EEE130 Digital Electronics I
Lecture #4_1
- Boolean Algebra and Logic Simplification -

By Dr. Shahrel A. Suandi

4-6 Standard Forms of Boolean Expressions

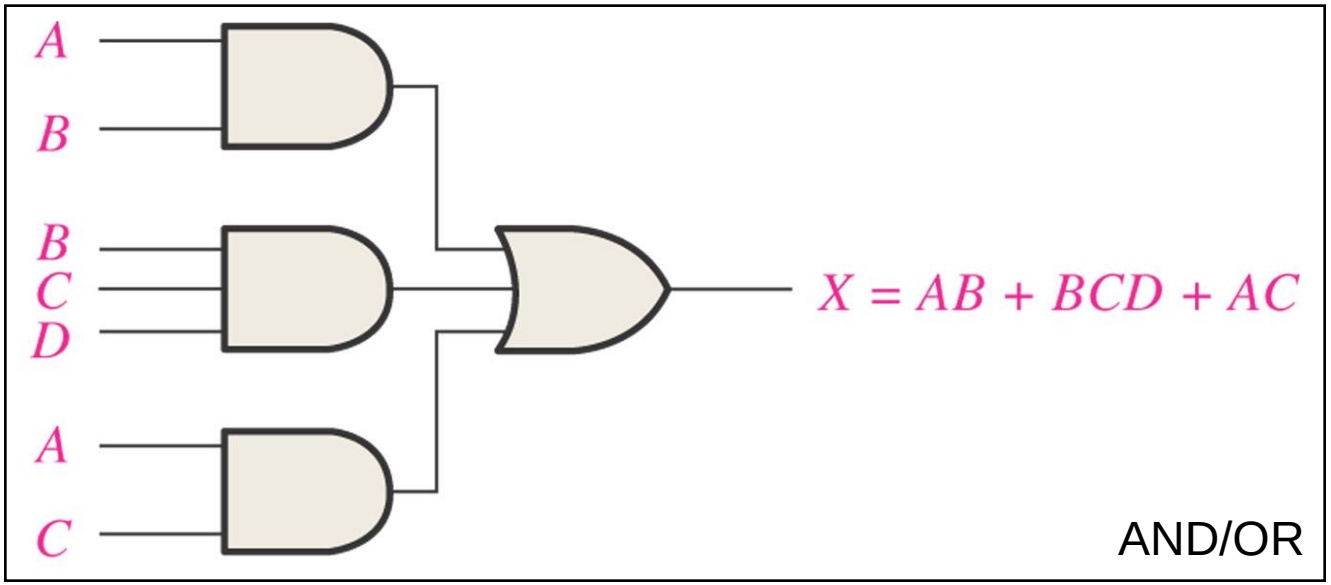
- There are two standard forms:
  - Sum-of-products form
  - Product-of-sums
- Sum-of-products (SOP) form
  - Can be expressed with one OR and two or more ANDs
  - Product – Boolean multiplication
  - Sum – Boolean addition
  - Overall meaning – the sum of a few products.
  - Examples: $AB + ABC$ $ABC + CDE + \bar{B}C\bar{D}$
  - Condition to be complied:
    - In an SOP expression, a single overbar cannot extend over more than one variable; but
      - More than one variable in a term can have an overbar
      - Example: $\bar{A}\bar{B}\bar{C}$ but not $\overline{ABC}$

