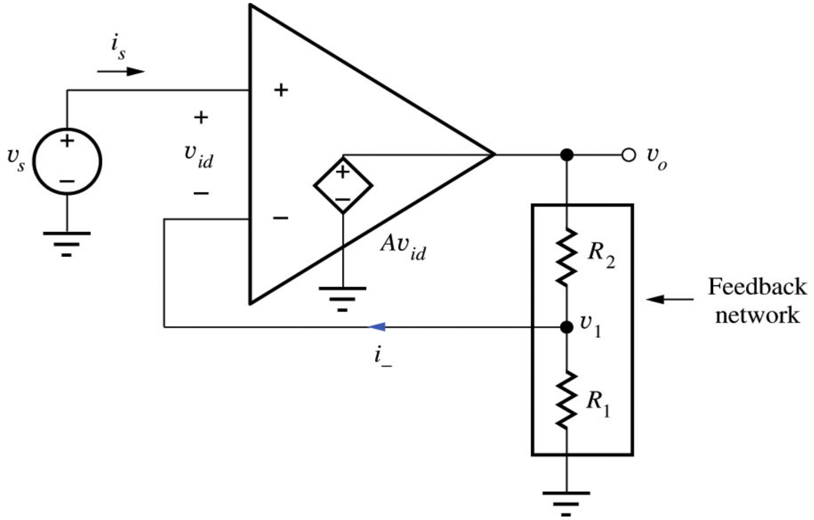
Non-ideal Operational Amplifier

- Various error terms arise in practical operational amplifiers due to non-ideal behavior.
- Some of the non-ideal characteristics include:
  1. Finite open-loop gain that causes gain error
  2. Nonzero output resistance
  3. Finite input resistance
  4. Finite CMRR
  5. Common-mode input resistance
  6. DC error sources
  7. Output voltage and current limits

Finite Open-loop Gain



$$v_1 = \frac{R_1}{R_1 + R_2} v_o = \beta v_o$$

$$\beta = \frac{R_1}{R_1 + R_2}$$

is called **feedback factor.**

$$v_o = Av_{id} = A(v_s - v_1) = A(v_s - \beta v_o)$$

$$A_v = \frac{v_o}{v_s} = \frac{A}{1 + A\beta}$$

Aβ is called loop gain.
For *Aβ* >>1,

$$A_{ideal} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$v_{id} = v_s - v_1 = v_s - \beta v_o$$

$$= v_s - \frac{A\beta}{1 + A\beta} v_s = \frac{v_s}{1 + A\beta}$$

No longer zero*, vid* is small for large *Aβ.*

Gain Error

Gain Error is given by
GE= (ideal gain)-(actual gain)
For non-inverting amplifier,

$$GE = \frac{1}{\beta} - \frac{A}{1 + A\beta} = \frac{1}{\beta(1 + A\beta)}$$

Gain error is also expressed as a fractional or percentage error.

$$FGE = \frac{\frac{1}{\beta} - \frac{A}{1 + A\beta}}{\frac{1}{\beta}} = \frac{1}{1 + A\beta} \cong \frac{1}{A\beta}$$

Gain Error: Example

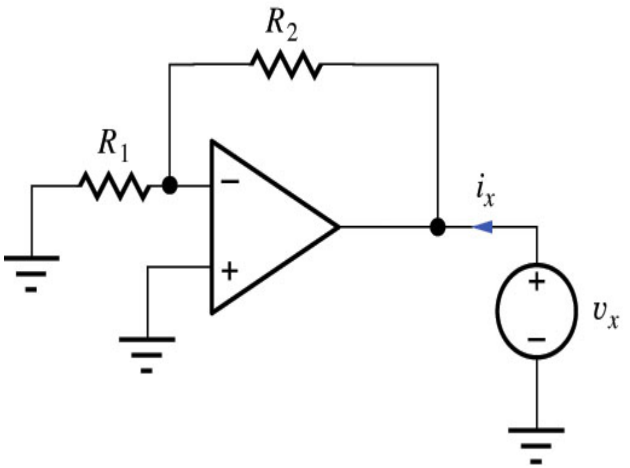
Problem: Find ideal and actual gain and gain error is percent
Given data: Closed-loop gain of 200 (46 dB), open-loop gain of op amp is 10,000 (80 dB).
Approach:Amplifier is designed to give ideal gain and deviations from ideal case are determined. Hence, .

