrm{A}}$ i.e. $\mathrm{N}_{\mathrm{D}} \gg \mathrm{N}_{\mathrm{A}}$.

Typical $\beta_{\mathrm{F}}$ is 50 to 500 for npn and for pnp is 10 to 100 .


$$
\begin{aligned}
& \mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}+\mathrm{I}_{\mathrm{E}}=0 \\
& \mathrm{I}_{\mathrm{E}}=-\left(\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}}\right) \\
& \mathrm{I}_{\mathrm{B}}=\mathrm{I}_{\mathrm{C}} / \beta_{\mathrm{F}} \\
& \mathrm{I}_{\mathrm{E}}=-\left[\left(\mathrm{I}_{\mathrm{C}} / \beta_{\mathrm{F}}\right)+\mathrm{I}_{\mathrm{C}}\right]=-\mathrm{I}_{\mathrm{C}}\left[1+\left(1 / \beta_{\mathrm{F}}\right)\right]=-\mathrm{I}_{\mathrm{C}}\left[\left(1+\beta_{\mathrm{F}}\right) / \beta_{\mathrm{F}}\right] \\
& \alpha_{\mathrm{F}}=\beta_{\mathrm{F}} /\left(1+\beta_{\mathrm{F}}\right) \\
& \mathrm{I}_{\mathrm{E}}=-\mathrm{I}_{\mathrm{C}} / \alpha_{\mathrm{F}}
\end{aligned}
$$

$$
\begin{align*}
& \alpha_{\mathrm{F}}=\beta_{\mathrm{F}} /\left(1+\beta_{\mathrm{F}}\right)=1 /\left[1+\left(1 / \beta_{\mathrm{F}}\right)\right] \\
& \beta_{\mathrm{F}}=\frac{1}{\left(\frac{\mathrm{~W}_{\mathrm{B}}^{2}}{2 \tau_{\mathrm{b}} \mathrm{D}_{\mathrm{n}}}+\frac{\mathrm{D}_{\mathrm{p}}}{\mathrm{D}_{\mathrm{n}}} \frac{\mathrm{~W}_{\mathrm{B}}}{\mathrm{~L}_{\mathrm{p}}} \frac{\mathrm{~N}_{\mathrm{A}}}{\mathrm{~N}_{\mathrm{D}}}\right)} \\
& \alpha_{\mathrm{F}}=\frac{1}{1+\left(\frac{\mathrm{W}_{\mathrm{B}}{ }^{2}}{2 \tau_{\mathrm{b}} \mathrm{D}_{\mathrm{n}}}+\frac{\mathrm{D}_{\mathrm{p}}}{\mathrm{D}_{\mathrm{n}}} \frac{\mathrm{~W}_{\mathrm{B}}}{\mathrm{~L}_{\mathrm{p}}} \frac{\mathrm{~N}_{\mathrm{A}}}{\mathrm{~N}_{\mathrm{D}}}\right)} \approx \alpha_{\mathrm{T}} \gamma \tag{1.51}
\end{align*}
$$

where,
$\alpha_{T}=$ base transport factor $=\frac{1}{1+\frac{W_{B}{ }^{2}}{2 \tau_{b} D_{n}}}$
$\gamma=$ emitter injection efficiency $=\frac{1}{1+\frac{D_{p}}{D_{n}} \frac{W_{B}}{L_{p}} \frac{N_{A}}{N_{D}}}$
(1.51) is valid if $\frac{W_{B}{ }^{2}}{2 \tau_{\mathrm{b}} \mathrm{D}_{\mathrm{n}}} \ll 1$ and $\frac{\mathrm{D}_{\mathrm{p}}}{\mathrm{D}_{\mathrm{n}}} \frac{W_{\mathrm{B}}}{L_{\mathrm{p}}} \frac{\mathrm{N}_{\mathrm{A}}}{N_{\mathrm{D}}} \ll 1$. These
conditions happen when $\beta_{\mathrm{F}}$ is large.

$$
\alpha_{\mathrm{F}}=\frac{1}{\frac{1}{\alpha_{\mathrm{T}}}+\frac{1}{\gamma}-1}=\frac{\gamma \alpha_{\mathrm{T}}}{\gamma+\alpha_{\mathrm{T}}-\gamma \alpha_{\mathrm{T}}}
$$

$$
\alpha_{\mathrm{F}}=\frac{1}{\frac{1}{\alpha_{\mathrm{T}}}+\frac{1}{\gamma}-1}=\frac{\gamma \alpha_{\mathrm{T}}}{\gamma+\alpha_{\mathrm{T}}-\gamma \alpha_{\mathrm{T}}}
$$

If $\gamma \rightarrow 1$ and $\alpha_{T} \rightarrow 1, \alpha_{F}=\alpha_{T} \gamma$

$\gamma=\frac{\mathrm{I}_{\mathrm{n}(\mathrm{E} \rightarrow \mathrm{B})}}{\mathrm{I}_{\mathrm{n}(\mathrm{E} \rightarrow \mathrm{B})}+\mathrm{I}_{\mathrm{p}(\mathrm{B} \rightarrow \mathrm{E})}}$
$\mathrm{I}_{\mathrm{n}(\mathrm{E} \rightarrow \mathrm{B})}=$ current contributed by the flow of electrons from E to B.
$\mathrm{I}_{\mathrm{p}(\mathrm{B} \rightarrow \mathrm{E})}=$ current contributed by the flow of holes from B to E.

