Sinusoids and Phasors

(Chapter 9 - Lecture #1)

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Outline of Chapter 9

- Introduction
- Sinusoids
- Phasors
- Phasor Relationships for Circuit Elements
- Impedance and Admittance
- Kirchhoff's Laws in the Frequency Domain
- Applications(optional)

9.1 Introduction

- So far, we have learnt about **DC(direct current)**. But from this lecture, we will learn about **AC(alternating current)**. Why??
  - Ac is more efficient and economical for long distance transmission
  - Many real applications use ac, e.g. electrical home appliances, factories, labs, etc.
- To deal with ac, we will only consider sinusoidally time-varying excitation $\rightarrow$ <u>*sinusoid*</u>
  - Sinusoid definition
    A sinusoid is a signal that has the form of the sine and cosine function

About Sinusoid

- A sinusoidal current is usually referred to as *alternating current (ac)*
- Circuits driven by sinusoidal current or voltage sources are called *ac circuits*
- Why sinusoids are of our interest??
  1. By nature, it is characteristically sinusoidal - anything that repeats its motions periodically, e.g., pendulum, ripples on the ocean surface, etc.
  2. Easy to generate and transmit - they are used in our homes, factories, etc.
  3. Can be analyzed using Fourier analysis - easy to manipulate signals $\rightarrow$ any practical periodic signal can be represented by a sum of sinusoids

4. Easy to be handled mathematically - the derivative and integral of a sinusoid are themselves sinusoids.

- When do a sinusoidal signal achieve a steady-state??
  When the transient response has died out (almost negligibly) and steady-state response remains - we are only interested on *sinusoidal steady-state response*.

- For today's lecture, we will look at and learn about ***sinuoids*** and ***phasors*** only. The remaining section will be done in the next class.

9.2 Sinusoid

- Sinusoid voltage is given as follows:

$$v(t) = V_m \sin \omega t \tag{1}$$

where,

$$\begin{aligned} V_m &= \text{the amplitude of the sinusoid} \\ \omega &= \text{teh angular frequency in radian/s} \\ \omega t &= \text{the argument of the sinusoid} \end{aligned}$$

- To understand more on these terms, let's see Figure 9.1 in our text book.
  → (a) shows a function of its argument.
  → (b) shows a function of time.

- Sinusoid repeats itself every T seconds $\Rightarrow$ *period* of the sinusoid, where T is

$$T = \frac{2\pi}{\omega} \tag{2}$$

- We may see that T repeats itself using the following equations:

$$\begin{aligned} v(t+T) &= V_m \sin\omega(t+T) &&= V_m \sin\omega(t+\frac{2\pi}{\omega}) \\ &= V_m \sin(\omega t + 2\pi) &&= V_m \sin\omega t = v(t) \end{aligned} \tag{3}$$

which consequently gives

$$v(t+T) = v(t) \tag{4}$$

which means

> A periodic function is one that satisfies $f(t) = f(t+nT)$, for all t and for all integers n.

- The reciprocal of T which is given as

$$f = \frac{1}{T} \tag{5}$$

is the *cyclic frequency*.

- From the equations introduced above, we know that

$$\omega = 2\pi f \tag{6}$$

in which the ω is in rad/s and f is in Hz.

- We will now consider phasor. Phasor is expressed using ϕ which is pronounced as '**fai**'. The general expression is given below:

$$v(t) = V_m \sin(\omega t + \phi) \tag{7}$$

Refer to Figure 9.2 for more intuitive presentation on this.

- To express sinusoid, we use sine and cosine form – trigonometry. Note that: "*when comparing two sinusoids, it is expedient (sesuai,elok,manfaat) to express both as either sine or cosine with positive amplitudes*".
- Using the relationships in Equations(9.9) and (9.10), we can transform a sinusoid from sine to cosine or vice versa.
- Instead of memorize all equations given in (9.9) and (9.10) (in trigonometry), we may also use *graphical approach* to relate or compare sinusoids. This can be done by remembering and understanding Figure 9.3.
- This graphical approach can also be used to add two sinusoids signals of same frequency.

$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta) \tag{8}$$

where

$$C = \sqrt{A^2 + B^2}, \qquad \theta = \tan^{-1} \frac{B}{A}$$

Example 9.1

Find the amplitude, phase, period and frequency of the sinusoid

$$v(t) = 12 \cos(50t + 10^\circ)$$

Solution

It is important to note that amplitude, phase, period and frequency of the given sinusoid are expressed as V_m, ϕ, T and f, respectively. Therefore, the answers can be straightforward taken from the sinusoid, that is,

$$\begin{aligned} V_m &= 12V. \\ \phi &= 10^\circ. \\ T &= \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257s, (\omega = 50\, rad/s.) \\ f &= \frac{1}{T} = 7.958Hz. \end{aligned} \tag{9}$$

9.3 Phasors

- Sinusoids are easily expressed in terms of phasors, which are more convenient to work with than sine and cosine functions.
- Definition of Phasor:

  A *phasor* is a complex number that represents the amplitude and phase of a sinusoid.
- Note here, we will use *complex number* when phasor is concerned in our circuit analysis.
- A complex number z (this is a usual expression for complex number in our future circuit analysis) is

$$z = x + jy$$

  where, $j = \sqrt{-1}$; $x \rightarrow$ real part and $y \rightarrow$ imaginary part of z. Note that: x and y are not a two-dimensional vector.

- z can also be written in polar or exponential form as

$$z = r\angle\phi = re^{j\phi}$$

where $r \rightarrow$ magnitude of z, $\phi \rightarrow$ phase of z.

- Due to this, z can be represented in these ways:

1. Rectangular form: $z = x + jy$.
2. Polar form: $z = r\angle\phi$.
3. Exponential form: $z = re^{j\phi}$

- Refer to Figure 9.6 for more details.