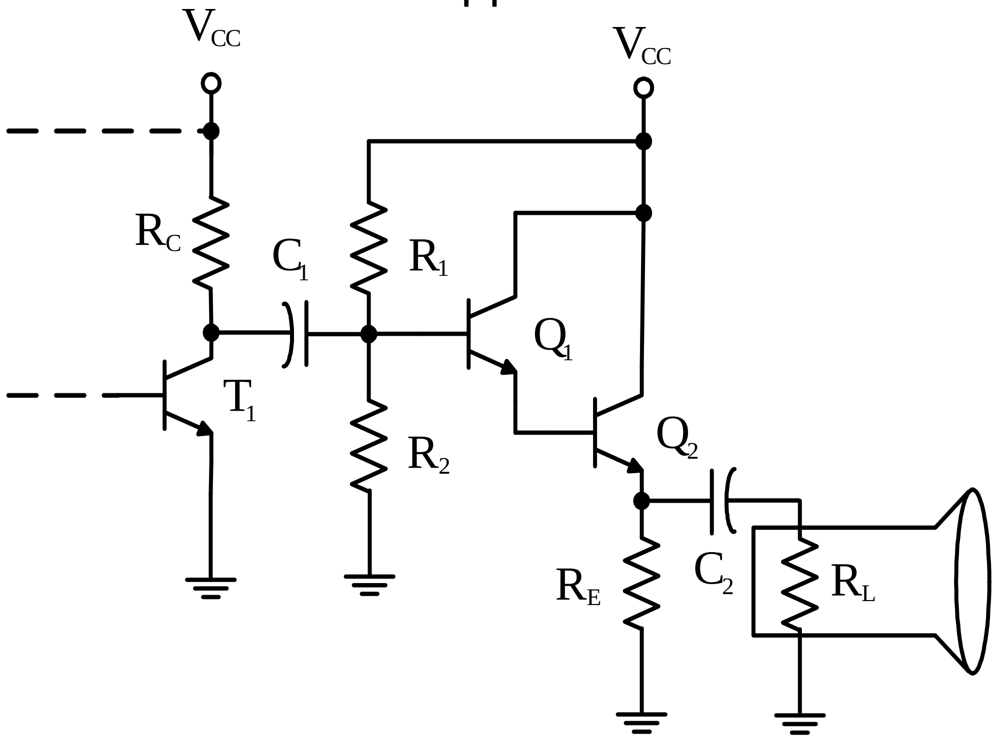
ASSIGNMENT 1

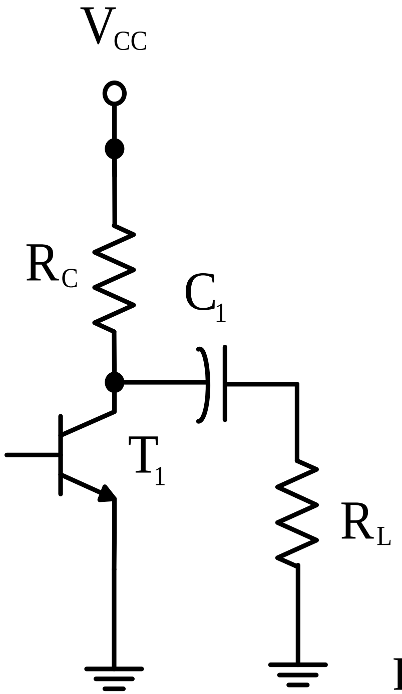
(1) Darlington emitter-follower application

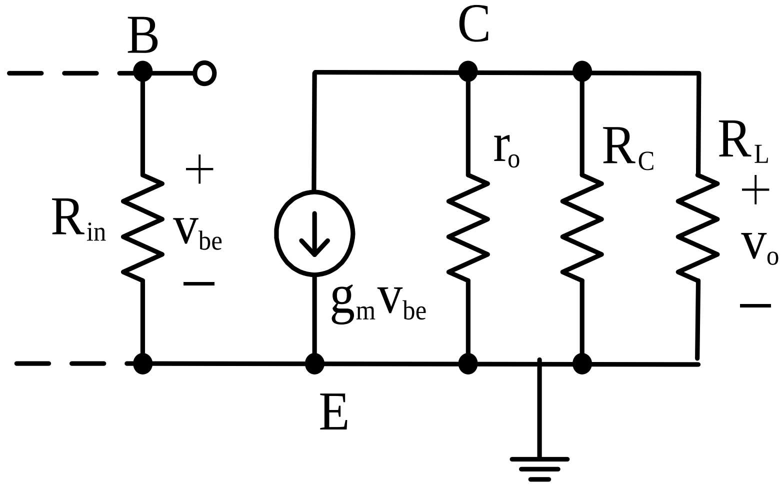


The emitter follower is often used as an interface between a circuit with a high output resistance and a low resistance load. In such an application, the emitter follower is called a buffer.