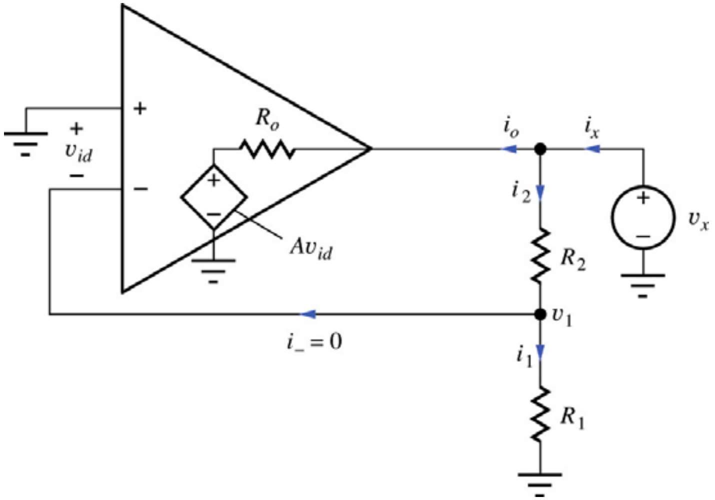
R1 and *R2* aren’t designed to compensate for finite open-loop gain of amplifier.
Analysis:

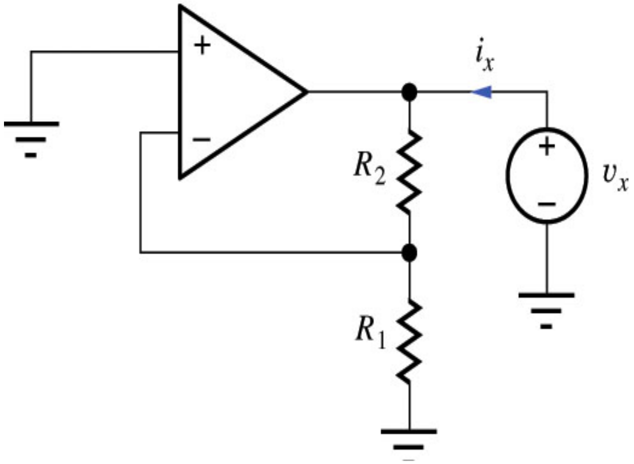
$$A_v = \frac{A}{1 + A\beta} = \frac{10^4}{1 + \frac{10^4}{200}} = 196$$

$$\mathrm{FGE} = \frac{200 - 196}{200} = 0.02$$

Nonzero Output Resistance







Output terminal is driven by test source *vx* and current *ix* is calculated to determine output resistance (all independent sources are turned off).The equivalent circuit is same For both inverting and non-inverting amplifiers.

$$R_{out} = \frac{v_x}{i_x}$$

Nonzero Output Resistance (contd.)

Analysis: $\mathrm{i_x} = \mathrm{i_o} + \mathrm{i_2}$

Also, vid= -v1 and $\mathrm{v_1} = \frac{R_1}{R_1 + R_2}\mathrm{v_x} = \beta \mathrm{v_x}$

$$\mathrm{i_o} = \frac{\mathrm{v_x} - A\mathrm{v_{id}}}{R_o} \quad \mathrm{i_2} = \frac{\mathrm{v_x}}{R_1 + R_2}$$

$$\therefore \frac{1}{R_{out}} = \frac{\mathrm{i_x}}{\mathrm{v_x}} = \frac{1 + A\beta}{R_o} + \frac{1}{R_1 + R_2}$$

