EEE130 Digital Electronics I
Lecture #4_2
- Boolean Algebra and Logic Simplification -

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4-7 Karnaugh Map

- Karnaugh map provides a systematic method for simplifying Boolean expressions – produce the simplest SOP or POS expression
  - Known as *minimum expression*
- Karnaugh map ≅ truth table
  - Presents all possible values of input variables and the outputs
- Simplification techniques:
  - Boolean algebra – we have discussed this. Remember all the laws(3), rules(12), DeMorgan's Theorem(2) and theorems of Boolean algebra
  - Karnaugh map – provides a 'cookbook' method for simplification

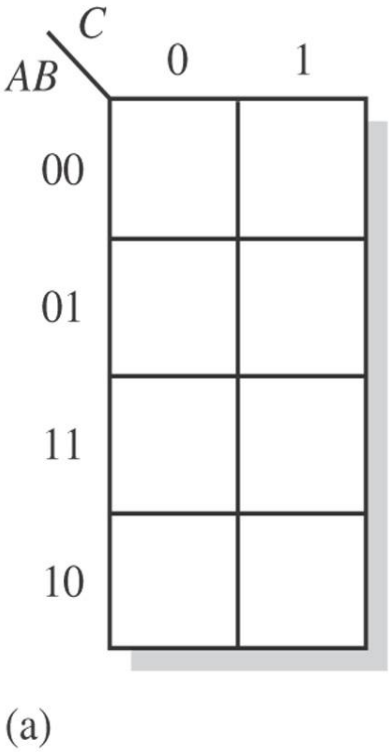
Introduction to Karnaugh Map

- It does not have rows and columns, instead, it is an array of cells
  - Each cell represents a binary value of the input variables
- Can be used for expressions with 2,3,4 or 5 variables
- Other option to Karnaugh map is Quine-McClusky – for higher numbers of variables
- Cell quantity = quantity of possible input variable combinations
  - Example: $2^3 = 8, \quad \text{or} \quad 2^4 = 16$

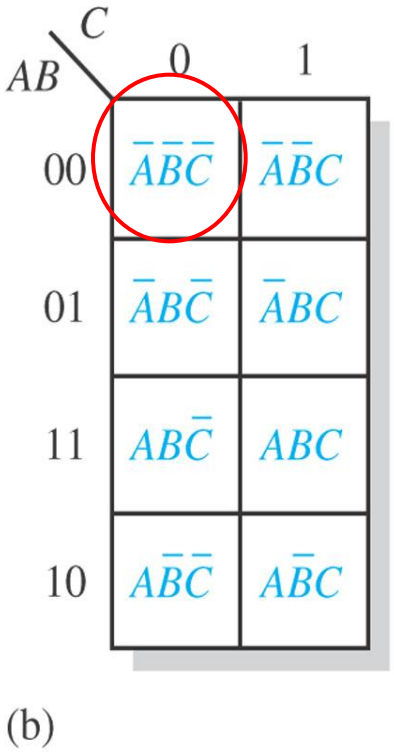
The 3-variable Karnaugh Map

- As it has 3 variables, the possible combinations are 8, so the total of cells is also 8
- Any letter can be used to represent the variables, eg. A, B, C, X, Y, Z, etc.
- For 3-variable Karnaugh map, the first two letters will be written on the left of the cell, while the remaining letter will be written at the top of the cell (refer Figure 4-21)
- The binary values of the inputs must be written in the sequence shown in the on the right
- How to read??
  - For cell ABC = 000, the value is

$$\bar{A}\bar{B}\bar{C}$$



(a)