For example, suppose a common-emitter amplifier with a 1kΩ collector resistance (output resistance) must drive a low resistance load such as an 8Ω low power speaker. C1 and C2 are coupling capacitors

Without the Darlington emitter follower :





$$R_O = r_o // R_C // R_L$$
$$a_v = -g_m \left(r_o // R_C // R_L \right)$$
$$\text{Since } r_o >> R_C,\ R_O \approx R_C // R_L$$

(a) When there is no, i.e. $R_L = \infty$

$\mathbf{a_v = -g_m R_C = -0.2(1k) = -200}$ (this is the open-circuit voltage gain)

(b) When the load R_L is connected, $R_O \approx R_C // R_L$

$a_v = -1.6$

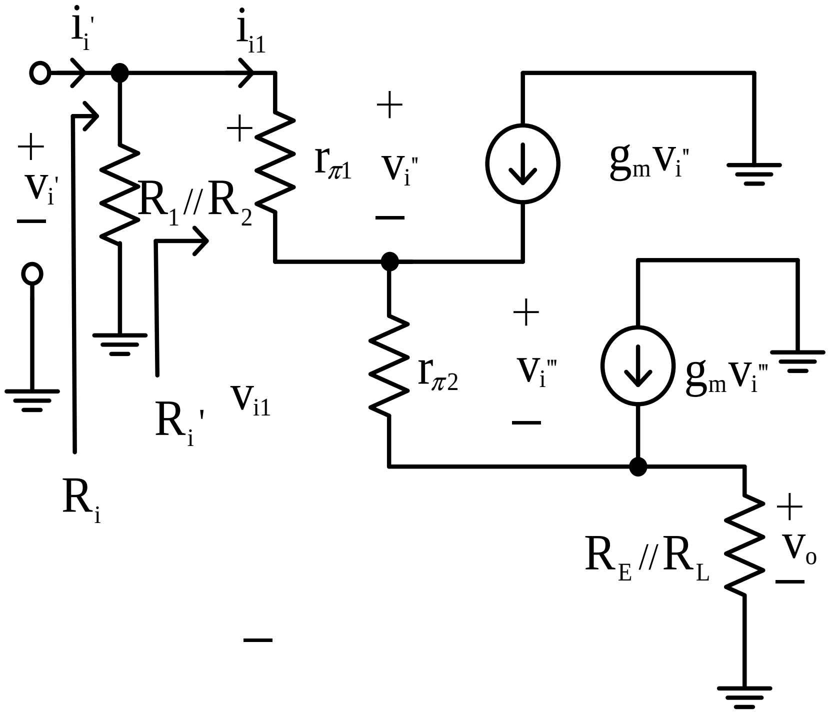
(c) The open-circuit voltage gain is -200 and the gain of the CE circuit when a load of 8 Ω is connected at the output is only -1.6.

Comments:

(i) This shows that the open-circuit voltage gain is the maximum gain that can be achieved by the amplifier circuit.

(ii) When a load with a much smaller resistance (as compared to the output resistance of the amplifier) is connected to the output of the amplifier circuit, the gain will tremendously reduced.

Hence, an intermediate circuit had to be placed between the CE amplifier and the load to prevent the small resistance load from affecting the CE gain. For this purpose, the Darlington emitter-follower is used to act as an impedance transformer. The Darlington emitter-follower has a large input resistance but a small output resistance to enable it to function as an impedance transformer.



(d) Determine the voltage gain of the Darlington emitter-follower

$$v_{i1} = i_{i1} r_{\pi1} + v_i''' + v_o$$

$$v_{i1} = i_{i1} r_{\pi1} + \left(i_{i1} + g_{m1} i_{i1} r_{\pi1}\right) r_{\pi2} + \left[\left(i_{i1} + g_{m1} i_{i1} r_{\pi1}\right) + g_{m2} v_i'''\right] R_E//R_L$$

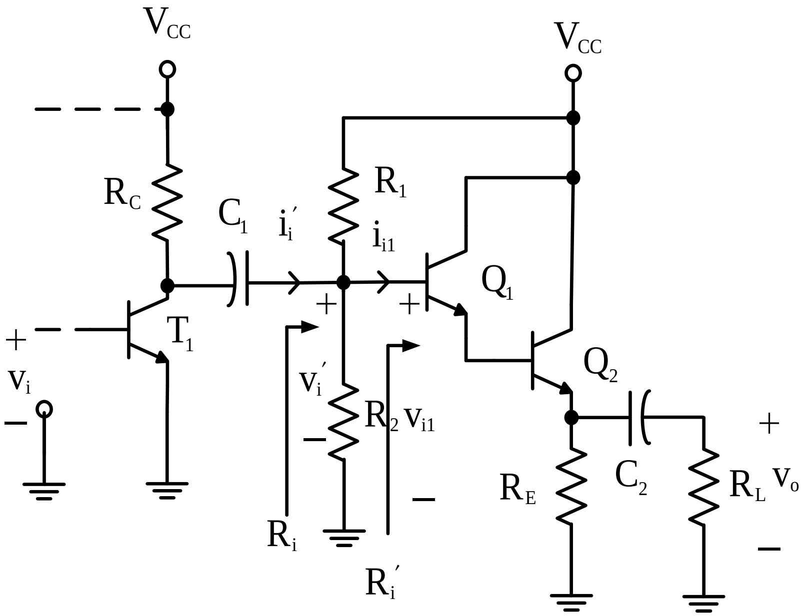
$$v_{i1} = i_{i1}\left[r_{\pi1} + \left(1 + g_{m1} r_{\pi1}\right) r_{\pi2}\right] + i_{i1}\left[\left(1 + g_{m1} r_{\pi1}\right) + g_{m2}\left(1 + g_{m1} r_{\pi1}\right) r_{\pi2}\right] R_E//R_L$$

