COHERENT BINARY MODULATION TECHNIQUES

As mentioned previously, binary modulation has three basic forms: amplitude-shift keying (ASK), phase-shift keying (PSK), and frequency-shift keying (FSK). In this section, we present the noise analysis for the coherent detection of PSK and FSK signals, assuming an *additive white Gaussian noise (AWGN) model.*

4.1 Coherent Binary PSK

- In a coherent binary PSK system, the pair of signals, $s_1(t)$ and $s_2(t)$, used to represent binary symbols 1 and 0, respectively, are defined by

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \qquad 4.1$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \qquad 4.2$$

where $0 \leq t < T_b$ and E_b is the *transmitted signal energy per hit.*

- In order to ensure that each transmitted bit contains an integral number of cycles of the carrier wave, the carrier frequency $f_c = n_c/T_b$ for some fixed integer n_c.

- A pair of sinusoidal waves that differ only in a relative phase-shift of 180 degrees, as defined above, are referred to as *antipodal signals.*

- From Eqs. 4.1 and 4.2, it is clear that there is only one basis function of unit energy, namely

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \qquad 0 \leqslant t < T_b \tag{4.3}$$

- Then we may expand the transmitted signals $s_1(t)$ and $s_2(t)$ in terms of $\phi(t)$ as follows

$$s_1(t) = \sqrt{E_b}\phi_1(t) \qquad 0 \leqslant t < T_b \tag{4.4}$$

$$s_2(t) = -\sqrt{E_b}\phi_1(t) \qquad 0 \leqslant t < T_b \tag{4.5}$$

- A coherent binary PSK system is therefore characterized by having a signal space that is one-dimensional (i.e., N = 1) and with two message points (i.e., M = 2), as shown in Figure. 4.1. The coordinates of the message points equal

$$\begin{aligned} s_{11} &= \int_0^{T_b} s_1(t)\phi_1(t)dt \\ &= +\sqrt{E_b} \end{aligned} \tag{4.6}$$

$$\begin{aligned} s_{21} &= \int_0^{T_b} s_2(t)\phi_1(t)dt \\ &= -\sqrt{E_b} \end{aligned} \tag{4.7}$$

- The message point corresponding to $s_1(t)$ is located at $s_{11} = +\sqrt{E_b}$, and the message point corresponding to $s_2(t)$ is located at $s_{11} = -\sqrt{E_b}$.

- We must partition the signal space of Fig. 4.1 into two regions:

1. The set of points closest to the message point at. $+\sqrt{E_b}$
2. The set of points closest to the message point at $-\sqrt{E_b}$

  - This is accomplished by constructing the midpoint of the line joining these two message points, and then marking off the appropriate decision regions. In Figure 4.1 these decision regions are marked Z_1 and Z_2 according to the message point around which they are constructed.

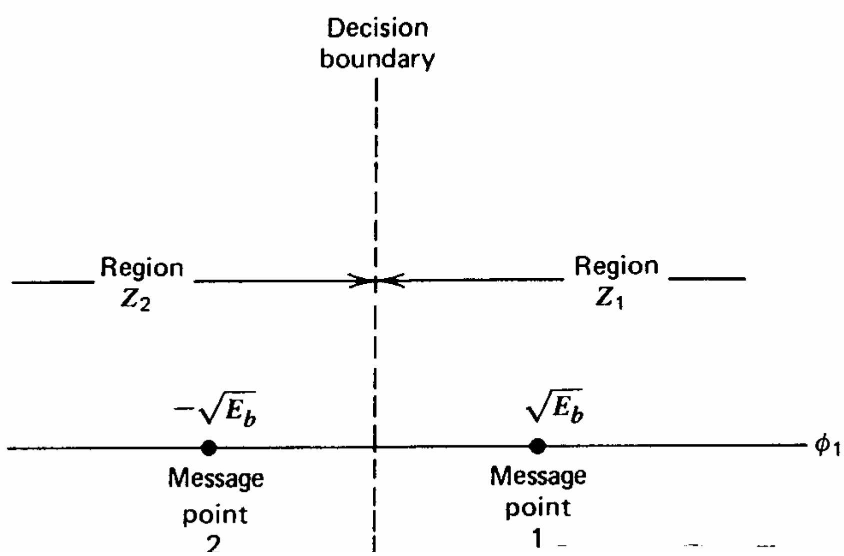

FIGURE 4.1 Signal space diagram for coherent binary PSK system

- The decision rule is now simply to guess signal $s_1(t)$ or binary symbol 1 was transmitted if the received signal point falls in region Z_1, and

- Signal $s_2(t)$ or binary symbol 0 was transmitted if the received signal point falls in region Z_2. Two kinds of erroneous decisions may, however, be made.

- Signal $s_2(t)$ is transmitted, but the noise is such that the received signal point falls inside region Z_1 and so the receiver decides in favor of signal $s_1(t)$.

- Alternatively, signal $s_1(t)$ is transmitted, but the noise is such that the received signal point falls inside region Z_2 and so the receiver decides in favor of signal $s_2(t)$.

- The average probability of symbol error for coherent binary PSK equals

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad 4.8$$

- To generate a binary PSK wave, the input binary sequence in polar form with symbols 1 and 0 represented by constant amplitude levels of $+\sqrt{E_b}$ and $-\sqrt{E_b}$, respectively.

- This binary wave and a sinusoidal carrier were ϕ_1 (t) (whose frequency f_c n_c/T_b for some fixed integer n_c) are applied to a product modulator, as in Figure 4.2b.

- The carrier and the timing pulses used to generate the binary wave are usually extracted from a common master clock. The desired PSK wave is obtained at the modulator output.

- To detect the original binary sequence of 1s and 0s, we apply the noisy PSK wave $x(t)$ (at the channel output) to a correlator, which is also supplied with a locally generated coherent reference signal $\phi_1(t)$, as in Figure 4.2b.

- The correlator output, x_1, is compared with a threshold of zero volts.

If $x_1 > 0$, the receiver decides in favor of symbol 1.

If $x_1 < 0$, it decides in favor of symbol 0.

