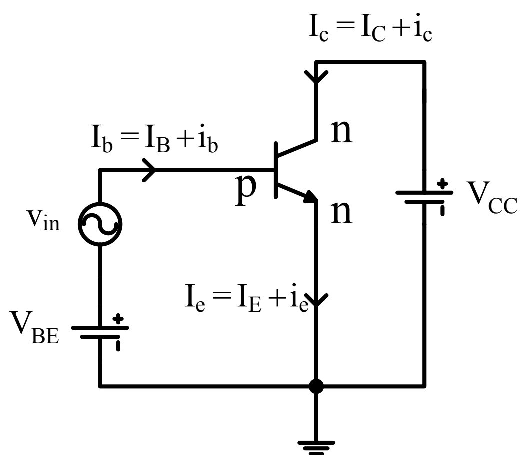
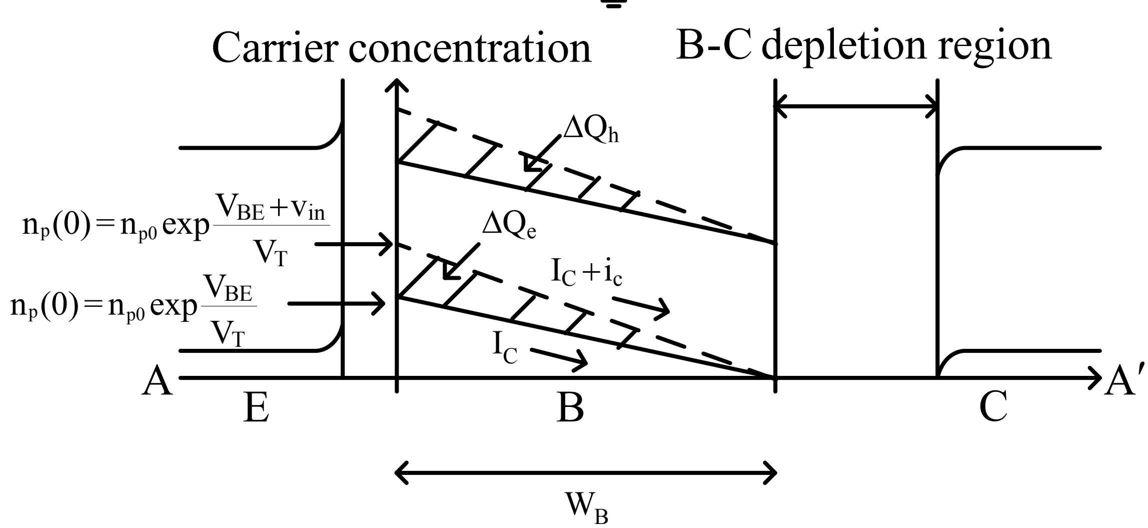
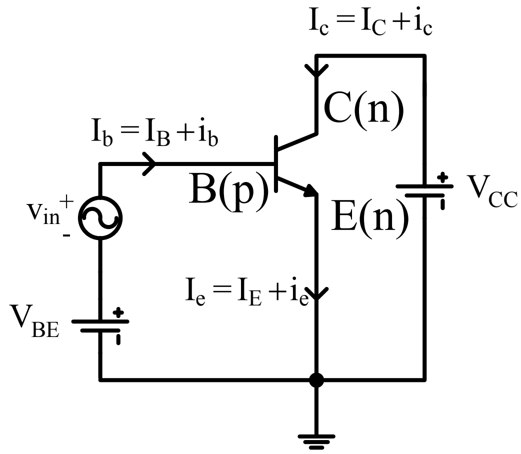
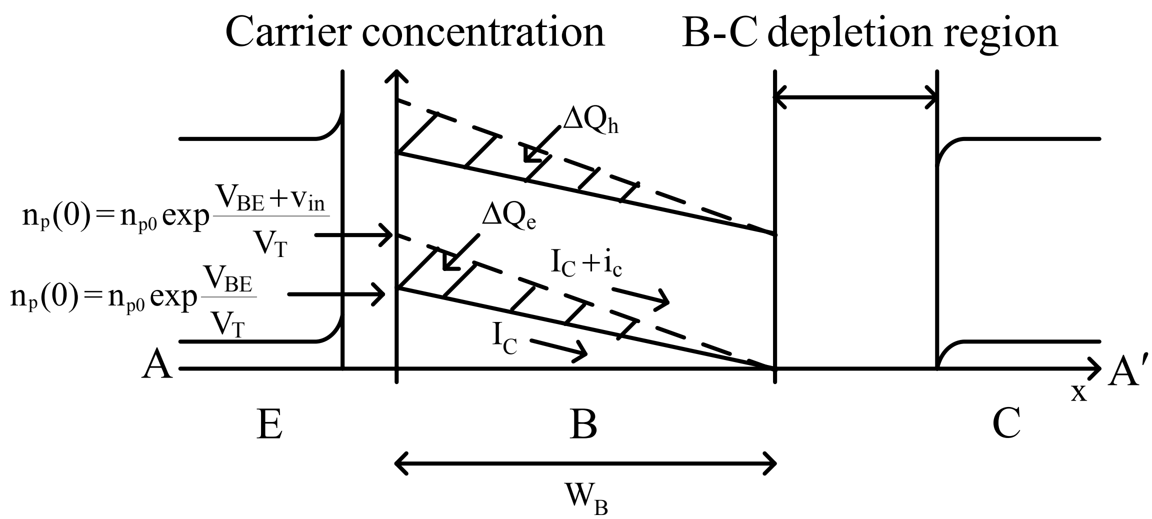
1.4 Small-signal models of BJT