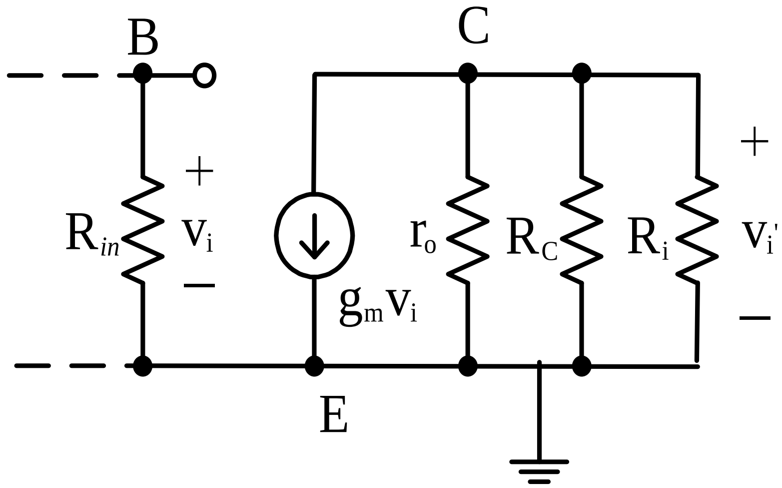
$$a_{v(\text{Darlington CC})} = \frac{v_o}{v_i'} = \frac{v_o}{v_{i1}}$$

$$\therefore a_{v(\text{Darlington CC})} = \frac{\left[\left(1 + g_{m1} r_{\pi1}\right) + g_{m2} r_{\pi2}\left(1 + g_{m1} r_{\pi1}\right)\right] R_E//R_L}{\left[r_{\pi1} + \left(1 + g_{m1} r_{\pi1}\right) r_{\pi2}\right] + \left[\left(1 + g_{m1} r_{\pi1}\right) + g_{m2} r_{\pi2}\left(1 + g_{m1} r_{\pi1}\right)\right] R_E//R_L}$$

$$= 0.9715$$

(e) Determine the a_v for the whole circuit:





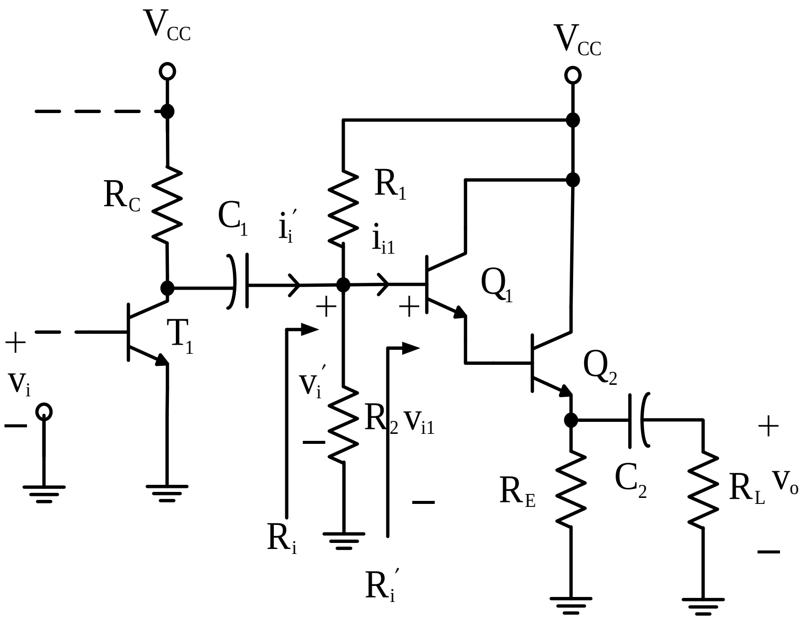
a_v is the gain of the CE with the darlington and R_L as load.

$$a_v = \frac{-g_m v_i r_o // R_C // R_i}{v_i}$$

$$a_v = -g_m r_o // R_C // R_i$$

Assuming that the effects of r_o are negligible, $a_v = -g_m R_C // R_i$

To determine a_v , we need to determine R_i .



For all transistors,

$\beta_F = \beta_o = 100$

$g_m = 0.2\,S$

$V_T = 26\,mV$

$V_{BE} = 0.7\,V$

Given :

$V_{CC} = 12V$

$R_C = 1k\Omega$

For the darlington emitter follower circuit :