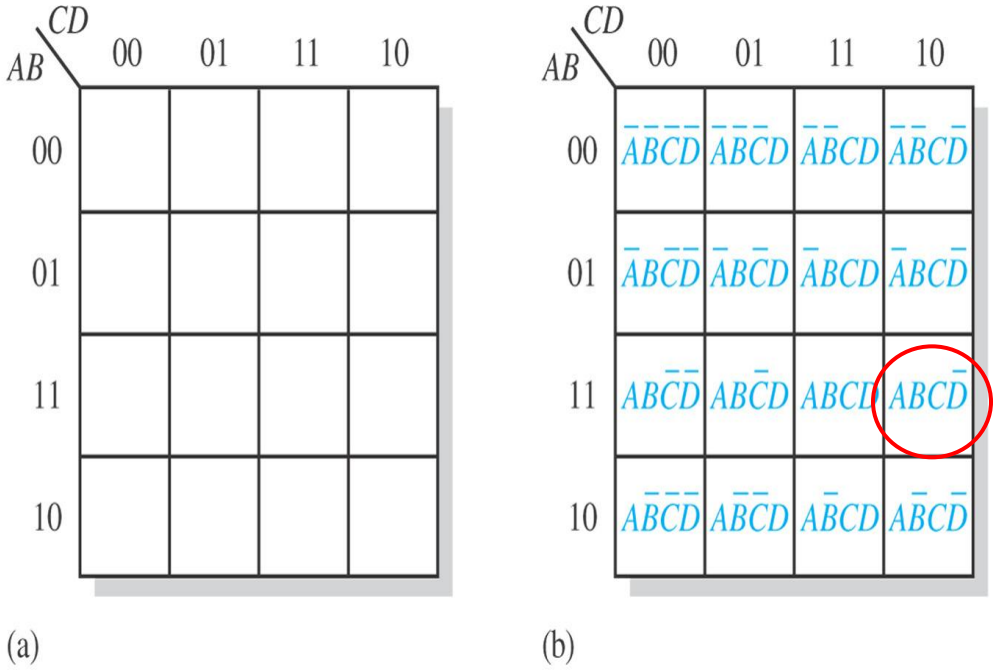


(b)

The 4-variable Karnaugh Map

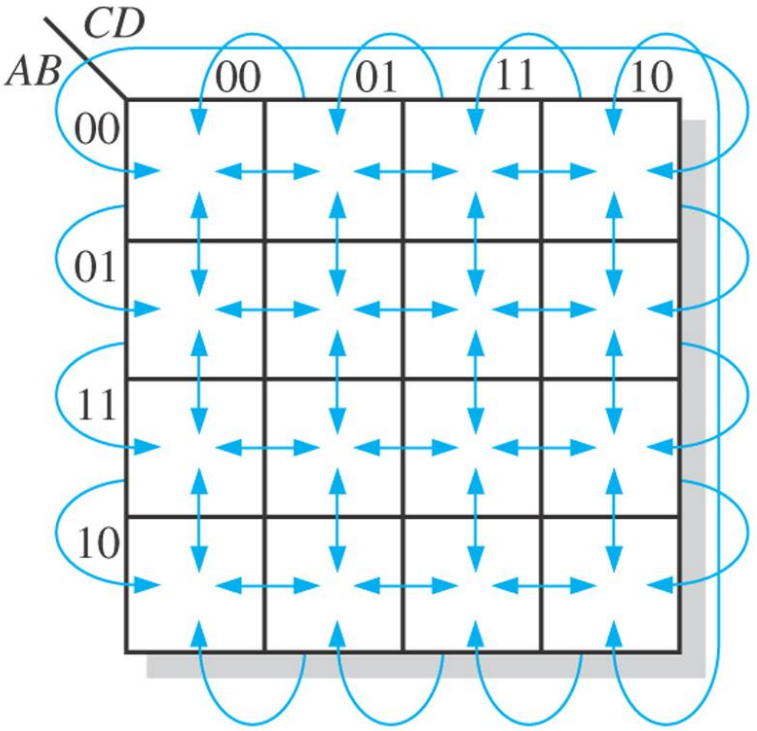
- As it has 4 variables, the possible combinations are 16, so the total of cells is also 16
- Any letter can be used to represent the variables, e.g., A, B, C, X, Y, Z, etc.
- For 4-variable Karnaugh map, the first two letters will be written on the left of the cell, while the remaining two letters will be written at the top of the cell (refer Figure 4-22)
- The binary values of the inputs must be written in the sequence shown in the figure on the right
- How to read??
  - For cell ABCD = 1110, the value is