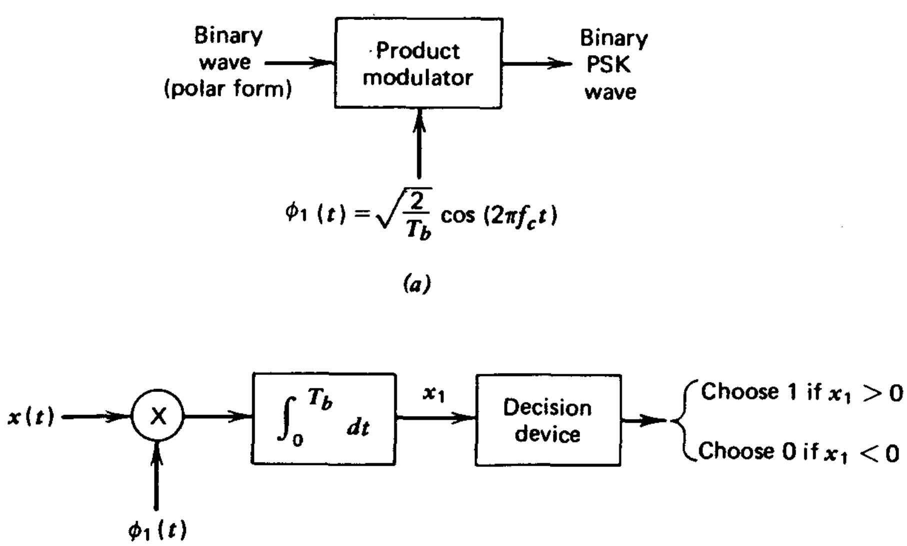

FIGURE 4.2 Block diagram for (a) binary PSK transmitter and (b) coherent binary PSK receiver

4.2 Coherent Binary FSK

In a binary FSK system, symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount. **A** typical pair of sinusoidal waves is described by

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases} \tag{4.9}$$

where i = 1, 2, and E_b is the transmitted signal energy per bit, and the transmitted frequency equals

$$f_i = \frac{n_c + i}{T_b} \quad \text{for some fixed integer } n_c \text{ and} \tag{4.10}$$

Thus symbol 1 is represented by $s_1(t)$, and symbol 0 by $s_2(t)$.

From Eq 4.9 we observe directly that the signals $s_1(t)$ and $s_2(t)$ are orthogonal, but not normalized to have unit energy. We therefore deduce that the moss useful form for the set of orthonormal basis functions is

$$\phi_i(t) = \begin{cases} \sqrt{\dfrac{2}{T_b}} \cos(2\pi f_i t) & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases} \tag{4.11}$$

where i = 1, 2. Correspondingly, the coefficient s_{ij} for i = 1, 2, and j = 1, 2. is definers by

$$
\begin{aligned}
s_{ij} &= \int_0^{T_b} s_i(t)\phi_j(t)dt \\
&= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_j t)dt \\
&= \begin{cases} \sqrt{E_b} & i = j \\ 0 & i \neq j \end{cases}
\end{aligned} \qquad 4.12
$$

Thus a coherent binary FSK system is characterized by having a signal space that is two-dimensional (i.e., N = 2) with two message points (i.e., M = 2), as in Figure 4-3. The two message points are defined by the signal vectors:

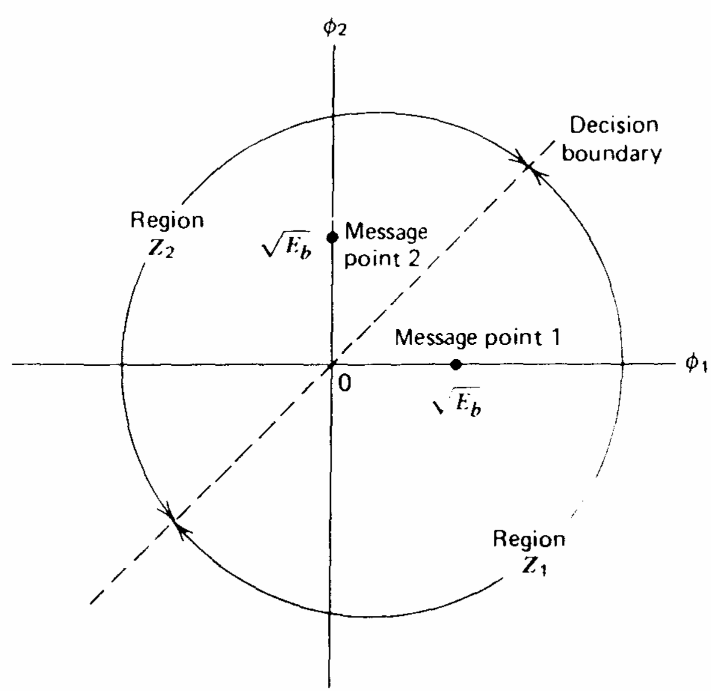

FIGURE 4-3 Signal space diagram for coherent binary FSK system.

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \quad 4.13$$

$$\mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix} \quad 4.14$$

- Note that the distance between the two message points is equal to $\sqrt{2E_b}$ The observation vector x has two elements, x_1 and x_2, that are defined by, respectively

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)dt \quad 4.15$$

$$x_2 = \int_0^{T_b} x(t)\phi_2(t)dt \quad 4.16$$

where *x(t)* is the received signal, the form of which depends on which symbol was transmitted.

- Given that symbol 1 was transmitted, x(t) equals $s_1(t)$ + w(t), where w(t) is the sample function of a white Gaussian noise process of zero mean and power spectral density $N_o/2$.

- Symbol 0 was transmitted, *x(t)* equals $s_2(t)$ + w(t).

- The receiver decides in favor of symbol 1 if the received signal point represented by the observation vector x falls inside region Z_1. This occurs when $x_1 > x_2$. If, on the other hand, we have $x_1 < x_2$, the received signal point falls inside region Z_2, and the receiver decides in favor of symbol 0. The decision boundary, separating region Z_1 from region Z_2, is defined by $x_1 = x_2$.

The average probability of symbol error for coherent binary FSK is

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}} \right) \tag{4.17}$$

- To generate a binary FSK signal, we may use the scheme shown in Figure 4-4a.

