T-model:
The s/c transconductance:
\[ G_m = \frac{i_o}{v_i} \bigg|_{v_o=0} = \frac{g_m v_e}{v_e} = g_m \]

The input resistance:
\[ R_i = \frac{v_i}{i_e} = \frac{v_e}{i_e} = r_e \]

The output resistance:
\[ R_o = \frac{v_o}{i_o'} \bigg|_{v_i=0} = R_C \]

o/c or unloaded voltage gain,
\[ a_v = \frac{v_o}{v_i} \bigg|_{i_o'=0} = \frac{g_m v_e R_C}{v_e} = g_m R_C \]
\[ a_i = \frac{i_o}{i_i} \bigg|_{v_o=0} = \frac{g_m v_e}{v_e/r_e} = g_m r_e \]
\[ a_i = \frac{i_o}{i_i} \bigg|_{v_o=0} = \frac{g_m v_e}{v_e/r_e} = g_m r_e \]

Since, \( r_e = \frac{\alpha_0}{g_m} \), then \( a_i = g_m \frac{\alpha_0}{g_m} \approx 1 \)

For the CE configuration: \( R_i = r_\pi = \frac{\beta_0}{g_m} \)

For the CB configuration: \( R_i = r_e = \frac{\alpha_0}{g_m} \)

\[ R_{i_{\text{CE}}} > R_{i_{\text{CB}}} \]

\[ R_{i_{\text{CB}}} = \frac{\alpha_0}{\beta_0} R_{i_{\text{CE}}} = \frac{1}{1+\beta_0} R_{i_{\text{CE}}} \]

\[ a_{i_{\text{CB}}} \approx \alpha_0 \]

\[ a_{i_{\text{CE}}} \approx \beta_0 \]

\[ a_{i_{\text{CB}}} < a_{i_{\text{CE}}} \]

\[ a_{i_{\text{CB}}} = \frac{\alpha_0}{\beta_0} a_{i_{\text{CE}}} = \frac{1}{1+\beta_0} a_{i_{\text{CE}}} \]

In terms of i/p resistance and current gain, the CE amplifier performs better than CB.
$C_\mu$ is between B-C. At high freqs., capacitive components are dominant.

For CE, $C_\mu$ is between i/p and o/p. Hence, at high-freqs., there will be a feedback from o/p to i/p.

For CB, i/p is at E and o/p is at C. Therefore, $C_\mu$ will not cause a feedback at high-freqs. CB circuits are used for high-freq. application.
Until now we have assumed that $r_b$ is negligible. In practice, $r_b$ has a significant effect on $G_m$ and $R_i$ when the CB stage is operated at sufficiently high current levels.

The s/c transconductance:

$$G_m = \frac{g_m v_e}{v_e + v_b}$$

KCL at B,

$$i_e = i_{rb} + g_m v_e$$

$$\frac{v_e}{r_e} = \frac{v_b}{r_b} + g_m v_e$$

$$v_b \left(\frac{v_e}{r_e} - g_m v_e\right) = r_b$$

$$G_m = \frac{g_m}{1 + \left(\frac{1}{r_e} - g_m\right)r_b}$$
From Equation (3.29), \( r_e = \frac{1}{g_m + \frac{1}{r_\pi}} \)

\[
\frac{1}{r_e} = g_m + \frac{1}{r_\pi}
\]

\[
G_m = \frac{g_m}{1 + \left( \frac{g_m}{r_\pi} - g_m \right) r_b}
\]

\[
= \frac{g_m}{1 + \frac{r_b}{r_\pi}}
\]

Comparing this with the transconductance of the CB circuit if \( r_b \) is neglected (i.e. \( G_m = g_m \)) shows that \( G_m \) becomes lower when \( r_b \) is included.

![Circuit Diagram]

The input resistance:

\[
R_i = \frac{v_i}{i_e} = \frac{v_b + v_e}{i_e} = \frac{\left( \frac{v_e}{r_e} - g_m v_e \right) r_b + v_e}{i_e} = \frac{\left( \frac{1}{r_e} - g_m \right) r_b + 1}{i_e}
\]

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\[
R_i = \frac{v_i}{i_e} = \frac{v_b + v_e}{i_e} = \left(\frac{v_e - g_m v_e}{r_e}\right) r_b + v_e = \left(\frac{1}{r_e} - g_m\right) r_b + 1
\]

\[
R_i = \left[\left(\frac{1}{r_e} - g_m\right) r_b + 1\right] r_e
\]

Since \(\frac{1}{r_e} = g_m + \frac{1}{r_\pi}\) and \(r_e = \frac{\alpha_o}{g_m}\), then

\[
R_i = \left[\left(g_m + \frac{1}{r_\pi} - g_m\right) r_b + 1\right] \frac{\alpha_o}{g_m} = \left[\frac{r_b}{r_\pi} + 1\right] \frac{\alpha_o}{g_m}
\]

Comparing this with the input resistance of the CB circuit if \(r_b\) is neglected (i.e. \(R_i = \frac{\alpha_o}{g_m}\)) shows that \(R_i\) becomes larger when \(r_b\) is included.