$$ABC\bar{D}$$



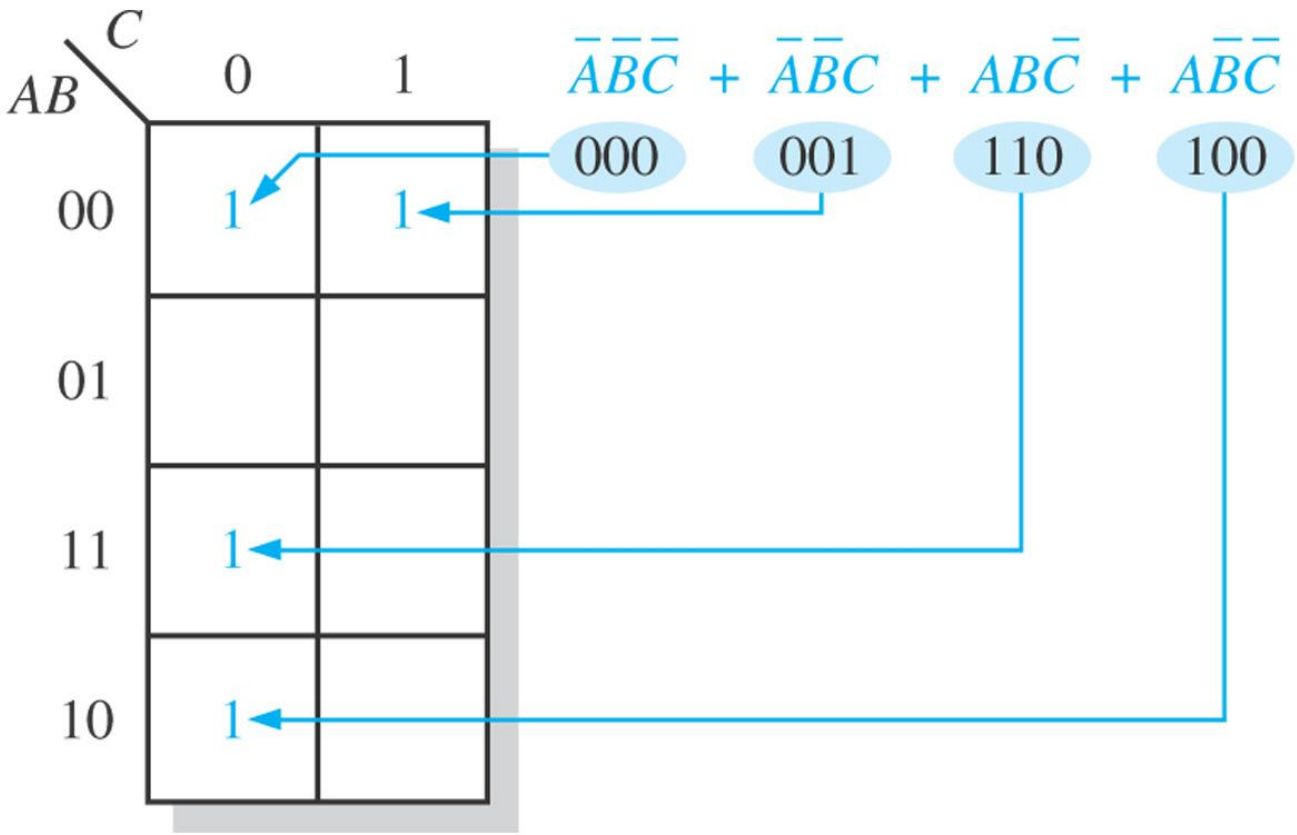
Cell adjacency

- Adjacency – is defined by a single-variable change
- There is only a single-variable change between adjacent cells
- To define a clearer picture of adjacency, please refer to the figure below (figure 4-23)
  - “wrap-around” adjacency



4-9 Karnaugh Map SOP Minimization (1)

- Mapping a standard SOP expression



4-9 Karnaugh Map SOP Minimization (2)