- The input binary sequence is represented in its on–off form, with symbol 1 represented by a constant amplitude of $\sqrt{E_b}$ volts and symbol 0 represented by zero volts.

- By using an *inverter* in the lower channel in Figure 4-4a we in effect make sure that when we have symbol 1 at the input, the oscillator with frequency f_1 in the upper channel is switched on while the oscillator with frequency f_2 in the lower channel is switched off, with the result that frequency f_1 is transmitted.

- Conversely, when we have symbol 0 at the input, the oscillator in the upper channel is switched off, and the oscillator in the lower channel is switched on, with the result that frequency f_2 is transmitted. The two frequencies f_1 and f_2 are chosen to equal integer multiples of the bit rate $1/T_b$.

- In the transmitter of Figure 4-4a, we assume that the two oscillators are synchronized, so that their outputs satisfy the requirements of the two orthonormal basis functions $\phi_1(t)$ and $\phi_2(t)$.

- Alternatively, we may use a single keyed (voltage-controlled) oscillator. In either case, the frequency of the modulated wave is shifted with *a continuous phase,* in accordance with the input binary wave.

- That is to say phase continuity is always maintained, including the inter-bit switching times. We refer to this form of digital modulation as *continuous-phase frequency-shift keying* (CPFSK).

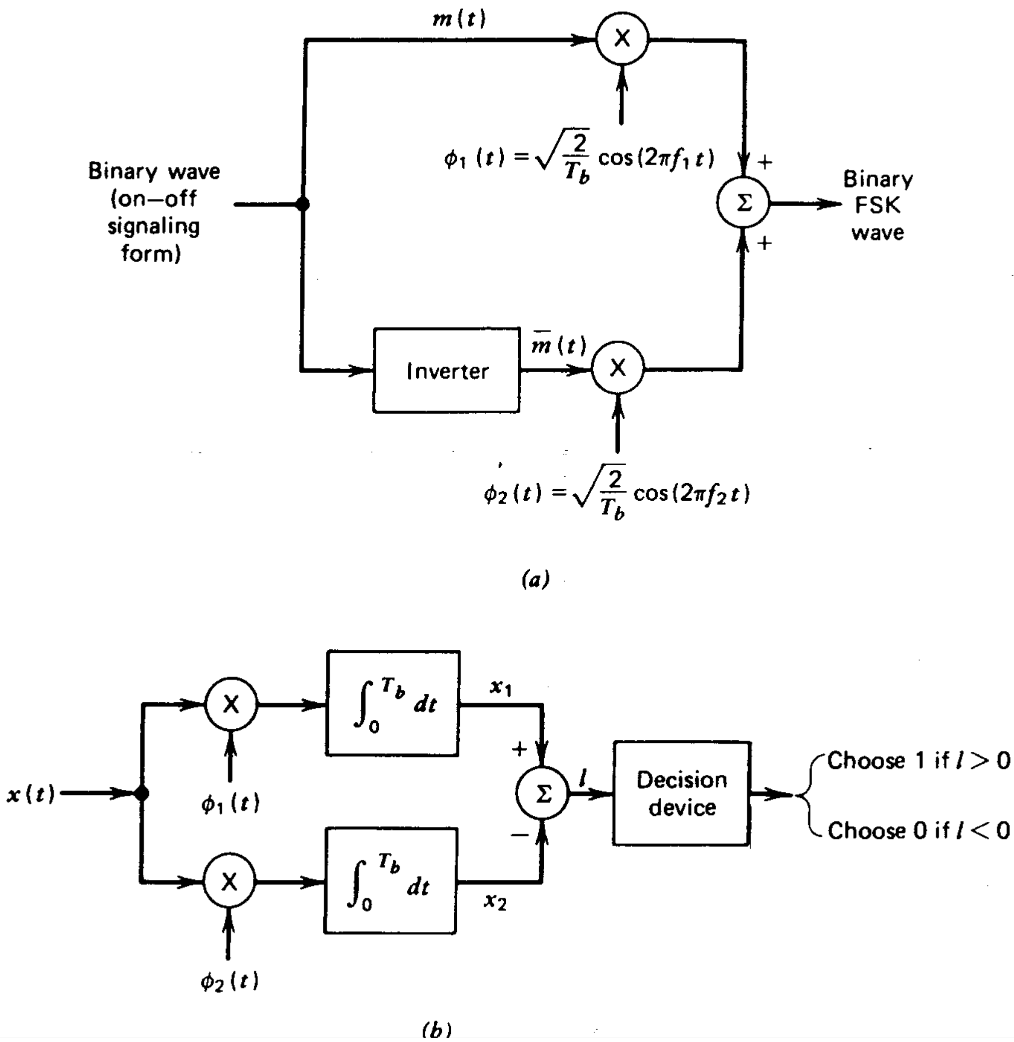

FIGURE 4-4 Block diagrams for (a) binary FSK transmitter, and (b) coherent binary FSK receiver.

- In order to detect the original binary sequence given the noisy received wave x(t), we may use the receiver shown in Figure 4-4 b.

- It consists of two correlators with a common input, which are supplied with locally generated coherent reference signals $\phi_1(t)$ and $\phi_2(t)$.

- The correlator outputs are then subtracted, one from the other, and the resulting difference, *l*, is compared with a threshold of zero volts. If $l > 0$, the receiver decides in favor of 1. On the other hand, if $l < 0$, it decides in favor of 0.

4.3 COHERENT QUADRATURE-MODULATION TECHNIQUES

- The provision of reliable performance:-

    1. Very low probability of error.

    2. Efficient utilization of channel bandwidth.

In this section, we study two bandwidth-conserving modulation schemes for the transmission of binary data. They are both examples of the *quadrature-carrier multiplexing system,* which produces a modulated wave described as follows:

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t) \quad 4.18$$

where $s_I(t)$ is the *in-phase component* of the modulated wave, and $s_Q(t)$ is the *quadrature component.*

4.3.1 Quadriphase-shift Keying

- In *quadriphase-shift keying* (QPSK), the phase of the carrier takes on one of four equally spaced values, such as π /4, 3π/4, 5π/4 and 7π/4 as shown by

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i - 1)\frac{\pi}{4}\right] & 0 \leq \\ 0 & \text{elsewhere} \end{cases} \qquad 4.19$$

- where i = 1, 2, 3, 4, and E is the transmitted signal energy per symbol, T is the symbol duration, and the carrier frequency f. equals n_c/T for some fixed integer n_c. Each possible value of the phase corresponds to a unique pair of bits called a *dibit.*
- Thus, for example, we may choose the foregoing set of phase values to represent the Gray encoded set of dibits: 10,00,01, and 11.