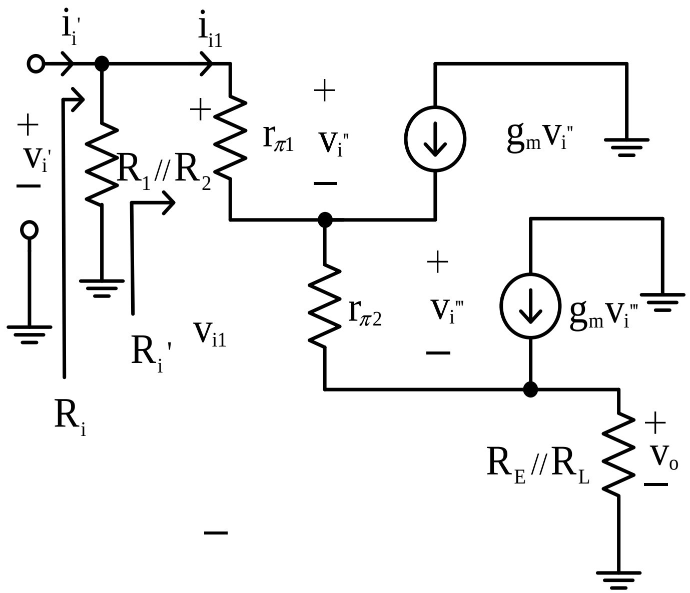
$R_1 = 10k\Omega$

$R_2 = 22k\Omega$

$R_E = 22\Omega$

$R_L = 8\Omega$

Assume that the effects of r_o in Q_1 and Q_2 are negligible,

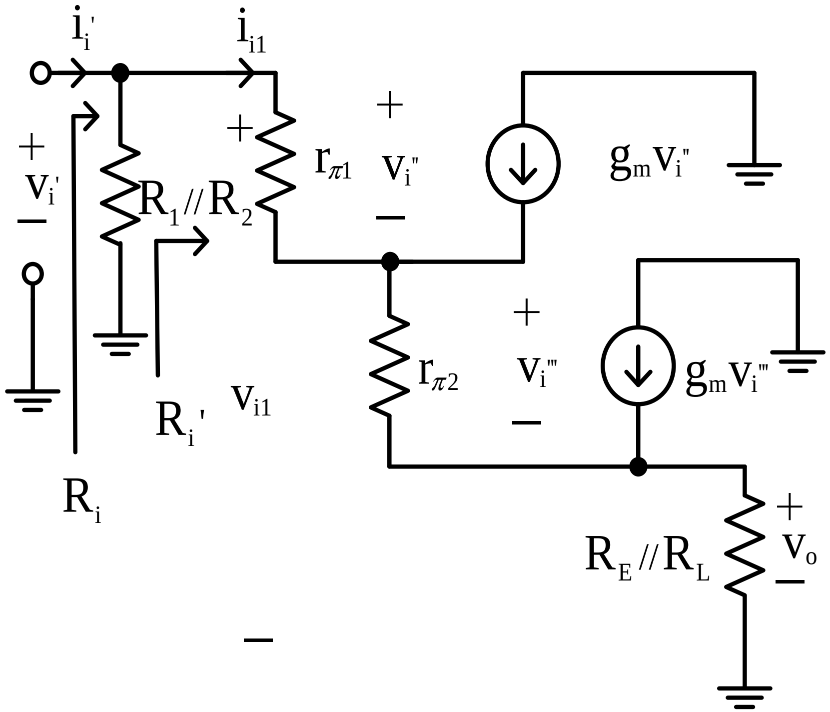


$$R_i = \frac{v_i'}{i_i'} = R_1//R_2//R_i'$$

$$R_i' = \frac{v_{i1}}{i_{i1}}$$

$$v_{i1} = i_{i1} r_{\pi 1} + v_i''' + v_o$$

$$v_{i1} = i_{i1} r_{\pi 1} + \left(i_{i1} + g_{m1} i_{i1} r_{\pi 1}\right) r_{\pi 2} + \left[\left(i_{i1} + g_{m1} i_{i1} r_{\pi 1}\right) + g_{m2} v_i'''\right] R_E//R_L$$



$$v_{i1} = i_{i1} r_{\pi 1} + \left(i_{i1} + g_{m1} i_{i1} r_{\pi 1}\right) r_{\pi 2} + \left[\left(i_{i1} + g_{m1} i_{i1} r_{\pi 1}\right) + g_{m2} v_i'''\right] R_E // R_L$$

$$v_{i1} = i_{i1}\left[r_{\pi 1} + \left(1 + g_{m1} r_{\pi 1}\right) r_{\pi 2}\right] + i_{i1}\left[\left(1 + g_{m1} r_{\pi 1}\right) + g_{m2}\left(1 + g_{m1} r_{\pi 1}\right) r_{\pi 2}\right] R_E // R_L$$

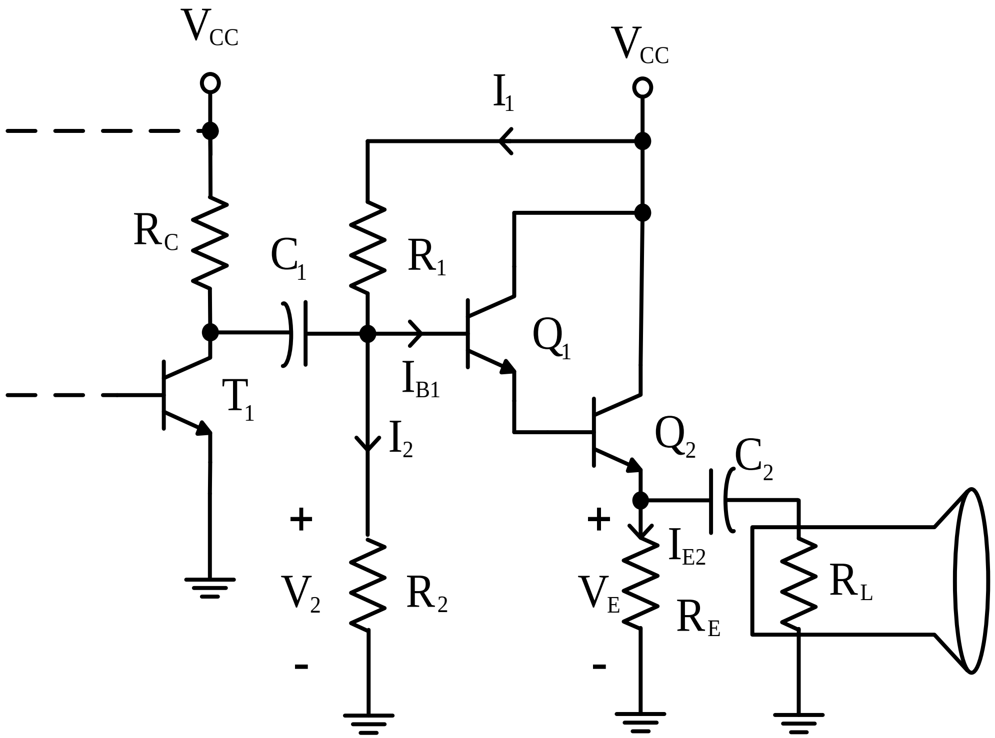
$$R_i' = \frac{v_{i1}}{i_{i1}} = r_{\pi 1} + \left(1 + g_{m1} r_{\pi 1}\right) r_{\pi 2} + \left[\left(1 + g_{m1} r_{\pi 1}\right) + g_{m2} r_{\pi 2}\left(1 + g_{m1} r_{\pi 1}\right)\right] R_E // R_L$$