Analog circuits often operate with signal levels that are small compared to the bias currents and voltages in the circuit. Under this condition, incremental or small-signal models can be derived that allow calculation of circuit gain and terminal impedances without the necessity of including the bias quantities.









I_C = quiescent C current

I_B = quiescent B current

v_i = small-signal i/p voltage

Total B current, $I_b = I_B + i_b$

Total C current, $I_c = I_C + i_c$

ΔQ_e = change in the minority carrier charge

ΔQ_h = change in the majority carrier charge

1.4.1 Transconductance (pg. 27)

$g_m = dI_C / dV_{BE}$

$\Delta I_C / \Delta V_{BE} = dI_C / dV_{BE}$

$g_m = \Delta I_C / \Delta V_{BE} = i_c / v_i$ (1.90)

$$g_m = \frac{dI_C}{dV_{BE}}$$

$$g_m = \frac{d}{dV_{BE}} I_S \exp\frac{V_{BE}}{V_T} = \frac{I_S}{V_T}\exp\frac{V_{BE}}{V_T} = \frac{I_C}{V_T}$$

$g_m = \frac{qI_C}{kT}$, i.e. g_m depends linearly on I_C.

If I_C = 1 mA and at 25°C,

$$g_m = \frac{1.6\times10^{-19}(C)\times1\times10^{-3}(A)}{1.38\times10^{-23}(J/K)\times298(K)} = 38.91\,mA/V$$

irrespective of the type of the BJT (npn or pnp), size and made of material (Si, Ge, GaAs).

$$I_c = I_S \exp\frac{V_{BE}+v_i}{V_T} = I_S \exp\frac{V_{BE}}{V_T}\exp\frac{v_i}{V_T}$$

Since $I_C = I_S \exp\frac{V_{BE}}{V_T}$,

then $I_c = I_C \exp\frac{v_i}{V_T}$ (1.94)

If $v_i < V_T$, equation (1.94) can be expanded in a power series,

$$I_c = I_C\left[1+\frac{v_i}{V_T}+\frac{1}{2}\left(\frac{v_i}{V_T}\right)^2+\frac{1}{6}\left(\frac{v_i}{V_T}\right)^3+......\right]$$

$i_c = I_c - I_C$

$$i_c = \frac{I_C v_i}{V_T}+\frac{I_C}{2}\left(\frac{v_i}{V_T}\right)^2+\frac{I_C}{6}\left(\frac{v_i}{V_T}\right)^3+......$$