Using a well-known trigonometric identity, we may rewrite Eq. 4.19 in the equivalent form:

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[(2i - 1)\frac{\pi}{4}\right] \cos(2\pi f_c & \\ \qquad - \sqrt{\frac{2E}{T}} \sin\left[(2i - 1)\frac{\pi}{4}\right] \sin(2\pi f_c t) & 0 \leq t \leq T \\ 0 & \text{elsewhere} \end{cases} \qquad 4.20$$

where i = 1, 2, 3, 4. Based on this representation, we can make the following observations:

1. There are only two orthonormal basis functions, $\phi_1(t)$ and $\phi_2(t)$ contained in the expansion of $s_i(t)$. The appropriate forms for $\phi_1(t)$ and $\phi_2(t)$ are defined by

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad 0 \leq t \leq T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \qquad 0 \leq t \leq T \tag{4.21}$$

2. There are four message points, and the associated signal vectors are defined by

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E} \cos\left((2i-1)\frac{\pi}{4}\right) \\ -\sqrt{E} \sin\left((2i-1)\frac{\pi}{4}\right) \end{bmatrix} \qquad i = 1, 2, \tag{4.22}$$

The elements of the signal vectors, namely, s_{i1} and s_{i2}, have their values summarized in Table 4.1. The first two columns of this table give the associated dibits and phase of the QPSK signal.

Table 4.1 Signal Space characterization of QPSK

Input dibit $0 \leq t \leq T$	Phase of QPSK signal (*radians*)	Coordinates of message points	
		s_{i1}	s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

Accordingly, a QPSK signal is characterized by having a two-dimensional signal constellation (i.e., $N = 2$) and four message points (i.e., $M = 4$), as illustrated in Figure 4-5.

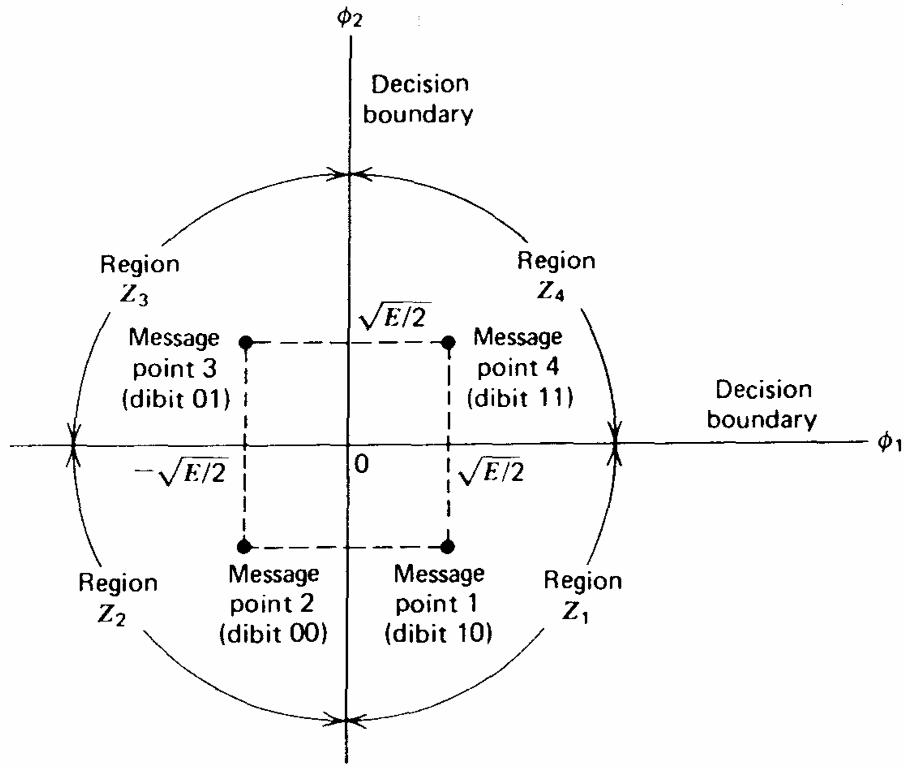

FIGURE 4-5 Signal Space diagram for coherent QPSK

The average probability of symbol error for coherent QPSK is therefore

$$P_e \simeq \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) \tag{4.23}$$

In a QPSK system, we note that there are two bits per symbol. This means that the transmitted signal energy per symbol is twice the signal energy per bit: that is,

$$E = 2E_b \tag{4.24}$$

Thus, expressing the average probability of symbol error in terms of the ratio E_b/No, we may write

$$P_e \simeq \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{4.25}$$

- Figure 4-6a shows the block diagram of a typical QPSK transmitter.

- The input binary sequence b(t) is represented in polar form, with symbols 1 and 0 represented by $+\sqrt{E_b}$ and $-\sqrt{E_b}$ volts, respectively.

- This binary wave is divided by means of a demultiplexer into two separate binary waves consisting of the odd- and even numbered input bits.

- These two binary waves are denoted by $b_1(t)$ and $b_2(t)$. The amplitudes of $b_1(t)$ and $b_2(t)$ equal s_{i1} and s_{i2} respectively depending on the particular dibit that is being transmitted.

- The two binary waves $b_1(t)$ and $b_2(t)$ are used to modulate a pair of quadrature orthonormal basis functions: $\phi_1(t) = \sqrt{2/T}\cos(2\pi f_c t)$ and $\phi_1(t) = \sqrt{2/T}\cos(2\pi f_c t)$. The result is a pair of binary PSK waves, which may detected independently due to the orthogonality of $\phi_1(t)$ and $\phi_2(t)$.