- Mapping a nonstandard SOP expression
  - Boolean expression must be in standard form
- Numerical expansion of a nonstandard product term
  - Refer to example 4-23
- Karnaugh map simplification of SOP expressions
  - This is the main reason why Karnaugh map is used – to get the fewest possible terms with the fewest variables (called '*minimization*')
  - Grouping the 1s
    - Goal: to *maximize the size of the groups* and to *minimize the number of groups*
  - How to implement this??
    - Let's look at the next slide…

Example 4-23

- Map the following SOP expression on a K map: $\bar{A} + A\bar{B} + AB\bar{C}$

$$\bar{A} + A\bar{B} + AB\bar{C}$$

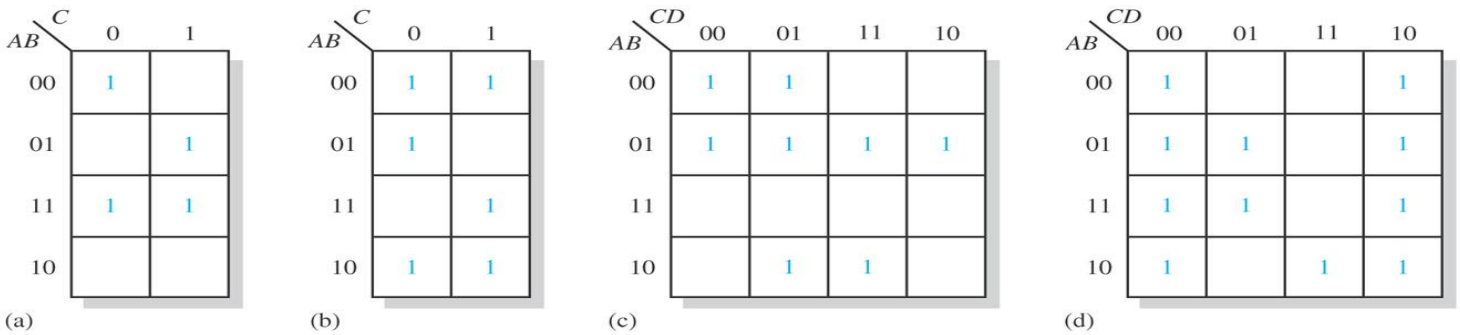
000
001
010
011

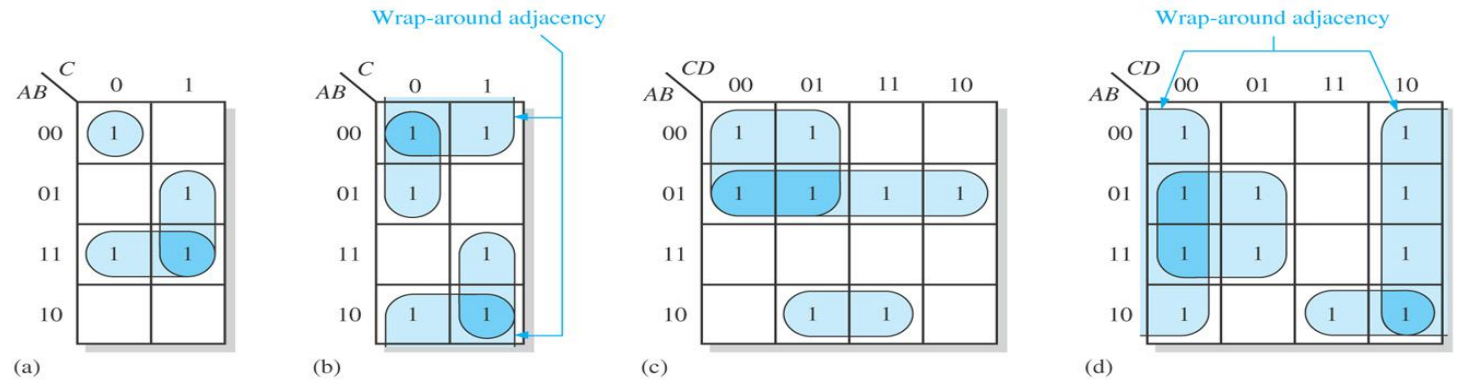
100
101

AB \ C	0	1
00	1	1
01	1	1
11	1	
10	1	1

4-9 Karnaugh Map SOP Minimization (3)

- How to do this (K-map simplification of SOP expressions)??
  - A group must contain either 1,2,4,8 or 16 cells, which are all powers of two. In the case of a 3-variable map, $2^3=8$ cells is the maximum group
  - Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group do not have to be adjacent to each other
  - Always include the largest possible number of 1s in a group in accordance with rule 1
  - Each 1 on the map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include noncommon 1s.