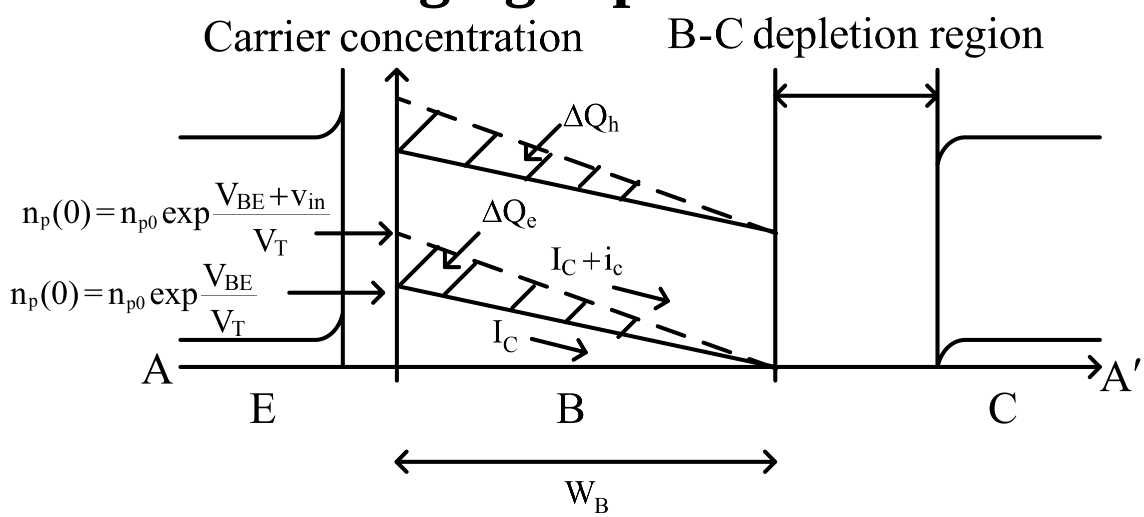
If $v_i << V_T$, $i_c = \frac{I_C v_i}{V_T} = g_m v_i$ which is the same as (1.90).

The small-signal analysis is valid.

The criterion for use of small-signal analysis is $v_i = \Delta V_{BE} << 26$ mV at 25°C.

In practice, if $\Delta V_{BE} < 10$ mV, the small-signal analysis is accurate within ≈ 10%.

1.4.2 Base-charging capacitance



The charge in BE voltage, $\Delta V_{BE} = v_i$, has caused a change in the minority-carrier charge in the B, i.e. $\Delta Q_e = q_e$. By charge neutrality requirements, there is an equal change, $\Delta Q_h = q_h$, in the majority carrier charge in the B. Since majority carriers are applied by the B lead, the application of voltage v_i requires the supply of charge q_h to the B. Due to this, the device has an input capacitance:

$$C_b = q_h / v_i \quad (1.98)$$

Minority-carrier charge in the B,

$$Q_e = \frac{1}{2} n_p(0) W_B q A \tag{1.39}$$

$$I_C = qAD_n \frac{n_p(0)}{W_B} \tag{1.33}$$

$$\frac{Q_e}{I_C} = \frac{\frac{1}{2} W_B{}^2}{D_n} = \tau_F \tag{1.99}$$

τ_F = B transit time in the forward direction
= average time per carrier spent in crossing the B

Typically, τ_F = 10 $\rightarrow$ 500 ps for npn
= 1 $\rightarrow$ 40 ns for pnp

From (1.99), $Q_e = \tau_F I_C$

$$\Delta Q_e = \Delta Q_n = q_n = \tau_F \Delta I_C = \tau_F i_c \tag{1.102}$$

From equations (1.98) i.e. $C_b = q_h / v_i$, and (1.102), i.e. $q_n = \tau_F i_c$,

$C_b = \tau_F i_c / v_i$.

From equation (1.90), i.e. $g_m = i_c / v_i$,

$i_c = g_m \tau_F i_c / C_b$

$$C_b = \tau_F g_m = \tau_F \frac{I_C}{V_T} = \tau_F \frac{qI_C}{kT}$$

Therefore, $C_b \propto I_C$.