- The QPSK receiver consists of a pair of correlators with a common input and supplied with a locally generated pair of coherent reference signals $\phi_1(t)$ and $\phi_2(t)$ as in Figure 4-6b. The correlator outputs, x_1 and x_2, are each compared with a threshold of zero volts.

- If $x_1 > 0$, a decision is made in favor of symbol 1 for the upper or in-phase channel output, but if $x_1 < 0$ a decision is made in favor of symbol 0**.**

- If $x_2 > 0$, a decision is made in favor of symbol 1 for the lower or quadrature channel output, but if $x_2 < 0$, a decision is made in favor of symbol 0.

- Finally, these two binary sequences at the in-phase and quadrature channel outputs are combined in *a multiplexer* to reproduce the original binary sequence at the transmitter input with the minimum probability of symbol error.

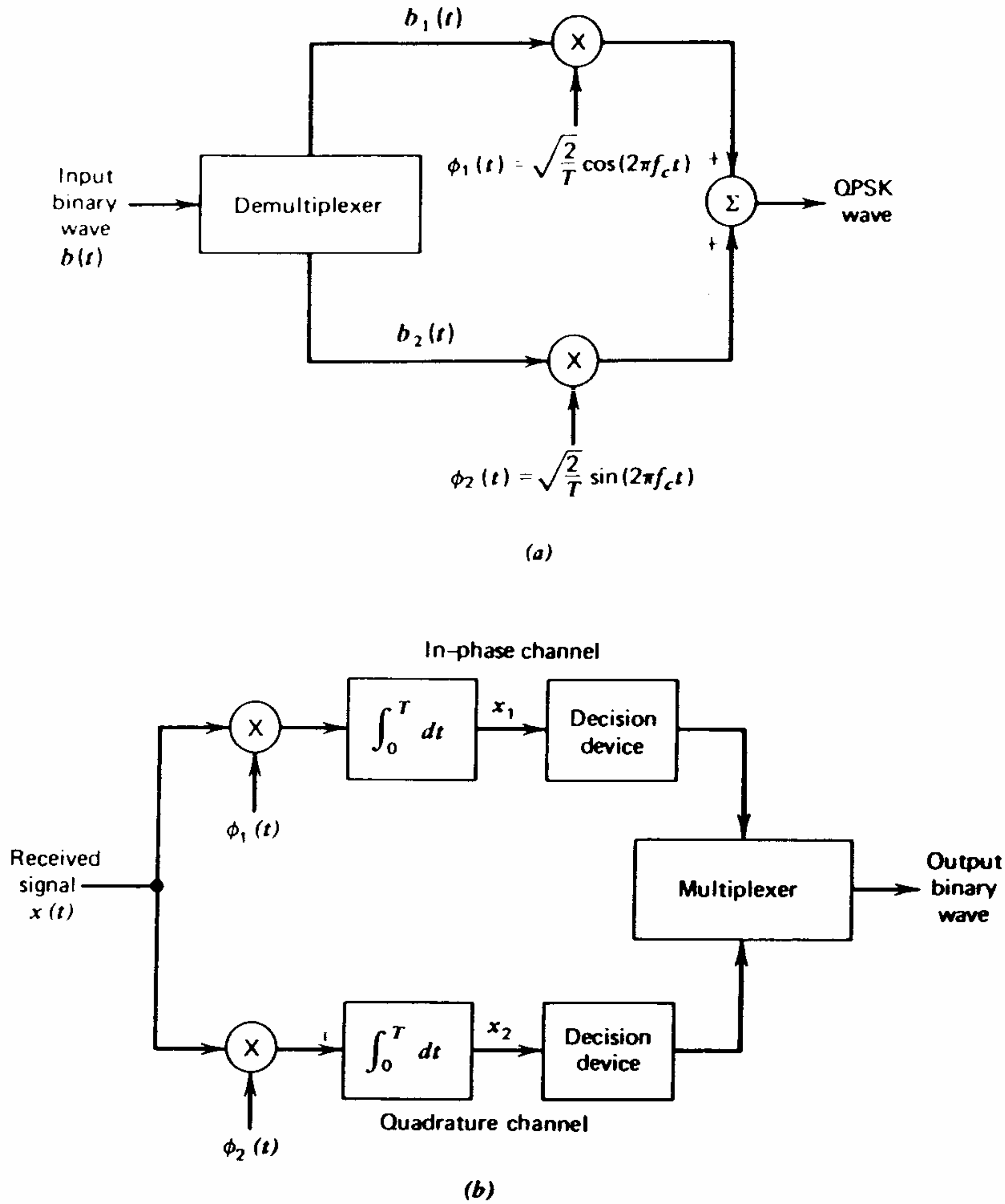

FIGURE 4-6 Block diagrams for (a) QPSK transmitter, and (b) QPSK receiver.

4.4 NONCOHERENT BINARY MODULATION TECHNIQUES

4.4.1 Noncoherent Orthogonal Modulation

- Consider a binary signaling scheme that involves the use of two orthogonal signals $s_1(t)$ and $s_2(t)$, which have equal energy.

- During the interval $0 \le t \le T$, one of these two signals is sent over an imperfect channel that shifts the carrier phase by an unknown amount.

- Let $g_1(t)$ and $g_2(t)$ denote the phase-shifted versions of $s_1(t)$ and $s_2(t)$, respectively.

- It is assumed that the signals $g_1(t)$ and $g_2(t)$ remain orthogonal and of equal energy, regardless of the unknown carrier phase. We refer to such a signaling scheme as *noncoherent orthogonal modulation.*

The channel also introduces an additive white Gaussian noise $W(t)$ of zero mean and power spectral density No/2. We may thus express the received signal $x(t)$ as

$$x(t) = \begin{cases} g_1(t) + w(t) & 0 \leq t \leq T \\ g_2(t) + w(t) & 0 \leq t \leq T \end{cases} \tag{4.26}$$

The requirement is to use $x(t)$ to discriminate between $s_1(t)$ and $s_2(t)$, regardless of the carrier phase.

