EEE443 Digital Signal Processing
Implementation of Discrete-Time Systems

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Introduction

- A linear-time invariant system (LTI) is described by linear constant coefficient difference equation (LCCDE)
  
  - If we are given a rational transfer function having the input \(x(n)\) and output \(y(n)\), the LTI is described as
    \[
    y(n) = \sum_{k=0}^{q} b(k)x(n - k) - \sum_{k=1}^{p} a(k)y(n - k)
    \]
  
  - What we understand from this equation?
    - Recurvise system – IIR (which have the terms of \(a(k)\))
    - Non-recursive system – FIR (which its all \(a(k) = 0\))
There are three basic computational elements:

- Adders
  \[ y(n) = x(n) + w(n) \]

- Multipliers
  \[ y(n) = ax(n) \]

- Delays
  \[ y(n) = x(n - 1) \]
Signal Flow Graph

• Originated from the basic computational elements

• Signal flow graph is drawn using nodes to show the network of directed branches

• The nodes correspond to two type of points
  – Adders – more than one incoming branches
  – Branch points – more than one outcoming branches
Signal Flow Graph Example

\[ x_j(n) \rightarrow \text{Node } j \rightarrow \text{Node } k \rightarrow x_k(n) \]
Facts About Signal Flow Graph

• Output of each branch is a linear transformation of the branch input and the linear operator is indicated next to the arrow
  – Notice that for these linear operators, multipliers and delays will be included

• Two types of special nodes:
  – Source nodes – these are nodes that have no incoming branches and are used for sequences that input to the filter
  – Sink nodes – these are nodes that have only entering branches and are used to represent output sequences
Structures of FIR Systems

• FIR – finite impulse response
• What we know about FIR?
  – All its $a(k)$ terms will be zero
  – Non-recursive
• A causal FIR filter/system has a transfer function:
  $$H(z) = \sum_{n=0}^{M-1} h(n)z^{-n}$$
  – Notice that this transfer function has no poles (non-recursive)
• An FIR system is described by the difference equation
  $$y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$$
  System Function
• There are several different realization block diagrams for this system:
  – Direct form, cascade form, frequency-sampling, and lattice
Structures of FIR Systems (cnt’d)

- Unit sample response of the FIR system is identical to the coefficient \( \{b_k\} \), ie.

\[
h(n) = \begin{cases} 
  b_n, & 0 \leq n \leq M - 1 \\
  0, & \text{otherwise}
\end{cases}
\]

- \( M \) is the length of the FIR filter
Direct Form (1)

- The most common way to implement an FIR
- Called a tapped delay line or a transversal system (below is shown the signal flow graph)
This structure requires
- M-1 memory locations for storing the M-1 previous inputs
- M multiplications
- M-1 additions per output point

When the FIR system has linear phase, the unit sample response of the system satisfies either the symmetry or asymmetry condition

\[ h(n) = \pm h(M - 1 - n) \]

- The advantage \(\rightarrow\) number of multiplications is reduced:
  - From M to M/2 for M even
  - From M to (M-1)/2 for M odd

**Figure 9.2.1** Direct-form realization of FIR system.
Cascade Form (1)

• The transfer function is factored into a product of first order factors

\[ H(z) = \sum_{n=0}^{N} h(n)z^{-n} = A \prod_{k=1}^{N} (a - \alpha_k z^{-1}) \]  

where, for \( \alpha_k, k = 1, 2, \ldots \)

• If \( h(n) \) is real, the complex roots of \( H(z) \) occur in complex conjugate pairs, and therefore these conjugate pairs may be combined to form second order factors with real coefficients

\[ H(z) = A \prod_{k=1}^{N} [1 + b_k(1)z^{-1} + b_k(2)z^{-2}] \]
Figure 9.2.3  Cascade realization of an FIR system.
Linear Phase Filter/Systems

• A linear phase filter/system is said to have linear phase if the frequency response has the form

\[ H(e^{j\omega}) = |H(e^{j\omega})|e^{-j\alpha\omega} \]