1.4.3 Input resistance, r_π (pg. 29)

$I_B = I_C / \beta_F$

$\Delta I_B / \Delta I_C = dI_B / dI_C = \dfrac{d}{dI_C} \dfrac{I_C}{\beta_F}$

$$\Delta I_B = \frac{d}{dI_C} \frac{I_C}{\beta_F} \Delta I_C$$

Small-signal current gain,

$$\beta_o = \frac{\Delta I_C}{\Delta I_B} = \frac{i_c}{i_b} = \left(\frac{d}{dI_C} \frac{I_C}{\beta_F} \right)^{-1}$$

If β_F is constant, $\beta_o = \beta_F$. Typically, $\beta_o \approx \beta_F$. A single value of β is often assumed for a transistor and used for both ac and dc calculations.

Small-signal input resistance,

$r_\pi = v_i / i_b = v_i \beta_o / i_c$.

Since $g_m = i_c / v_i$, $r_\pi = \beta_o / g_m = \beta_o V_T / I_C$

Hence, $r_\pi \propto 1/ I_C$.

1.4.4 Output resistance, r_o (pg. 29)

Small changes ΔV_{CE} in V_{CE} produce corresponding changes ΔI_C in I_C, where

$$\Delta I_C = \frac{\partial I_C}{\partial V_{CE}} \Delta V_{CE}$$

$$\frac{\Delta V_{CE}}{\Delta I_C} = \frac{\partial V_{CE}}{\partial I_C}$$

From equation (1.55), $\frac{\partial I_C}{\partial V_{CE}} = -\frac{I_C}{W_B}\frac{dW_B}{dV_{CE}}$

From equation (1.57), $V_A = -W_B \frac{dV_{CE}}{dW_B}$

$$\frac{\partial I_C}{\partial V_{CE}} = \frac{I_C}{V_A}$$

Hence, $\frac{\Delta V_{CE}}{\Delta I_C} = \frac{V_A}{I_C} = r_o$

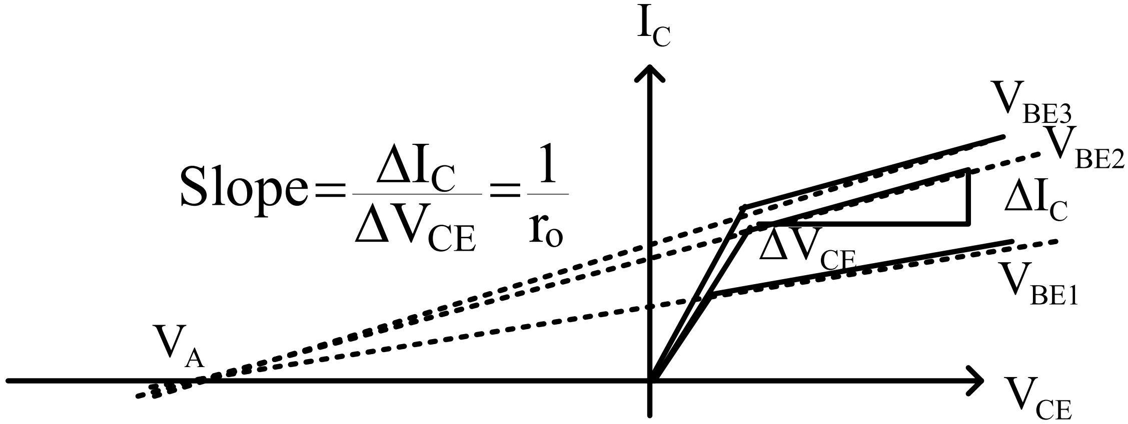
V_A = Early voltage = 50→100 V.

r_o = small-signal output resistance

For I_C = 1 mA, r_o = 50→100 kΩ.