At high current level, i.e. \(I_C \uparrow\), \(r_\pi \downarrow\) as \(r_\pi = \frac{\beta_o}{g_m} = \frac{\beta_o V_T}{I_C}\). So, if \(r_\pi\) is small enough that it is comparable with \(r_b\), then \(r_b\) should be included in the analysis.

Example: If \(r_b = 100 \Omega\), \(\beta_o = 100\) and \(I_C = 26\) mA, then \(r_\pi = \frac{\beta_o}{g_m} = \frac{100 \times 26}{26} = 100 \Omega\). Hence, \(r_\pi = r_b\) when \(I_C\) is high.
In CB configuration, $R_o = R_C$

In CE configuration, $R_o = R_C // r_o$

If $R_C \to \infty$, $R_{o\_CB} = \infty$ and $R_{o\_CE} = r_o$

Under this condition, $R_{o\_CB} > R_{o\_CE}$

Besides using the CB as high freq. amplifier, it can also be used as a current source whose current is nearly independent of the voltage across it (i.e. $i_o = g_m v_e$)

### 3.3.4 Common-gate (CG) configuration.
I/p signal is applied to the S. O/p is taken from the D. G is connected to the ac gnd.

The analysis of CG amplifiers can be simplified if the model is changed from a hybrid-π to a T-model.

The body (B) is ac gnd, $v_{bs} = v_{gs}$ because G is also at gnd.
Figure (b):
Node 1: \( i_{ro} = i_S + (g_m + g_{mb})v_{sg} \)
Node 2: \( i_d + (g_m + g_{mb})v_{sg} = i_{ro} \)

Figure (c):
Node 1: \( i_{ro} = i_S + (g_m + g_{mb})v_{sg} \)
Node 2: \( i_d + (g_m + g_{mb})v_{sg} = i_{ro} \)

Equal currents are pushed into and pulled out of the G as the equations that describe the operation of the circuits are identical.
If \( r_o \) is finite, the circuit is bilateral because of the feedback. If \( r_o \to \infty \), the cct. is unilateral.

For \( r_o \to \infty \):
\[ G_m = \left. \frac{i_o}{v_i} \right|_{v_o=0} = \frac{(g_m + g_{mb})v_{sg}}{v_{sg}} = g_m + g_{mb} \]

\[ R_i = \left. \frac{v_i}{i_i} \right|_{i_i=0} = \frac{1}{g_m + g_{mb}} \]

\[ R_o = \left. \frac{v_o}{i_o'} \right|_{v_i=0} = R_D \]

\[ a_v = \left. \frac{v_o}{v_i} \right|_{i_o'=0} = \frac{(g_m + g_{mb})v_{sg}R_d}{v_{sg}} = (g_m + g_{mb})R_d \]

\[ a_i = \left. \frac{i_o}{i_i} \right|_{v_o=0} = \frac{(g_m + g_{mb})v_{sg}}{\frac{v_{sg}}{1} \frac{1}{g_m + g_{mb}}} = (g_m + g_{mb}) \left( \frac{1}{g_m + g_{mb}} \right) = 1 \]
3.3.6 Common-collector (CC) configuration (Emitter follower)

I/p signal applied to the B.
O/p signal taken from the E.
\[ v_S = i_i (R_S + r_\pi) + v_o \]
\[ v_i = i_i r_\pi + v_o \]
\[ R_i = \frac{v_i}{i_i} \]

Hence, this circuit is not unilateral as the input resistance depends on the load resistor \( R_L \) and the output resistance depends on the source resistance \( R_S \).

To determine \( R_i \):
\[ R_i = \frac{v_i}{i_i} \]
\[ i_i + \beta_o i_i = \frac{v_o}{R_L} + \frac{v_o}{r_o} \]
\[ i_1 + \beta_0 i_i = \frac{v_o}{R_L} + \frac{v_o}{r_o} \]

\[ i_i (1 + \beta_o) = (v_i - i_i r_\pi) \left( \frac{1}{R_L} + \frac{1}{r_o} \right) \]

\[ i_i \left[ 1 + \beta_o + r_\pi \left( \frac{1}{R_L} + \frac{1}{r_o} \right) \right] = v_i \left( \frac{1}{R_L} + \frac{1}{r_o} \right) \]

\[ R_i = \frac{v_i}{i_i} = \frac{1 + \beta_o + r_\pi \left( \frac{1}{R_L} + \frac{1}{r_o} \right)}{\left( \frac{1}{R_L} + \frac{1}{r_o} \right)} \quad \leftarrow \text{enough.} \]
\[ R_i = \frac{1 + \beta_o + r_\pi \left( \frac{1}{R_L} + \frac{1}{r_o} \right)}{\left( \frac{1}{R_L} + \frac{1}{r_o} \right)} = \frac{1 + \beta_o}{1 + \beta_o} + r_\pi \]