$$R_E // R_L = 5.87\Omega$$

$$R_i = R_1 // R_2 // R_i'$$

$$R_1 // R_2 = 6875\Omega$$

$$r_{\pi 1} = \frac{\beta_o}{g_{m1}} ? \quad g_{m1} = \frac{I_{C1}}{V_T} ? \quad r_{\pi 2} = \frac{\beta_o}{g_{m2}} ? \quad g_{m2} = \frac{I_{C2}}{V_T} ?$$



$$I_1R_1 + I_2R_2 = V_{CC}$$
$$(I_{B1} + I_2)R_1 + I_2R_2 = V_{CC}$$
$$I_{B1}R_1 + I_2(R_1 + R_2) = V_{CC}$$
$$I_{B1}R_1 + \frac{V_2}{R_2}(R_1 + R_2) = V_{CC}$$
$$V_2 = (V_{CC} - I_{B1}R_1)\frac{R_2}{R_1 + R_2} \qquad (1)$$

$$-V_2 + 0.7 + 0.7 + V_E = 0$$
$$I_E = I_C + I_B = (\beta + 1)I_B$$
$$I_{E1} = (\beta + 1)I_{B1} = I_{B2}$$
$$V_E = I_{E2}R_E$$
$$= (\beta + 1)I_{B2}R_E$$
$$= (\beta + 1)(\beta + 1)I_{B1}R_E$$

Hence, $V_2 = 1.4 + (\beta + 1)^2 I_{B1}R_E \qquad (2)$

Darlington emitter-follower application cont'd

$$V_2 = (V_{CC} - I_{B1}R_1)\frac{R_2}{R_1+R_2} \qquad (1) \qquad\qquad V_2 = 1.4 + (\beta+1)^2 I_{B1}R_E \qquad (2)$$

(1)=(2)

$$(V_{CC} - I_{B1}R_1)\frac{R_2}{R_1+R_2} = 1.4 + (\beta+1)^2 I_{B1}R_E$$

$$V_{CC}\frac{R_2}{R_1+R_2} = 1.4 + \left[(\beta+1)^2 R_E + \frac{R_1R_2}{R_1+R_2}\right] I_{B1}$$

$$I_{B1} = \frac{\frac{R_2}{R_1+R_2}V_{CC} - 1.4}{\left[(\beta+1)^2 R_E + \frac{R_1R_2}{R_1+R_2}\right]} = \frac{6.85}{231297} = 29.6156\mu A$$

$$I_{E2} = (\beta+1)^2 I_{B1} = (101)^2\, 29.6156\mu = 302.11mA$$

$$I_{E2} = \frac{I_{C2}}{\alpha}$$

$$I_{C2} = \frac{\beta}{\beta+1} I_{E2} = 299.12mA$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{299.12m}{26m} = 11.5046$$

$$R_1//R_2 = 6875\Omega$$

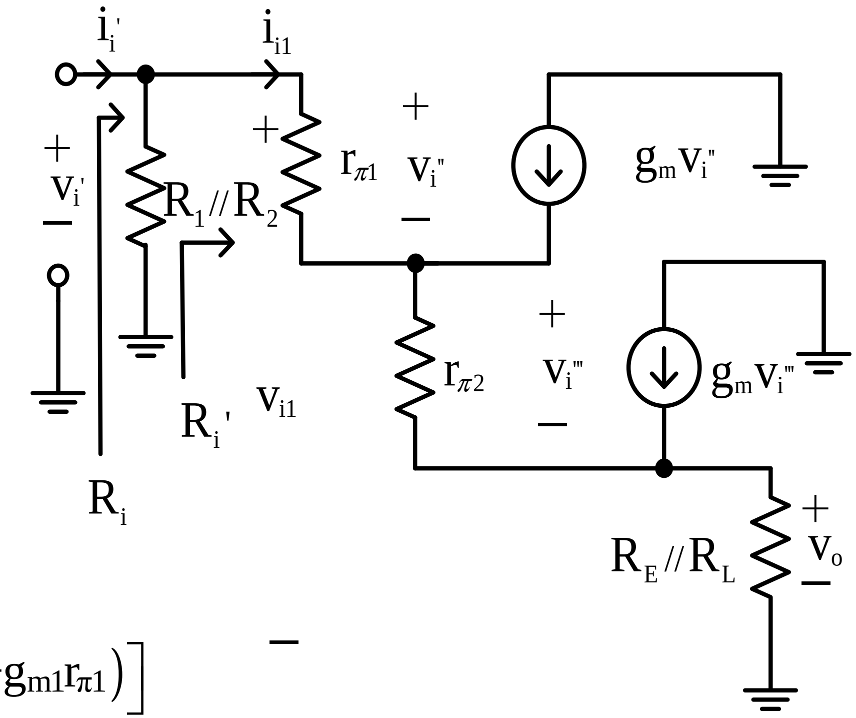
$$R_E//R_L = 5.87\Omega$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{299.12m}{26m} = 11.5046$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{\beta I_{B1}}{V_T} = \frac{100(29.6156\mu)}{26m} = 0.1139$$

$$r_{\pi1} = \frac{\beta_o}{g_{m1}} = \frac{100}{0.1139} = 877.96\Omega$$

$$r_{\pi2} = \frac{\beta_o}{g_{m2}} = \frac{100}{11.5046} = 8.6922\Omega$$



$$R_i' = r_{\pi1} + (1 + g_{m1} r_{\pi1}) r_{\pi2} + R_E//R_L \left[1 + g_{m1} r_{\pi1} + g_{m2} r_{\pi2}(1 + g_{m1} r_{\pi1})\right]$$

$$R_i' = 877.96 + (1 + 0.1139(877.96))8.6922 +$$

$$5.87\left[1 + 0.1139(877.96) + 11.5046(8.6922)(1 + 0.1139(877.96))\right]$$

$$R_i' = 1755.6469 + 59879.8274 = 61635.4743$$

$$R_i = R_1//R_2//R_i' = 6875//61635.4743 = 6185.0964$$

$$a_{v(\text{CE\_with Darlington and } R_L \text{ as load})} = -g_m R_C//R_i = -0.2(1k//6185.0964) \approx -172$$