Ideally, $\gamma \approx 1$. From the expression above, this condition is achieved when $\mathrm{I}_{\mathrm{p}(\mathrm{B} \rightarrow \mathrm{E})} \ll \mathrm{I}_{\mathrm{n}(\mathrm{E} \rightarrow \mathrm{B})}$.

As $\gamma=\frac{1}{1+\frac{D_{\mathrm{p}}}{\mathrm{D}_{\mathrm{n}}} \frac{\mathrm{W}_{\mathrm{B}}}{L_{\mathrm{p}}} \frac{\mathrm{N}_{\mathrm{A}}}{\mathrm{N}_{\mathrm{D}}}}, \gamma \approx 1$ can also be achieved by making
$\mathrm{N}_{\mathrm{A}} / \mathrm{N}_{\mathrm{D}} \ll 1\left(\right.$ or $\left.\mathrm{N}_{\mathrm{D}} \gg \mathrm{N}_{\mathrm{A}}\right)$ and $\mathrm{W}_{\mathrm{B}}$ small.

$\alpha_{\mathrm{T}}$ represents the fraction of carriers injected into $B$ from $E$ that reaches C .

$$
\alpha_{\mathrm{T}}=\frac{\mathrm{I}_{\mathrm{n}(\mathrm{~B} \rightarrow \mathrm{C})}}{\mathrm{I}_{\mathrm{n}(\mathrm{E} \rightarrow \mathrm{~B})}}
$$

where $\mathrm{I}_{\mathrm{n}(\mathrm{B} \rightarrow \mathrm{C})}$ is the current contributed by the amout of electrons swept from B to C.


$$
\alpha_{\mathrm{T}}=\frac{\mathrm{I}_{\mathrm{n}(\mathrm{~B} \rightarrow \mathrm{C})}}{\mathrm{I}_{\mathrm{n}(\mathrm{E} \rightarrow \mathrm{~B})}}
$$

Ideally, $\alpha_{T} \approx 1$ and this happens when $\mathrm{I}_{\mathrm{n}(\mathrm{B} \rightarrow \mathrm{C})} \approx \mathrm{I}_{\mathrm{n}(\mathrm{E} \rightarrow \mathrm{B})}$. This can happen when $W_{B}$ is small as can be seen from

$$
\alpha_{\mathrm{T}}=\frac{1}{1+\frac{\mathrm{W}_{\mathrm{B}}^{2}}{2 \tau_{\mathrm{b}} \mathrm{D}_{\mathrm{n}}}} .
$$

## Large-signal models of npn transistors for use in bias

## calculations.



Collector voltage ideally has no influence on the collector current and the collector node acts as a high impedance current source.

In the forward-active region, $\mathrm{V}_{\mathrm{BE}}$ varies very little because of the steep slope of the exponential characteristic.

### 1.3.2 Effects of the collector voltage on large-signal characteristics in the forward active region.

The rb CB junction has ideally no effect on the collector current. This approximation is useful for $1^{\text {st }}$ order calculation, but not true in practice. There are occasions when the influence of the collector voltage on the collector current is important.

Collector voltage has dramatic effect on the collector current in 2 regions of device operation:

1. saturation region $\left(\mathrm{V}_{\mathrm{CE}} \rightarrow 0\right)$
2. breakdown region ( $\mathrm{V}_{\mathrm{CE}}$ very large)

Between these two extremes, $\mathrm{I}_{\mathrm{C}}$ increases slowly with the increase of $\mathrm{V}_{\mathrm{CE}}$.


If $\mathrm{V}_{\mathrm{BE}}$ is constant, the change in $\mathrm{V}_{\mathrm{CB}}$ equals the change in $\mathrm{V}_{\mathrm{CE}}$. This causes an increase in the CB depletion layer width.

The change in the B width $=\Delta \mathrm{W}_{\mathrm{B}}=$ change in depletion layer width.