Thus, shunt feedback at output reduces *Rout.*

$$\therefore R_{out} = \frac{R_o}{1 + A\beta} \Big\| \left(R_1 + R_2\right)$$

Since, *Ro/(1+Aβ)<<(R*1+*R*2*),* $R_{out} \cong \frac{R_o}{1 + A\beta}$

If A is infinite, *Rout*=0

Open-loop Gain Design: Example

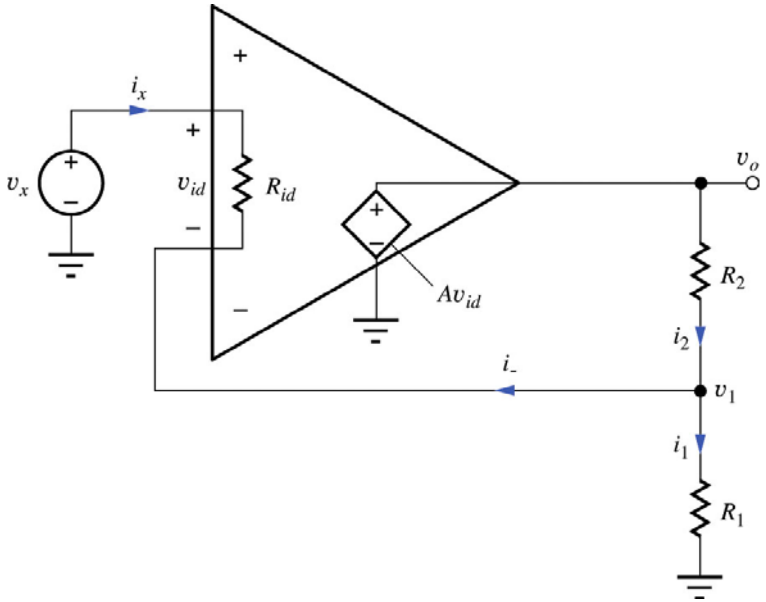
- **Problem**: Design non-inverting amplifier and find open-loop gain
- **Given Data**: A_v=35 dB, R_{out} =0.2Ω, R_o = 250Ω
- **Analysis:**

$$\left|A_v\right| = 10^{35\text{dB}/20\text{dB}} = 56.2 \qquad \beta = \frac{1}{A_v} = \frac{1}{56.2}$$

$$R_{out} = \frac{R_o}{1 + A\beta} \leq 0.2\Omega$$

$$\therefore A \geq \frac{1}{\beta}\left(\frac{R_o}{R_{out}} - 1\right) = 56.2\left(\frac{250}{0.2} - 1\right) = 7.03 \times 10^4 = 96.9\text{dB}$$

Finite Input Resistance: Non-inverting Amplifier



Test voltage source *vx* is applied to input and current *ix* is calculated.

$$i_x = \frac{v_x - v_1}{R_{id}} \qquad v_1 = i_1 R_1 \cong i_2 R_1$$

Assuming i-<<i2 implies i1 = i2.

$$v_1 \cong \frac{R_1}{R_1 + R_2} v_o = \beta v_o$$

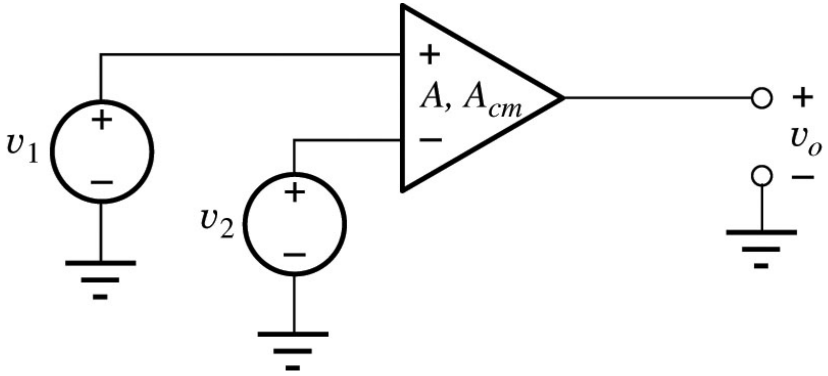
$$= \beta(A v_{id}) = A\beta(v_x - v_1)$$