• The constant group delay \( \tau_h(\omega) = \alpha \)

• The unit impulse response of a linear phase system is either (taking into account \( N = M - 1 \))
  – Symmetric \( h(n) = h(N - n) \)
  – Non-symmetric \( h(n) = -h(N - n) \)
Type of FIR Filter with Linear Phase

• The symmetry can be exploited to simplify the system/filter structure
• Therefore, there are four types of FIR filter
  – Type I: N is even and $h(n)$ is symmetric
  – Type II: N is odd and $h(n)$ is symmetric
  – Type III: N is even and $h(n)$ is antisymmetric
  – Type IV: N is odd and $h(n)$ is antisymmetric
Structures of IIR Filters

- As being described before, a causal IIR filter with rational system or transfer function is expressed as
  \[
  H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^{q} b(k)z^{-k}}{1 + \sum_{k=1}^{p} a(k)z^{-k}}
  \]

- Describing it in LCCDE
  \[
  y(n) = \sum_{k=0}^{q} b(k)x(n - k) - \sum_{k=1}^{p} a(k)y(n - k)
  \]

- Similar to FIR, there are four types of IIR structures or realizations
  - Direct form, cascade form, lattice, and lattice-ladder
Direct-Form (1)

- The rational system function that characterizes an IIR system can be viewed as two systems in cascade

\[ H(z) = H_1(z)H_2(z) \]

\[ H_1(z) = \sum_{k=0}^{M} b_k z^{-k} \quad H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} \]

- In Section 2.5, we have come across this where the cases were \( H_1(z) \) precedes \( H_2(z) \), or vice versa

- Since the top equation is an FIR system, we know how write the diagram for this (Figure 9.2.1)

- To finalize the structure for an IIR, we attach the all-pole system in cascade with all-zero system (FIR system) as shown in Figure 9.3.1
Figure 9.3.1  Direct form I realization.
Direct-Form (3)

• Figure 9.3.1 can be redrawn as shown in Figure 9.3.2 by considering linear equation for all-pole filter

\[ w(n) = - \sum_{k=1}^{N} a_k w(n-k) + x(n) \]

• Since \( w(n) \) is the input to the all-zero system, its output is

\[ y(n) = \sum_{k=0}^{M} b_k w(n-k) \]

• Looking at both equations above, we may note that both involve delayed versions of the sequence \( \{w(n)\} \)
  – Resulting only a single delay line or single set of memory locations is required for storing the past values of \( \{w(n)\} \)
  – This structure type is called a direct form II
  – Requires \( M+N+1 \) multiplications, \( M+N \) additions and max of \( \{M,N\} \) memory locations
  – Is said to be *canonic* → as it minimizes the number of memory locations
• “Direct-form” is given due to it can be obtained directly from the rational function or system function
  – BUT SENSITIVE TO PARAMETER QUANTIZATION → NOT RECOMMENDED IN PRACTICAL APPLICATIONS
Direct-Form (4)

Figure 9.3.2  Direct form II realization ($N = M$).
About Signal Flow Graphs and Transposed Structures

**Signal Flow Graph**

![Signal Flow Graph](Image)

**Transposed Structures**

![Transposed Structures](Image)

**Figure 9.3.3** Second-order filter structure (a) and its signal flow graph (b).

**Figure 9.3.4** Signal flow graph of transposed structure (a) and its realization (b).
Let’s try using the transposition theorem to the direct form II

Figure 9.3.2  Direct form II realization \((N = M)\).

Figure 9.3.5  Transposed direct form II structure.
Cascade-Form

Figure 9.3.8 Cascade structure of second-order systems and a realization of each second-order section.
Parallel-Form

Performing partial fraction expansion of \( H(z) \), we obtain

\[
H(z) = C + \sum_{k=1}^{N} \frac{A_k}{a - p_k z^{-1}}
\]

Figure 9.3.9  Parallel structure of IIR system.