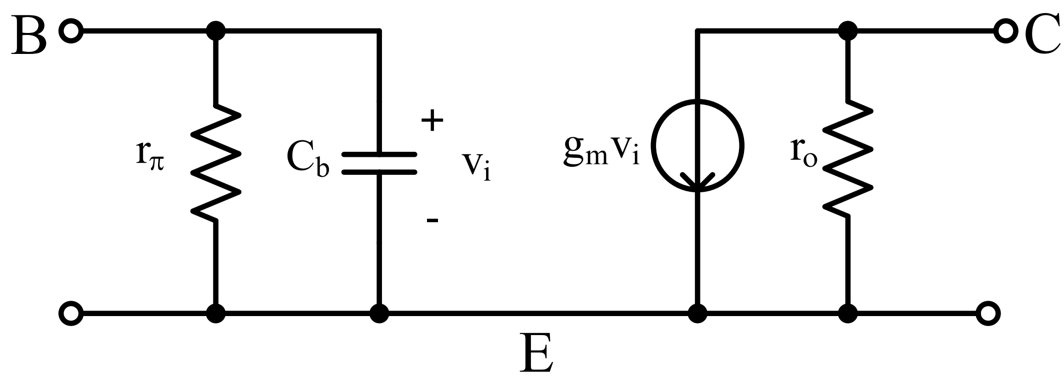
$r_o \propto 1/I_C$ and $g_m = I_C / V_T$

$$r_o = \frac{V_A}{g_m V_T} = \frac{qV_A}{g_m kT} = \frac{1}{g_m \frac{kT}{qV_A}} = \frac{1}{g_m \eta}$$



1.4.5 Basic small-signal model of the BJT

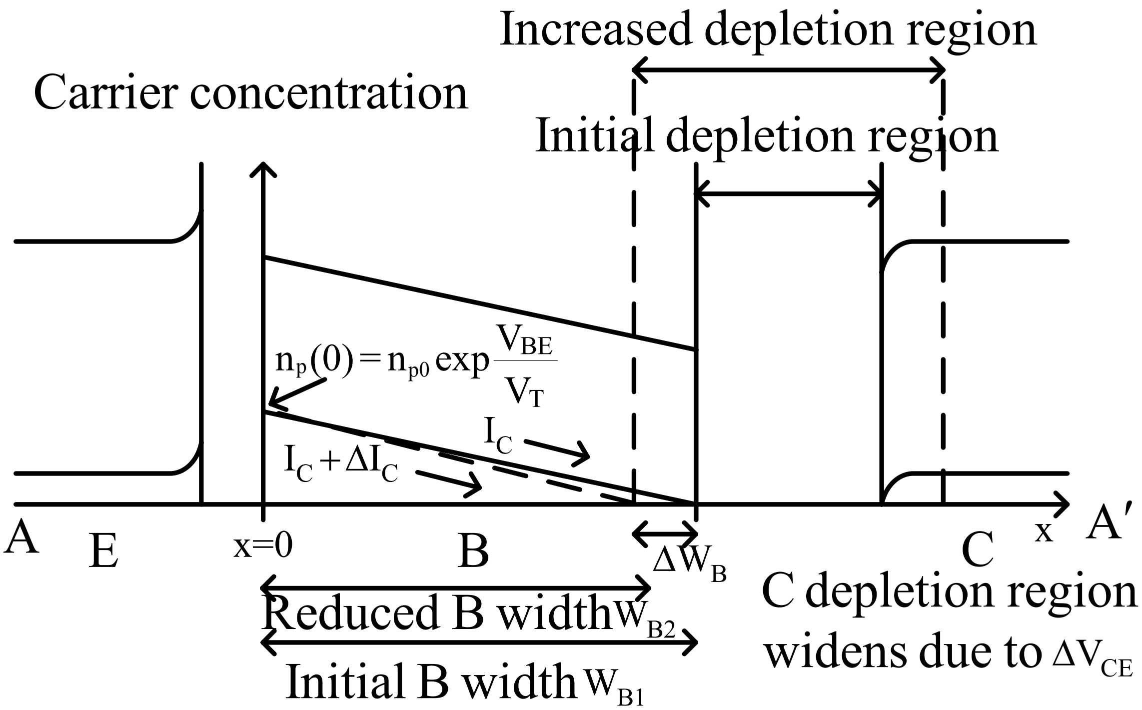
Hybrid-π model:



Valid for both npn and pnp in the forward active region.