The voltage gain of the CE amplifier with Darlington emitter-follower and R_L as load is -172.

$$a_{v(\text{Darlington CC})} = \frac{\left[(1+g_{m1}r_{\pi1})+g_{m2}r_{\pi2}(1+g_{m1}r_{\pi1})\right]R_E//R_L}{\left[r_{\pi1}+(1+g_{m1}r_{\pi1})r_{\pi2}\right]+\left[(1+g_{m1}r_{\pi1})+g_{m2}r_{\pi2}(1+g_{m1}r_{\pi1})\right]R_E//R_L}$$
$$=0.9715$$

$$a_{v(\text{CE\_with Darlington and } R_L \text{ as load})} = -g_m R_C//R_i = -0.2(1k//6185.0964) \approx -172$$

Overall voltage gain:

$$a_{v(\text{CE\_with Darlington and } R_L \text{ as load})} \times a_{v(\text{Darlington CC})} = -172 \times 0.9715 \approx -167$$

The open-load voltage gain of the CE amplifier is -200. If the CE amplifier drives the speaker directly, the voltage gain is -1.6. With the Darlington emitter-follower connected, the voltage gain of the whole circuit is -167. This shows that:

(i) the maximum voltage gain is the open-circuit voltage gain

(ii) when the CE circuit, which has a large output resistance, is connected to a low resistance load, the voltage gain will drop drastically

(iii) the Darlington emitter-follower will help in maintaining the voltage gain of the CE as it has a large input resistance but a low output resistance (impedance transformer). Hence, the Darlington emitter-follower acts as a voltage buffer.

(2) (a)What is the minimum value of V_{BIAS} required for a cascode amplifier operating at I=100μA? The cascode circuit is shown in Figure 2.

Let $\mu_n C_{ox} = 300\ \mu A/V^2$, $\frac{W}{L} = 10$ and $V_t = 0.6V$.

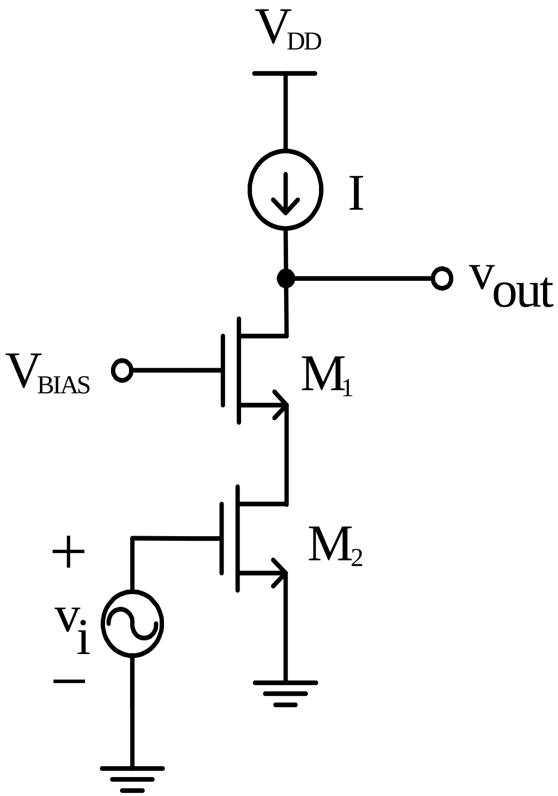


Figure 2

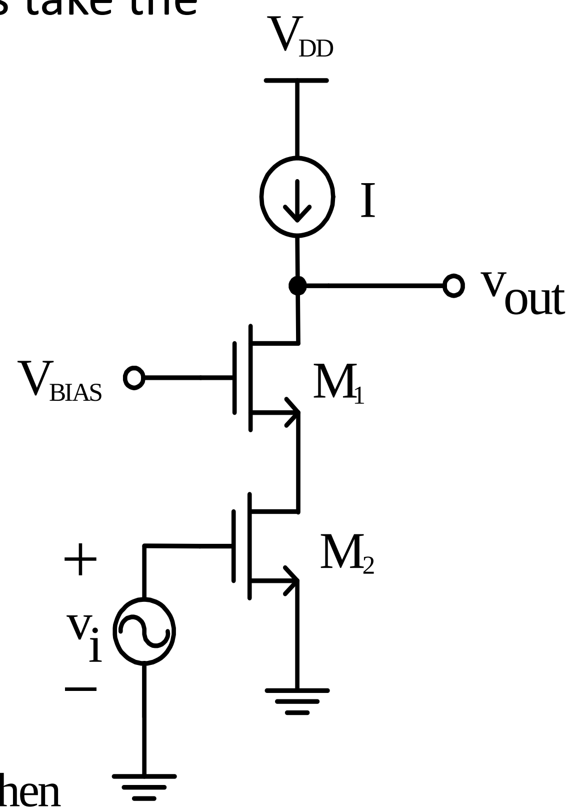
Solution :

$$I = \frac{\mu_n C_{ox}}{2}\frac{W}{L}\left(V_{GS} - V_t\right)^2$$

$$\left(V_{GS} - V_t\right) = \sqrt{\frac{100\mu(2)}{300\mu(10)}}$$

$$\left(V_{GS} - V_t\right) = V_{ov} = 0.2582V$$

To enable M_1 to be operating in the active/saturation region, $V_{DS} > V_{GS} - V_t$. As $V_{GS} - V_t = 0.2582V$, $V_{DS} > 0.2582V$. Lets take the minimum V_{DS_2} as 0.2582 V.