The change in width causes the change in the collector current as much as $\Delta \mathrm{I}_{\mathrm{C}}$.

$$
\mathrm{I}_{\mathrm{C}}=\frac{\mathrm{qAD}_{\mathrm{n}} \mathrm{n}_{\mathrm{p} 0}}{\mathrm{~W}_{\mathrm{B}}} \exp \frac{\mathrm{~V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{T}}}
$$

From this equation, $\mathrm{I}_{\mathrm{C}}$ will increase when $\mathrm{W}_{\mathrm{B}}$ decreases at a fixed $V_{B E}$.

$$
\begin{aligned}
\mathrm{I}_{\mathrm{C}} & =\frac{\mathrm{qAD}_{\mathrm{n}} \mathrm{n}_{\mathrm{p} 0}}{\mathrm{~W}_{\mathrm{B}}} \exp \frac{\mathrm{~V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{T}}} \\
& =\frac{\mathrm{qAD}_{\mathrm{n}}}{\mathrm{~W}_{\mathrm{B}}} \frac{\mathrm{n}_{\mathrm{i}}^{2}}{N_{\mathrm{A}}} \exp \frac{\mathrm{~V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{T}}} \\
& =\frac{q A D_{\mathrm{n}} n_{i}^{2}}{\mathrm{Q}_{\mathrm{B}}} \exp \frac{\mathrm{~V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{T}}}
\end{aligned}
$$

where $\mathrm{Q}_{\mathrm{B}}=\mathrm{W}_{\mathrm{B}} \mathrm{N}_{\mathrm{A}}=$ no of doping atoms in B per unit area of E
$\frac{\partial \mathrm{I}_{\mathrm{C}}}{\partial \mathrm{Q}_{\mathrm{B}}}=-\frac{\mathrm{qAD}_{\mathrm{n}} \mathrm{n}_{\mathrm{i}}{ }^{2}}{\mathrm{Q}_{\mathrm{B}}{ }^{2}} \exp \frac{\mathrm{~V}_{\mathrm{BE}}}{\mathrm{V}_{\mathrm{T}}}$

$$
\begin{align*}
& \frac{\partial I_{C}}{\partial V_{C E}} \frac{d V_{C E}}{d Q_{B}}=-\frac{q A D_{n} n_{i}^{2}}{Q_{B}^{2}} \exp \frac{V_{B E}}{V_{T}} \\
& \frac{\partial I_{C}}{\partial V_{C E}}=-\frac{I_{C}}{Q_{B}} \frac{d Q_{B}}{d V_{C E}} \\
& \frac{\partial I_{C}}{\partial V_{C E}}=-\frac{I_{C}}{W_{B} N_{A}} \frac{d W_{B} N_{A}}{d V_{C E}} \\
& =-\frac{I_{C}}{W_{B}} \frac{d W_{B}}{d V_{C E}} \tag{1.55}
\end{align*}
$$

Since $W_{B}$ decreases when $V_{C E}$ increases, then $\frac{d W_{B}}{d V_{C E}}$ is negative. Hence, $\frac{\partial I_{C}}{\partial V_{C E}}$ is positive, i.e. when $V_{C E}$ increases $I_{C}$ will also increase.
$\frac{\partial \mathrm{I}_{\mathrm{C}}}{\partial \mathrm{V}_{\mathrm{CE}}}=-\frac{\mathrm{I}_{\mathrm{C}}}{\mathrm{W}_{\mathrm{B}}} \frac{d \mathrm{~W}_{\mathrm{B}}}{\mathrm{dV}_{\mathrm{CE}}}$
$\frac{\partial \mathrm{I}_{\mathrm{C}}}{\partial \mathrm{V}_{\mathrm{CE}}} \propto \mathrm{I}_{\mathrm{C}}$ or $\frac{\partial \mathrm{I}_{\mathrm{C}}}{\partial \mathrm{V}_{\mathrm{CE}}} \propto \mathrm{I}_{\mathrm{C}}$
Narrow-base transistors show a greater dependence of $I_{C}$ on $\mathrm{V}_{\mathrm{CE}}$ in the forward-active region.


Early voltage,

$$
\begin{equation*}
\mathrm{V}_{\mathrm{A}}=\frac{\mathrm{I}_{\mathrm{C}}}{\frac{\partial \mathrm{I}_{\mathrm{C}}}{\partial \mathrm{~V}_{\mathrm{CE}}}}=-\mathrm{W}_{\mathrm{B}} \frac{\mathrm{dV}_{\mathrm{CE}}}{\mathrm{dW}_{\mathrm{B}}} . \tag{1.57}
\end{equation*}
$$

Hence, $\mathrm{V}_{\mathrm{A}}$ independent of $\mathrm{I}_{\mathrm{C}}$.
Variation of $I_{C}$ with $V_{C E}$ is called the Early effect. $V_{A}$ is typically 15 to 100 V .

The influence of the Early effect on the transistor largesignal characteristics in the forward-active region is shown by the following equation:

$$
\mathrm{I}_{\mathrm{C}}=\mathrm{I}_{\mathrm{S}}\left(1+\mathrm{V}_{\mathrm{CE}} / \mathrm{V}_{\mathrm{A}}\right) \exp \left(\mathrm{V}_{\mathrm{BE}} / \mathrm{V}_{\mathrm{T}}\right)
$$

When no Early effect considered:
$\mathrm{I}_{\mathrm{C}}=\mathrm{I}_{\mathrm{S}} \exp \left(\mathrm{V}_{\mathrm{BE}} / \mathrm{V}_{\mathrm{T}}\right)$