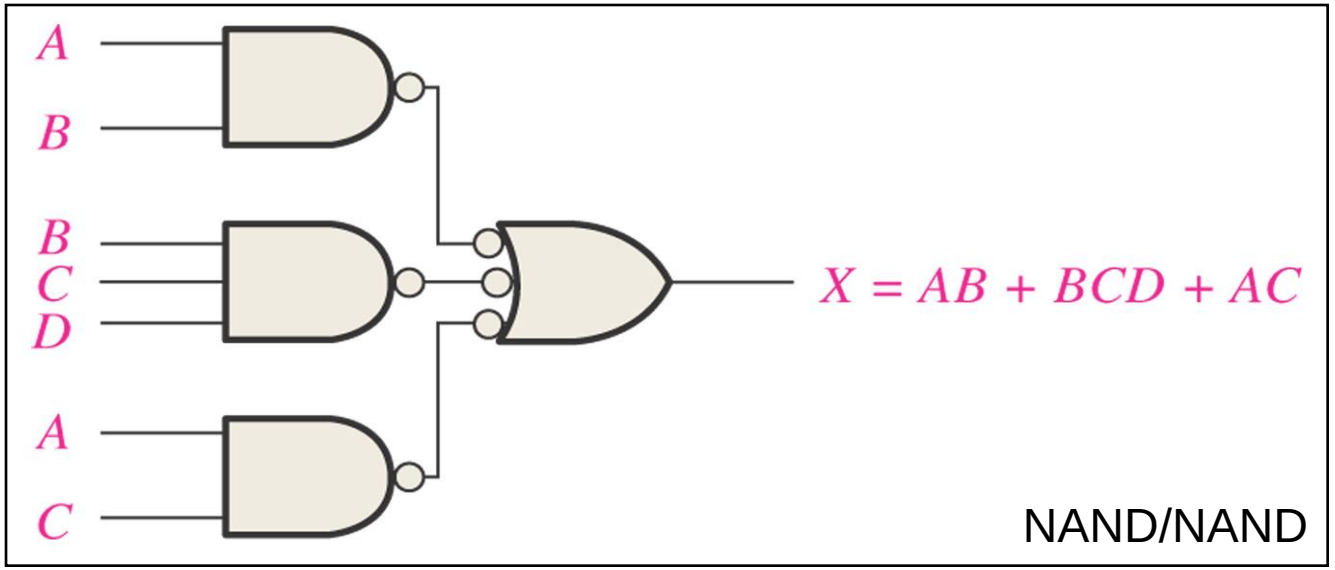
OK NG

More about SOP forms (1)

- Domain of a Boolean expression
  - ***Domain***: is the set of variables contained in the expression in either complemented or uncomplemented
  - Example: for $ABC + CDE + \bar{B}C\bar{D}$ then the domain is the set of variables of A, B, C, D, E
- AND/OR implementation of an SOP expression
  - Use AND and OR to get SOP form
- NAND/NAND implementation of an SOP expression
  - Use NAND or Negative OR to get SOP form

AND/OR SOP & NAND/NAND SOP





More about SOP forms (2)

- Conversion of a general expression to SOP form
  - To convert, we can simply use Boolean algebra techniques
    - Example: $A(B + CD) = AB + ACD$
- The standard SOP form
  - *All variables* in the domain *must appear* in each product term in the expression
  - This is important in constructing truth tables
  - A nonstandard SOP is usually referred to as SOP
  - Converting product terms to standard SOP (ex. 4-13)
    - Basic → using rule 6, that is,$(A + \bar{A}) = 1$
    - Method:
      - Multiply each nonstandard product term by a term made up of the sum of a missing variable and its complement (rule 6)
      - Repeat until we get all product terms contain all variables in the domain
      - Indirectly, we know that the process finishes when
        - » The number of product terms is doubled for each of missing variable

More about SOP forms (3)

- Binary representation of a standard product term (Ex 4-14)
  - A standard product term is *equal to 1 for only one combination of variable values*
  - Example: $A\bar{B}C\bar{D}$
- Remember that SOP is based on products and then sum, therefore
  - It equals to one if one or more of the product terms in the expression is equal to 1

The Product-of-Sums (POS) Form

- Meaning: multiplication of two or more sum terms
- Examples:

$$\boxed{(\bar{A}+B)(A+\bar{B}+C)} \qquad \boxed{(A+B)(A+\bar{B}+C)(\bar{A}+C)}$$

- Conditions to be complied:
  - A single overbar cannot extend over more than one variable; but
    - More than one variable in a term can have an overbar
    - Example: $\bar{A}+\bar{B}+\bar{C}$ but not $\overline{A+B+C}$

OK NG

More about POS form (1)

- Implementation of a POS expression
  - ANDing the outputs of two or more OR gates
- Similar to SOP form, there is a standard expression of POS
  - Each sum term *must contain all variables in the domain*
  - Usually, when it is written POS, it means nonstandard POS
- Converting a sum term to standard POS
  - Use rule 8 and apply this to the sum term which does not contain all variables in the domain
  - Rule 8: $(A \cdot \bar{A}) = 0$
  - Method:
    - Add to each nonstandard product term a term made up of the product of the missing variable and its complement, giving results in two sum terms
    - Apply rule 12: $A + BC = (A + B)(A + C)$
    - Repeat until all sum terms contain all variables in the domain

More about POS form (2)

- Binary representation of a standard sum term
  - Equals to 0 for only one combination of variable values
  - Example: $A + \bar{B} + C + \bar{D}$
- Remember this:
  - A POS expression is equal to 0 only if one or more of the sum terms in the expression is equal to 0

About SOP and POS

- Converting SOP to POS
  1. Evaluate each product term in the SOP expression, i.e., determine the binary numbers that represent the product terms
  2. Determine all of the binary numbers not included in the evaluation in step 1
  3. Write the equivalent sum term for each binary number from step 2 and express in POS form
- Converting POS to SOP
  - Use similar steps as described above

4-7 Boolean Expressions and Truth Tables (1)

- Truth table – a common way of presenting, in concise format, the logical operation of a logic circuit
- Let's look at example 4-18 for SOP
  - SOP → *equals to 1 only if at least one of the product terms is equal to 1*
- Let's look at example 4-19 for POS
  - POS → *equals to 0 only if at least one of the sum terms is equal to 0*

4-7 Boolean Expressions and Truth Tables (2)

- Determining standard expressions from a truth table
  - We need to know that SOP relates to 1 and POS relates to 0
    - Which means we need to list out the binary values of the input variables for which the output is 1 for SOP and 0 for POS
- Example 4-20 illustrates good explanation on this

Next class

- 4.7 The Karnaugh Map