To let 100μA of current flowing through M_2,

$$V_{DS_2} = 0.2582\text{ V}$$

$$V_{DS_2} = V_{S_1}$$

Since $(V_{GS}\text{-}V_t) = 0.2582\text{ V}$ when $I = 100\,\mu\text{A}$ of current is flowing, then

$$V_{GS_1} = 0.2582\text{ V} + 0.6\text{ V} = 0.8582\text{ V}$$

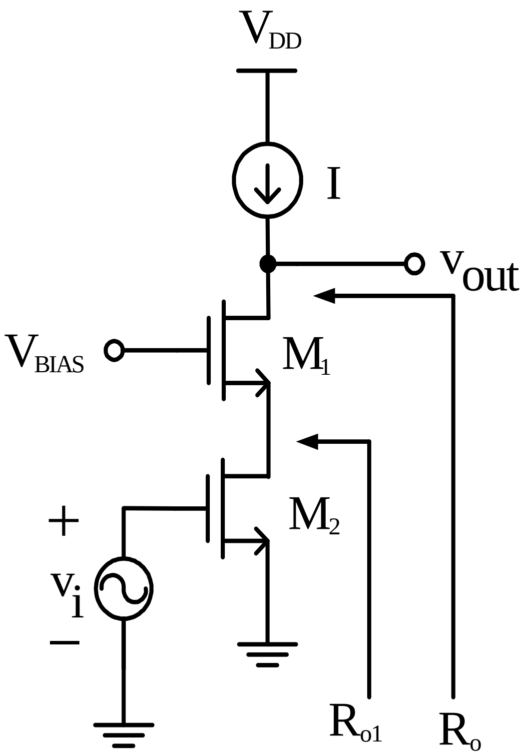
$$V_{G_1} = V_{S_1} + V_{GS_1} = 0.2582\text{ V} + 0.8582\text{ V}$$

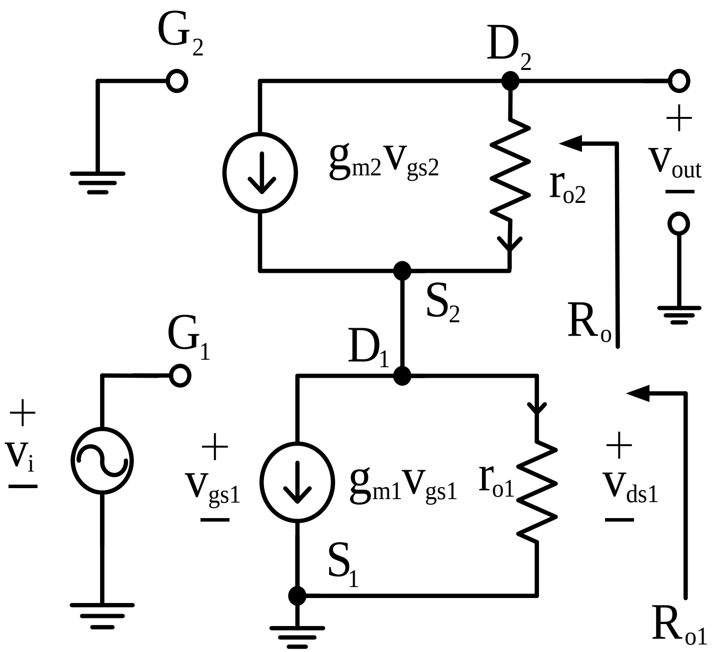
$$V_{G_1} = V_{BIAS} = 1.1164\text{ V}$$

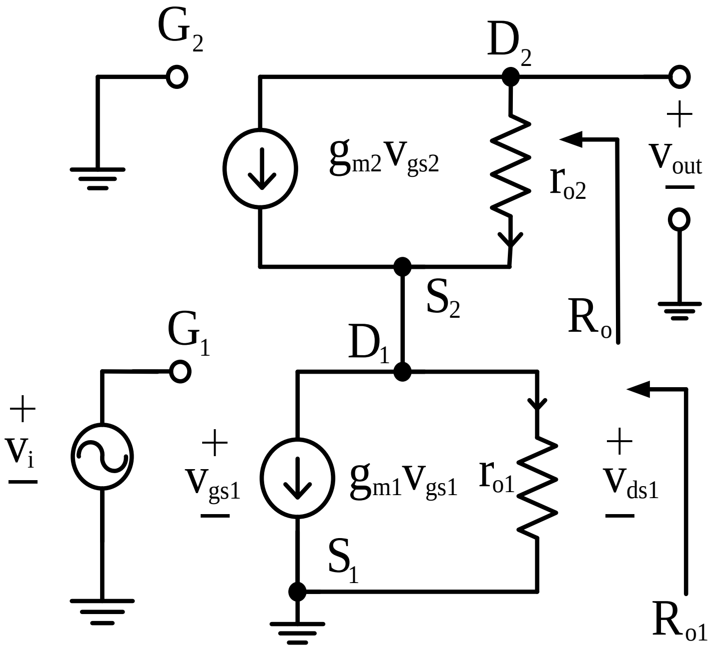
(b) Consider a cascode amplifier as shown below :