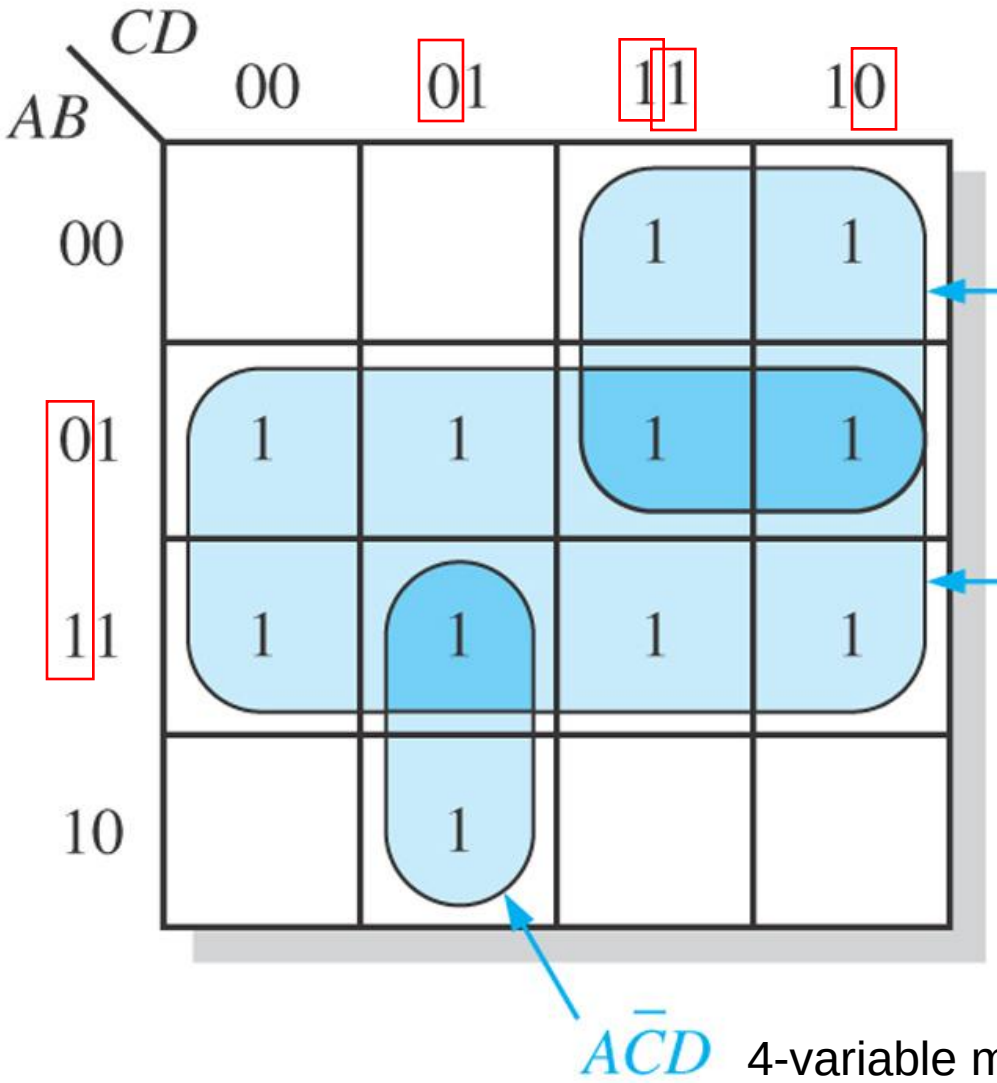




4-9 Karnaugh Map SOP Minimization (4)