$$v_1 = \frac{A\beta}{1 + A\beta} v_x$$

$$\therefore i_x = \frac{v_x - \frac{A\beta}{1+A\beta} v_x}{R_{id}} = \frac{v_x}{(1 + A\beta) R_{id}}$$

$$R_{in} = R_{id}(1 + A\beta)$$

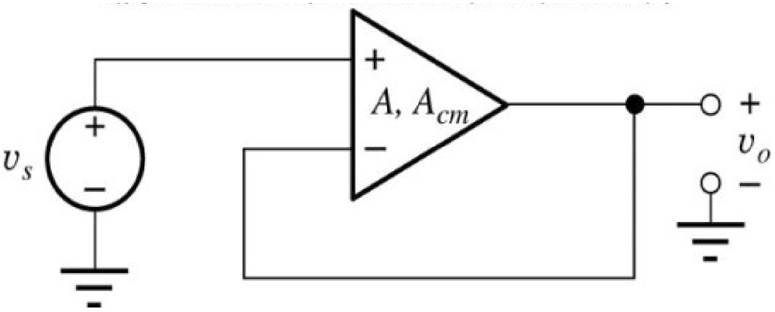
Finite Common-Mode Rejection Ratio (CMRR)



A real amplifier responds to signal common to both inputs, called common-mode input voltage. In general,

$$v_o = A(v_1 - v_2) + A_{cm}\left(\frac{v_1 + v_2}{2}\right)$$
$$= A(v_{id}) + A_{cm}(v_{ic})$$

Voltage Follower Gain Error due to CMRR



$$v_{id} = v_s - v_o \qquad v_{ic} = \frac{v_s + v_o}{2}$$

$$v_o = A\left((v_s - v_o) + \frac{(v_s + v_o)}{2\text{CMRR}}\right)$$

$$A_v = \frac{v_o}{v_s} = \frac{A\left(1 + \frac{1}{2\text{CMRR}}\right)}{1 + A\left(1 - \frac{1}{2\text{CMRR}}\right)}$$

Ideal gain for voltage follower is unity, gain error

$$\text{GE} = 1 - A_v = \frac{1 - \frac{A}{2\text{CMRR}}}{1 + A\left(1 - \frac{1}{2\text{CMRR}}\right)}$$

Since, both A and CMRR are normally >>1,

$$\text{GE} \cong \frac{1}{A} - \frac{1}{\text{CMRR}}$$

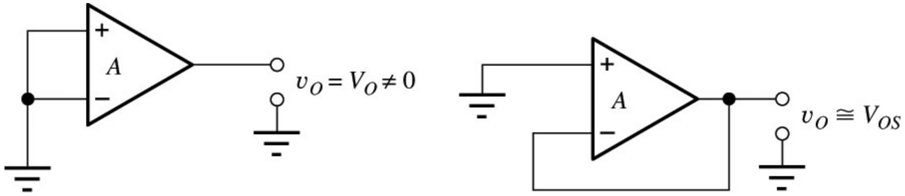
First term is due to finite amplifier gain, second term shows that CMRR may introduce an even larger error.

DC Error Sources: Input-Offset Voltage

With inputs being zero, the amplifier output rests at some dc voltage level instead of zero. The equivalent dc **input offset voltage** is

$$V_{OS} = \frac{V_O}{A}$$

The amplifier is connected as voltage-follower to give output voltage equal to offset voltage.



To include effect of offset voltage

$$v_O = A\left(v_{id} + \frac{v_{ic}}{\text{CMRR}} + V_{OS}\right)$$

If *v*id =0,

$$v_O = A\left(\frac{v_{ic}}{\text{CMRR}} + V_{OS}\right) = A(v_{OS})$$

$$\therefore \text{CMRR} = \frac{\text{v}_{\text{ic}}}{\text{v}_{\text{OS}}}\,\mu\text{V/V}$$

Thus, CMRR is a measure of how total offset voltage changes from its dc value when common-mode voltage is applied.