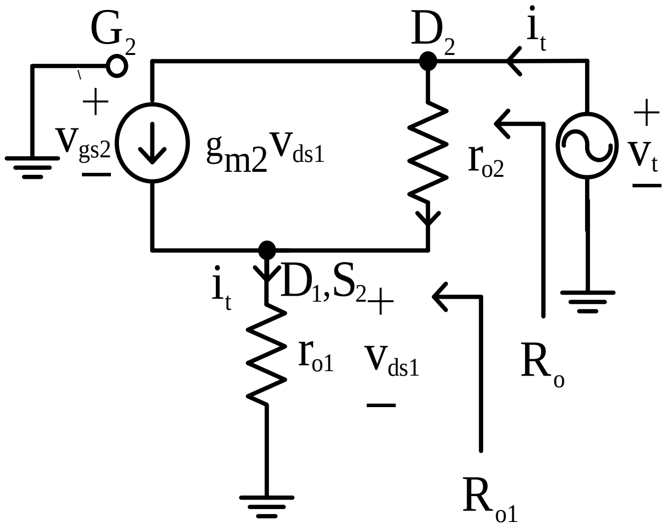
I = 100μA and for each transistor, $\frac{W}{L} = \frac{5\mu m}{0.5\mu m} = 10$, V_A = 10V, and $\mu_n C_{ox} = 190 \mu A / V^2$.

Find R_{O1} and R_O.









$$R_O = \left.\frac{v_t}{i_t}\right|_{v_i=0}$$

KCL at node D_2,

$$i_t = -g_{m2}v_{ds1} + \frac{v_t - v_{ds1}}{r_{o2}}$$

$$v_{ds1} = i_t r_{o1}$$

$$i_t = -g_{m2}i_t r_{o1} + \frac{v_t - i_t r_{o1}}{r_{o2}}$$

$$i_t\left[1 + g_{m2}r_{o1} + \frac{r_{o1}}{r_{o2}}\right]r_{o2} = v_t$$

$$R_O = \frac{v_t}{i_t} = \left[1 + g_{m2}r_{o1}\right]r_{o2} + r_{o1}$$

$$R_{O1} = r_{o1}$$

$$r_{o1} = \frac{1}{\lambda I_D}$$

$$\lambda = \frac{1}{V_A}$$

$$\therefore R_{O1} = r_{o1} = \frac{V_A}{I_D} = \frac{10}{100\mu} = 100k\Omega$$

Assume $r_{o1} = r_{o2}$

$$R_O = \frac{v_t}{i_t} = \left[1 + g_{m2} r_{o1}\right] r_{o2} + r_{o1}$$

$$g_{m2} = \sqrt{2 I_D \mu_n C_{ox} \frac{W}{L}} = \sqrt{2(100\mu)(190\mu)10}$$

$$g_{m2} = 6.1644 \times 10^{-4}$$

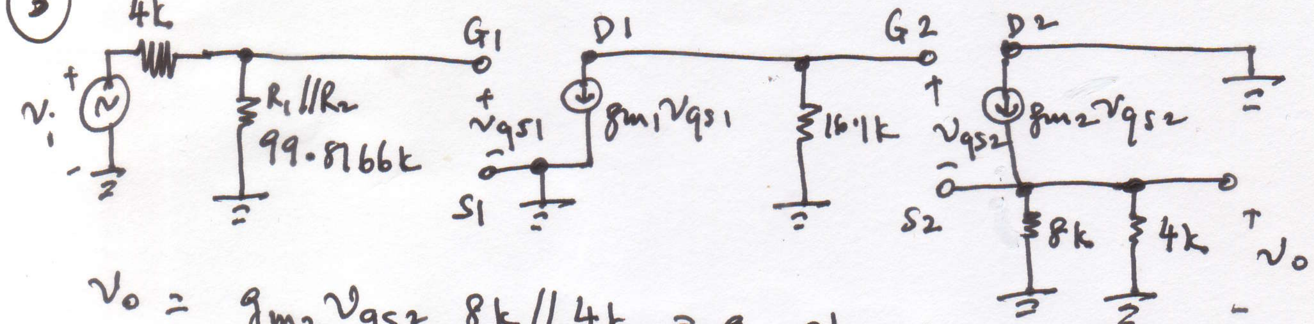
$$R_{O1} = r_{o1} = 100k\Omega$$

$$R_O = \left[1 + \left(6.1644 \times 10^{-4}\right)(100k)\right] 100k + 100k$$

$$R_O = 6.3644\, M\Omega$$

The cascode configuration has increased the output resistance by a factor of 63.644 (comparing R_o and R_{o1}). As a rough estimation, we can take that the cascode has increased the output resistance by a factor of $g_m r_o$ from the output resistance of a CS

(3)



$$v_o = g_{m2} v_{gs2}\ 8k \| 4k = g_{m2} v_{gs2} (2.6667k)$$

$$v_{ds1} = v_{gs2} + v_o$$

$$v_{ds1} = -g_{m1} v_{gs1} (16.1k)$$

$$v_{gs1} = \frac{99.8166k}{103.8166k} v_i = 0.9615\, v_i$$

$$g_{m1} = 2\sqrt{K_{n1} I_{D1}} = 2\sqrt{0.5m(0.2m)} = 0.6324\ mS$$

$$g_{m2} = 2\sqrt{K_{n2} I_{D2}} = 2\sqrt{0.2m(0.5m)} = 0.6324\ mS$$

$$\therefore\ v_{ds1} = -(0.6324m)(0.9615\, v_i)(16.1k) = -9.7896\, v_i$$

$$v_{gs2} = v_{ds1} - v_o$$

$$v_o = g_{m2}(v_{ds1} - v_o)(2.6667k)$$

$$= (0.6324m)(-9.7896 v_i - v_o)(2.6667k)$$

$$= 1.6864(-9.7896 v_i - v_o)$$

$$= -16.5092 v_i - 1.6864 v_o$$

$$(1 + 1.6864) v_o = -16.5092 v_i$$

$$a_v = \frac{v_o}{v_i} = \frac{-16.5092}{2.6864} = -6.1455 \#$$