- Determining the minimum SOP expression from the map
  - When we finish with all the 1s representing the standard product terms, the process of determining the resulting minimum SOP begins
- The rules are:
  - Group the cells that have 1s. Each group of cells containing 1s creates one product term composed of all variables that occur in only one form (either uncomplemented or complemented) within the group. Variables that occur both uncomplemented and complemented within the group are eliminated. These are called *contradictory variables*
  - Determine the minimum product term for each group
    - For a 3-variable map:
      - 1-cell group → 3-variable product term
      - 2-cell group → 2-variable "
      - 4-cell group → 1-variable "
      - 8-cell group → value of 1 for the expression
    - For a 4-variable map:
      - 1-cell group → 4-variable product term
      - 2-cell group → 3-variable "
      - 4-cell group → 2-variable "
      - 8-cell group → 1-variable "
      - 16-cell group → value of 1 for the expression
  - When all the minimum product terms are derived from the Karnaugh map, they are summed to form the minimum SOP expression

Example 4-26



4-variable map No.(3)

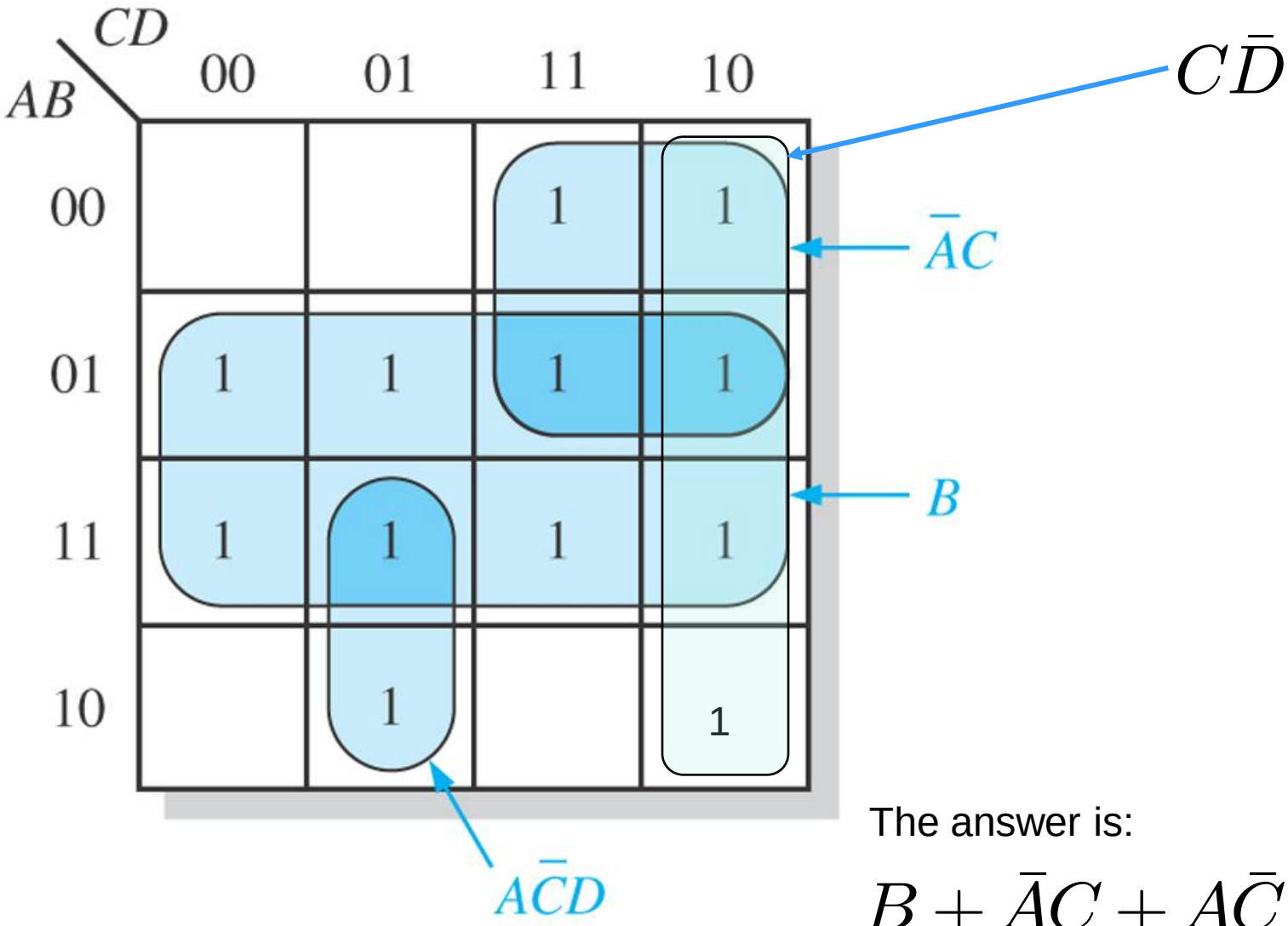
4-variable map No.(4)