Base-charging capacitance, $C_b = \tau_F \, g_m$
Input resistance, $r_\pi = \beta_o / g_m$
Output resistance, $r_o = 1/\eta g_m$
Transconductance, $g_m = I_C / V_T$

1.4.6 C-B resistance



$V_{CE} \uparrow$ C-B depletion-layer width $\uparrow$

Base width (W_B) $\downarrow$

Total minority-carrier charge in B $\downarrow$.

$I_{B1} \downarrow$ (as I_{B1} represents recombination of electron and holes in B)

$I_B \downarrow$

Since an increase ΔV_{CE} in V_{CE} causes a decrease ΔI_B in I_B, this effect can be modeled by the inclusion of a resistor r_μ from C to B. If V_{BE} is assumed constant,

$$r_\mu = \frac{\Delta V_{CE}}{\Delta I_{B1}} = \frac{\Delta V_{CE}}{\Delta I_C}\frac{\Delta I_C}{\Delta I_{B1}} = r_o \frac{\Delta I_C}{\Delta I_{B1}}$$

If $I_B = I_{B1}$,

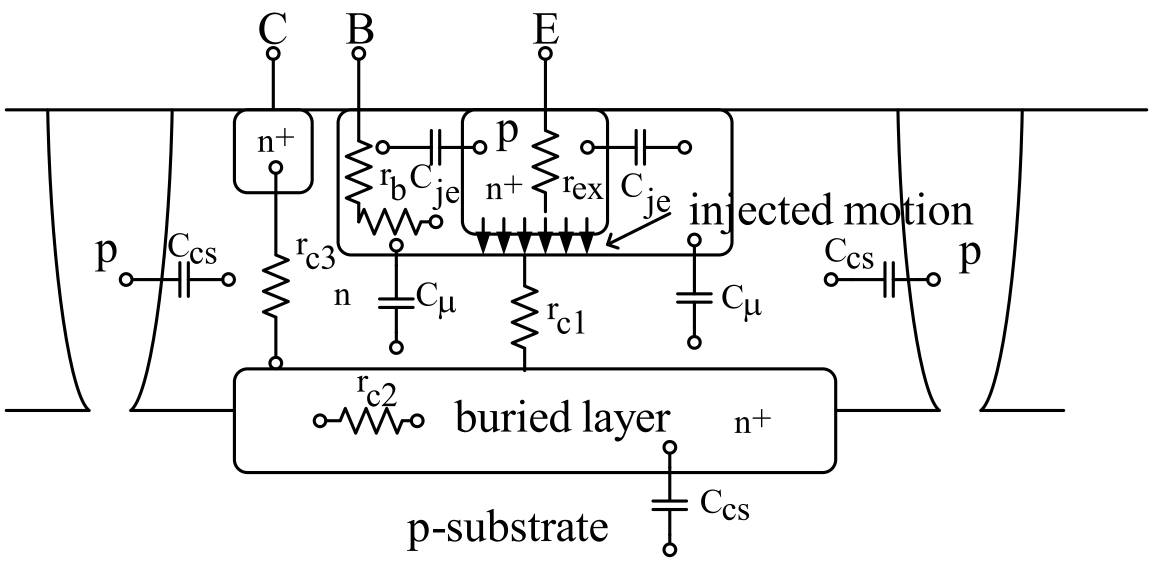
$$r_\mu = r_o \frac{\Delta I_C}{\Delta I_B} = r_o \frac{i_c}{i_b} = r_o \beta_o$$

This is a lower limit for r_μ as I_{B1} under this condition is at its maximum value I_B. In practice, $I_{B1} = 10\% I_B$ (as I_{B2} is the one that dominates). Since I_{B1} is very small, the change ΔI_{B1} in I_{B1} for a given ΔV_{CE} and ΔI_C is also very small.

$$r_\mu = r_o \frac{i_c}{0.1 i_b} = 10\beta_o r_o$$

1.4.7 Parasitic elements in the small-signal model

Technological limitations in the fabrication of transistors give rise to a number of parasitic elements that must be added to the equivalent circuit.
Cross-section of a typical npn transistor:



All pn junctions have a voltage-dependence capacitance associated with the depletion region.
C_{je} = depletion region capacitance at BE junction
C_μ = BC junction capacitance
C_{cs} = C-substrate junction capacitance