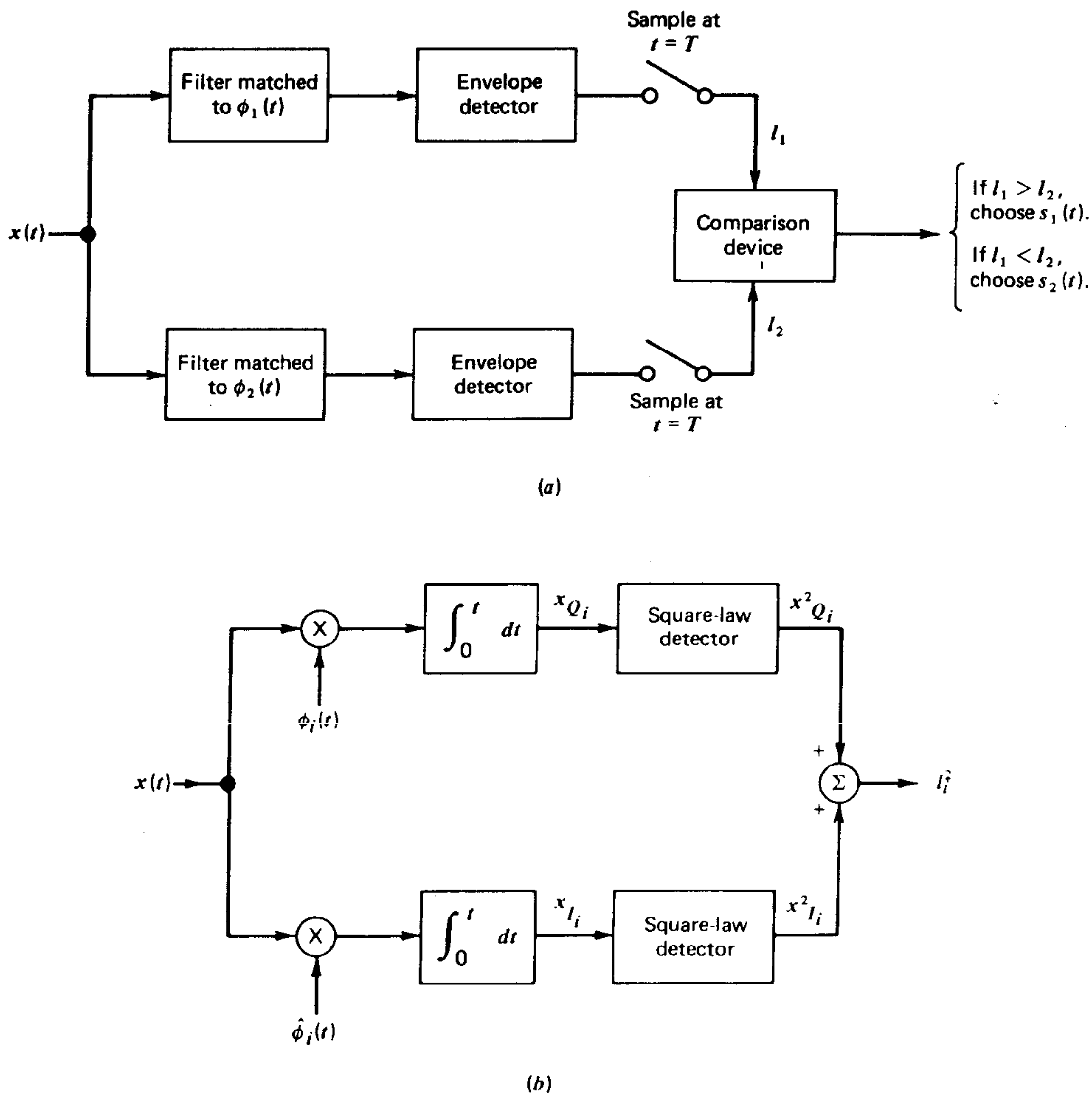

FIGURE 4-7 (a) Generalized binary receiver for noncoherent orthogonal modulation. (b) Quadrature receiver equivalent to either one of the two matched filters in part a; the index i = 1, 2.

- Figure 4.7a has an output amplitude l_1 greater than the output amplitude l_2 of the lower path, the receiver makes a decision in favor of $s_1(t)$. If the converse is true, it decides in favor of $s_2(t)$.
- When they are equal, the decision may be made by flipping a fair coin.

- In any event, a decision error occurs when the matched filter that rejects signal component of the received signal *x(t)* has a larger output amplitude (due to noise alone) than the matched filter that passes it.

- The quadrature receiver is shown in Figure 4-7b. In the upper path, called the *in-phase path,* the received signal *x(t) is* correlated against the basis function $\phi_i(t)$, representing a scaled version of the transmitted signal $s_1(t)$ or $s_2(t)$ with zero carrier phase.

- In the lower path, called the *quadrature path,* on the other hand, *x(t) is* correlated against another basis function $\hat{\phi}_i(t)$ representing the version of $\phi_i(t)$, that results from shifting the carrier phase by -90°. Naturally, $\phi_i(t)$, and $\hat{\phi}_i(t)$ are orthogonal to each other.

The average probability error for the noncoherent receiver is given as

$$P_e = \frac{1}{2}\exp\left(\frac{-E}{2N_o}\right) \tag{4.27}$$

4.4.2 Noncoherent Binary FSK

In the binary FSK case, the transmitted signal is defined by

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_i t) & 0 \leq t \leq \\ 0 & \text{elsewhere} \end{cases} \tag{4.28}$$

- where the carrier frequency f_i equals one of two possible values f_1 and f_2.

- The transmission of frequency f_1 represents symbol 1, and the transmission of frequency f_2 represents symbol 0.

- For the noncoherent detection of this frequency-modulated wave, the receiver consists of a pair of matched filters followed by envelope detectors, as in Fig. 4-8.

- The filter in the upper path of the receiver is matched to $\sqrt{2/T_b}\cos(2\pi f_1 t)$ and the filter in the lower path is matched to $\sqrt{2/T_b}\cos(2\pi f_2 t)$ and $0 \le t \le T_b$.

- The resulting envelope detector outputs are sampled at $t = T_b$, and their values are compared. The envelope samples of the upper and lower paths in Figure 4-8 are shown as l_1 and l_2, signals respectively; then, if $l_1 > l_2$, the receiver decides in favor of symbol 1, and if $l_1 < l_2$ it decides in favor of symbol 0.