4-variable map No.(2)

The answer is:

$$B + \bar{A}C + A\bar{C}D$$

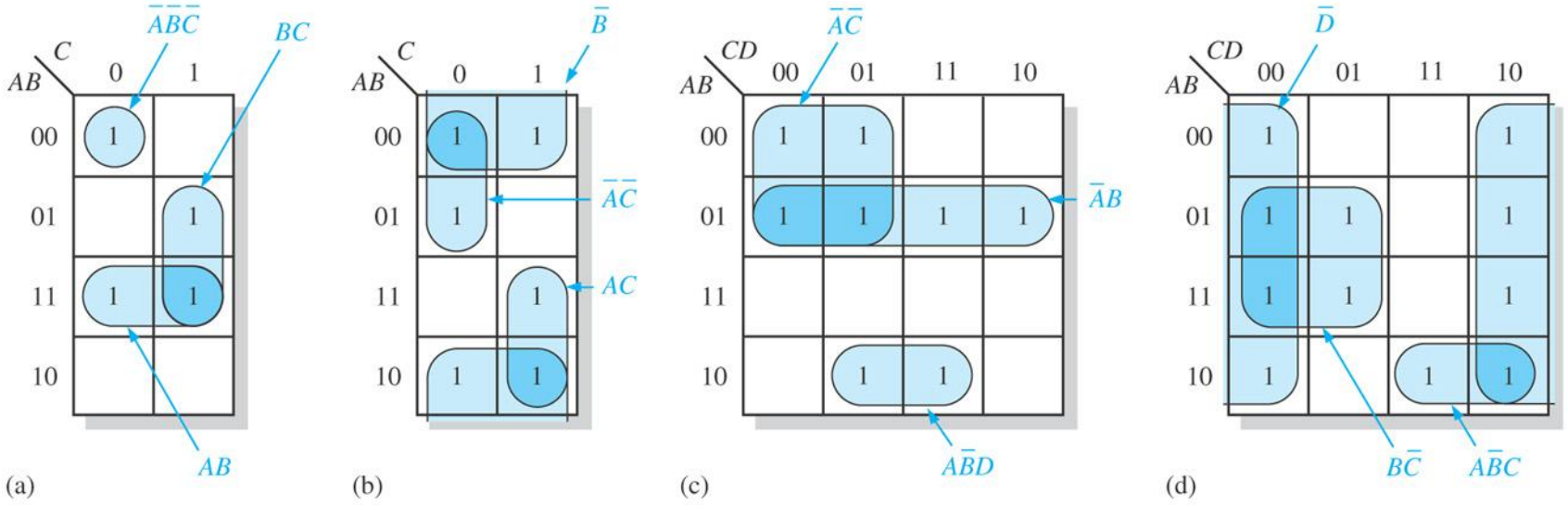
Example 4-26 (Related Problem)



The answer is:

$$B + \bar{A}C + A\bar{C}D + C\bar{D}$$

Example 4-27


$\bar{A}\bar{B}\bar{C}$
BC
AB
(a)
$\bar{B}$
$\bar{A}\bar{C}$
AC
(b)
$\bar{A}\bar{C}$
$\bar{A}B$
$A\bar{B}D$
(c)
$\bar{D}$
$B\bar{C}$
$A\bar{B}C$
(d)

Example and exercises

- Let’s try to do some examples and exercises