For \( R_i \) with no load, i.e. \( R_i = \frac{v_i}{i_i} \bigg|_{R_L=\infty} \),

\[ R_i = (1 + \beta_o)r_o + r_\pi \]

To determine \( a_v \):

Overall voltage gain: \( a_v = v_o / v_s \)
At node E,

\[ \frac{v_s - v_o}{R_s + r_\pi} + g_m v_1 = v_o \left( \frac{1}{R_L} + \frac{1}{r_o} \right) \]

\[ \frac{v_s}{R_s + r_\pi} + g_m \frac{r_\pi(v_s - v_o)}{R_s + r_\pi} = v_o \left( \frac{1}{R_L} + \frac{1}{r_o} + \frac{1}{R_s + r_\pi} \right) \]

Since \( g_m r_\pi = \beta_o \),

\[ \frac{v_s + \beta_o(v_s - v_o)}{R_s + r_\pi} = v_o \left( \frac{1}{R_L} + \frac{1}{r_o} + \frac{1}{R_s + r_\pi} \right) \]

\[ v_s \left( \frac{1}{R_s + r_\pi} + \frac{\beta_o}{R_s + r_\pi} \right) = v_o \left( \frac{1}{R_L} + \frac{1}{r_o} + \frac{1}{R_s + r_\pi} + \frac{\beta_o}{R_s + r_\pi} \right) \]
\[ a_v = \frac{v_o}{v_s} = \frac{(1+\beta_o)\left( \frac{1}{R_s+r_\pi} \right)}{\frac{1}{R_L} + \frac{1}{r_o} + \frac{1+\beta_o}{R_s+r_\pi}} \quad \leftarrow \text{enough.} \]

\[ = \frac{1}{\frac{R_s+r_\pi}{(1+\beta_o)R_L} + \frac{R_s+r_\pi}{(1+\beta_o)r_o} + 1} \]

\[ = \frac{1}{\frac{r_o(R_s+r_\pi) + R_L(R_s+r_\pi)}{(1+\beta_o)R_Lr_o} + 1} \]

\[ = \frac{1}{\frac{(R_s+r_\pi)(r_o+R_L)}{(1+\beta_o)R_Lr_o} + 1} \]

\[ = \frac{1}{\frac{(R_s+r_\pi)}{(1+\beta_o)R_L // r_o} + 1} \]

Open-circuit overall voltage gain,

\[ a_v = \frac{v_o}{v_s} \bigg|_{R_L=\infty} = \frac{1}{\left( \frac{R_s+r_\pi}{(1+\beta_o)r_o} + 1 \right)} \]
If the B resistance, $r_b$, is significant, it can be simply be added to $R_S$ in the expression. From the $a_v$ expression, $a_v$ will always be less than unity.

If $\beta_o R_L/\pi \gg R_s + r_\pi$, then $a_v \approx 1$. This means that the output signal follows the input signal. Hence, this topology is also known as the emitter follower.

If $r_\pi \gg R_s$, $\beta_o \gg 1$ and $r_o \gg R_L$, then

$$a_v = \frac{1}{r_\pi} \left( \frac{r_\pi}{(\beta_o)R_L} + 1 \right)$$

Since $g_m r_\pi = \beta_o$, then

$$a_v = \frac{1}{g_m R_L} \left( \frac{1}{g_m R_L} + 1 \right) = \frac{g_m R_L}{1 + g_m R_L}$$
To determine the short circuit current gain, \(a_i\):

\[
a_i = \frac{i_o}{i_i} \bigg|_{v_o=0}
\]

\[i_o = i_i + g_m v_1\]

\[v_1 = i_i r_\pi\]

\[i_o = i_i + g_m i_i r_\pi\]

\[
a_i = \frac{i_o}{i_i} = \frac{i_i (1 + g_m r_\pi)}{i_i} = 1 + g_m r_\pi = 1 + \beta_o
\]

or

\[g_m v_1 = g_m i_i r_\pi = \beta_o i_i\]

\[i_o = i_i + \beta_o i_i = i_i (1 + \beta_o)\]

\[a_i = \frac{i_o}{i_i} = 1 + \beta_o\]