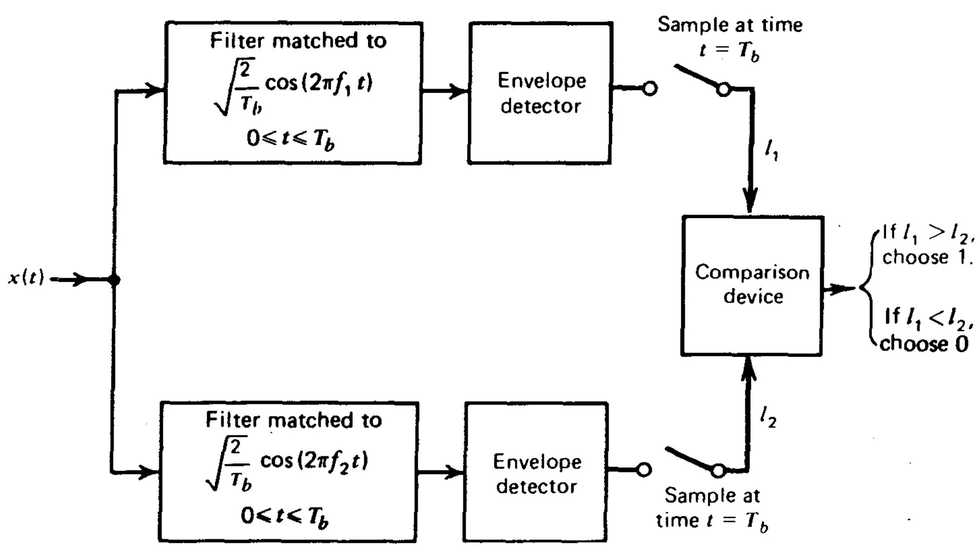

FIGURE 4-8 Noncoherent receiver for detection of binary FSK

- The noncoherent binary FSK described is a special case of noncoherent orthogonal modulation with

$$T = T_b \tag{3.57}$$

and

$$E = E_b \tag{3.58}$$

The average probability of error for noncoherent binary FSK is given by

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right) \tag{4.29}$$

3.5.3 Differential Phase-shift Keying

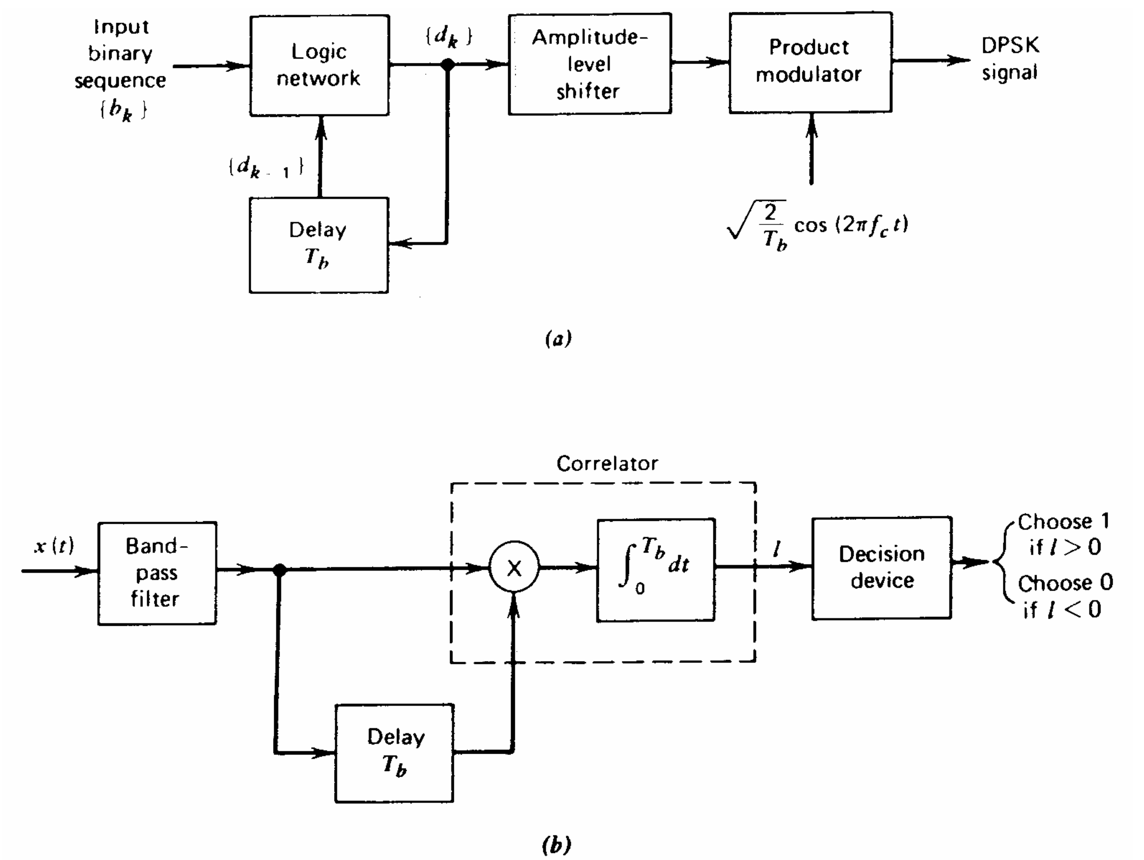

FIGURE 4-9 Block diagrams for (a) DPSK transmitter, and (b) DPSK receiver.

- The transmitted DPSK signal equals $\sqrt{E_b / 2T_b}\cos(2\pi f_c t)$ for $0 \le t \le T_b$, where T_b is the bit duration and E_b is the signal energy per bit.

- Let $s_1(t)$ denote the transmitted DPSK signal for $0 \le t \le 2T_b$ for the case when we have binary symbol 1 at the transmitter input for the second part of this interval, namely, $T_b \le t \le 2T_b$.

- The transmission of 1 leaves the carrier phase unchanged, and so we define $s_1(t)$ as

$$s_1(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}} \cos(2\pi f_c t) & 0 \leqslant t \leqslant T \\ \sqrt{\dfrac{E_b}{2T_b}} \cos(2\pi f_c t) & T_b \leqslant t \leqslant 2T_b \end{cases} \qquad 4.30$$

Let $s_2(t)$ denote the transmitted DPSK signal for $0 \leq t \leq 2T_b$ for the case when we have binary symbol 0 at the transmitter input for $T_b \leq t \leq 2T_b$.

The transmission of 0 advances the carrier phase by 180°, and so we define $s_2(t)$ as

$$s_2(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}} \cos(2\pi f_c t) & 0 \leqslant \\ \sqrt{\dfrac{E_b}{2T_b}} \cos(2\pi f_c t + \pi) & T_b \leqslant t \leqslant 2T_b \end{cases} \qquad 4.31$$

DPSK is a special case of noncoherent orthogonal modulation with

$$T = 2T_b$$

and

$$E = 2E_b$$

The average probability of error for DPSK is given by

$$P_e = \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right) \qquad 4.32$$

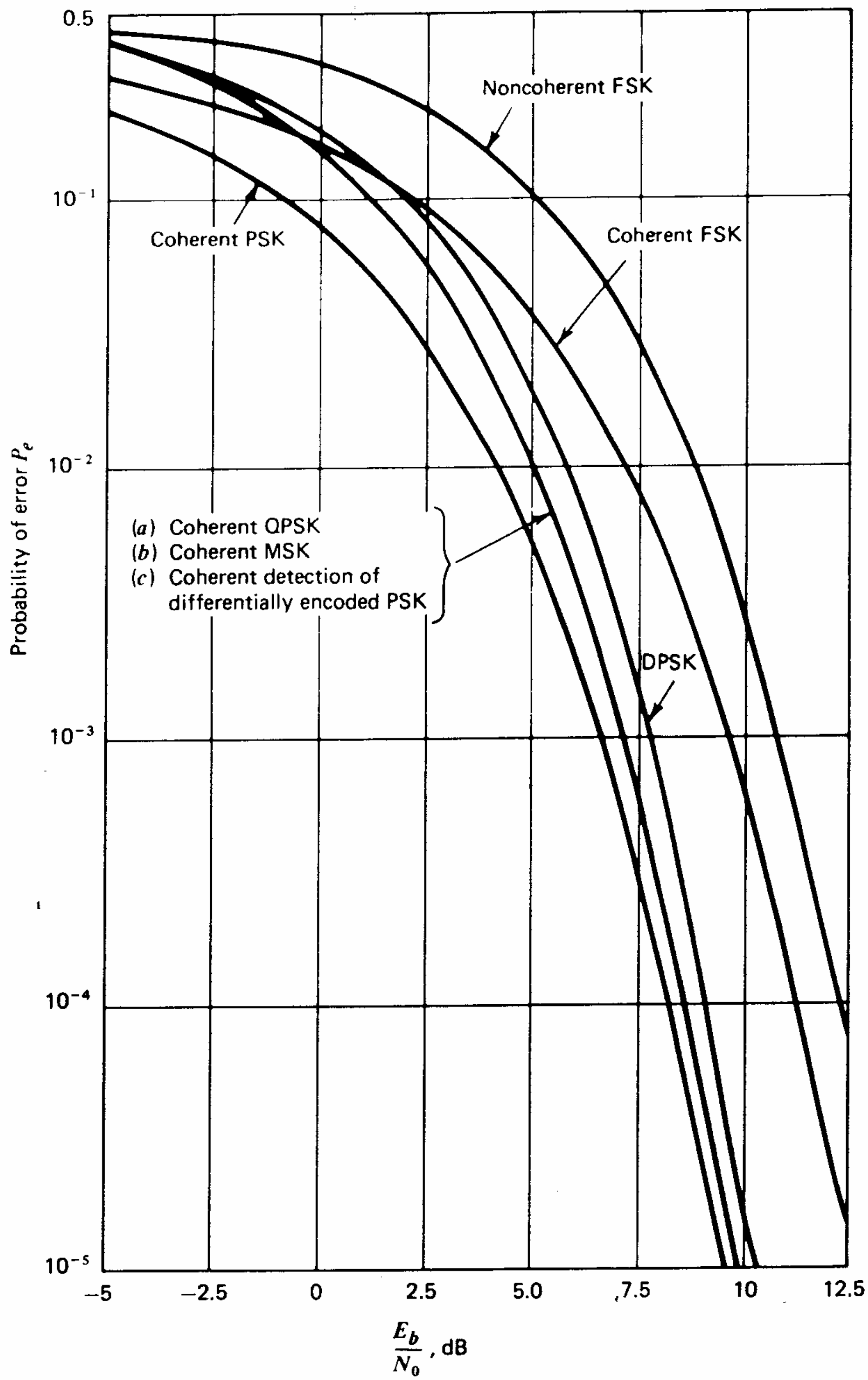

FIGURE 4-10 Comparison of the noise performances of different PSK and FSK schemes.

Summary:

1. The error rates for all the systems decrease monotonically with increasing values of E_b/N_o.

2. For any value of E_b/N_o, coherent PSK produces a smaller error rate than any of the other systems. Indeed, it may be shown that in the case of systems restricted to one-bit decoding, perturbed by additive white Gaussian noise, coherent PSK system is the optimum system for transmitting binary data in the sense that it achieves the minimum probability of symbol error for a given value of E_b/N_o.

3. Coherent PSK and DPSK require an E_b/N_o that is 3 dB less than the corresponding values for conventional coherent FSK and noncoherent FSK, respectively, to realize the same error rate.

4. At high values of E_b/N_o, DPSK and noncoherent FSK perform almost as well (to within about 1 dB) as coherent PSK and conventional coherent FSK, respectively, for the same bit rate and signal energy per bit.

5. In QPSK two orthogonal carriers $\sqrt{2/T}\cos(2\pi f_c t)$ and $\sqrt{2/T}\sin(2\pi f_c t)$ are used, where the carrier frequency f_c is an integral multiple of the symbol rate 1/T, with the result that two independent bit streams can he transmitted and subsequently detected in the receiver. At high values of E_b/N_o coherently detected binary PSK and QPSK have about the same error rate performance for the same value of E_b/N_o.