Sinusoids and Phasors

(Chapter 9 – Lecture #2)

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Outline of Chapter 9

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- Sinusoids
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- ***9.4* Phasor Relationships for Circuit Elements**
- ***9.5* Impedance and Admittance**
- Kirchhoff's Laws in the Frequency Domain
- Applications(optional)

9.4 Phasor Relationships for Circuit Elements

- In this section, we will learn to apply voltage and current phasor (or frequency) domain to R, L and C.
- Our duty $\Rightarrow$ transform the voltage-current relationship from the time domain to the frequency domain.

$$\textbf{time domain} \overset{T}{\rightarrow} \textbf{phasor(frequency) domain}$$

- Resistor

  Given current through a resistor R is $i = I_m \cos(\omega t + \phi)$, then the voltage, v

  $$v = iR = RI_m \cos(\omega t + \phi)$$

  Therefore, the phasor form of voltage is

  $$\mathbf{V} = RI_m \angle \phi$$

  But the phasor representation of the current $\mathbf{I}$ is $\mathbf{I} = I_m \angle \phi$.

Hence,

$$\mathbf{V} = R\mathbf{I}$$

This indicates that the voltage-current relation for the resistor in the phasor domain remains to be Ohm's law.

Notes:

One should note that the equation given above are in <u>*phase*</u>*.*

- Inductor

  Using the same $i = I_m \cos(\omega t + \phi)$, the voltage across the inductor is

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$$

And since $-\sin A = cos(A + 90^\circ)$, the voltage becomes

$$v = \omega L I_m \cos(\omega t + \phi + 90^\circ)$$

which transforms to phasor

$$\begin{aligned}\mathbf{V} &= \omega L I_m e^{j(\phi+90^\circ)} \\ &= \omega L I_m e^{j\phi} e^{j90^\circ} \\ &= \omega L I_m \angle \phi + 90^\circ\end{aligned}$$

From $I_m \angle \phi = \mathbf{I}$ and e^{j90°, $\mathbf{V}$ for the inductor is

$$\mathbf{V} = j\omega L\mathbf{I}$$

Refer to Figure 9.12 for the phasor diagram.

- Capacitor
  Similarly, for capacitor we assume the voltage across it is $v = V_m \cos(\omega t + \phi)$. Then the current is

$$i = C\frac{dv}{dt}$$

Taking the same steps for inductor, we obtain

$$\mathbf{I} = j\omega\mathbf{C} \quad \Rightarrow \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

- Refer to Table 9.2 for the summary of the voltage-current relationships. From this table, we might note that in phasor(frequency) domain, we need to write the voltage($\mathbf{V}$) and current($\mathbf{I}$) in bold as phasors are vector-like quantities.

9.5 Impedance and Admittance

- The equations that we have seen in previous section (refer to Table 9.2 for the summary) can also be written as the ratio of the phasor voltage to the phasor current like this $\rightarrow \frac{\mathbf{V}}{\mathbf{I}}$.
- This ratio is known as *impedance*, which is written as $\mathbf{Z}$ or explicitly

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

where $\mathbf{Z}$ is a *frequency-dependent* quantity.

- Definition of impedance
The impedance $\mathbf{Z}$ *of a circuit is the ratio of the phasor voltage* $\mathbf{V}$ *to the phasor current* $\mathbf{I}$, *measured in ohms(*Ω*).*
- Impedance is not a phasor $\rightarrow$ not correspond to a sinusoidally varying quantity.

- Check Table 9.3 for the summary, where the expressions are changed to $\mathbf{Z}$ instead of $\mathbf{V}$ and $\mathbf{I}$
- Conditions for impedance
  1. When $\omega = 0$ (this is actually dc) – $\mathbf{Z}_L = 0$ and $\mathbf{Z}_C \to \infty$.
  2. When $\omega \to \infty$ – $\mathbf{Z}_L \to \infty$ and $\mathbf{Z}_C = 0$.
- Impedance can be expressed in rectangular form as a complex quantity

$$\mathbf{Z} = R + jX$$

where,

$$\begin{aligned} R &= \mathrm{Re}\mathbf{Z} \Rightarrow \textbf{resistance} \\ X &= \mathrm{Im}\mathbf{Z} \Rightarrow \textbf{reactance}. \end{aligned}$$

- Impedance in polar form

$$\mathbf{Z} = |\mathbf{Z}|\angle\theta$$

- From above equations, we infer that

$$\mathbf{Z} = R + jX = |\mathbf{Z}|\angle\theta$$

where

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R}$$

and

$$R = |\mathbf{Z}|\cos\theta, \qquad X = |\mathbf{Z}|\sin\theta.$$

- Reciprocal of impedance is called *admittance*. It is given by following equation

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}}$$

- Definition of admittance:
The admittance $\mathbf{Y}$ *is the reciprocal of impedance, measure in siemens (S) or mhos.*

- Admittance can be written in rectangular form as a complex quantity.

$$\mathbf{Y} = G + jB$$

where

$$G = \mathrm{Re}\mathbf{Y} \Rightarrow \textbf{conductance}$$
$$B = \mathrm{Im}\mathbf{Y} \Rightarrow \textbf{susceptance}.$$

- R and B come from the relationship between admittance and impedance in rectangular form. Each of them are given below.

$$G = \frac{R}{R^2 + B^2}, \quad B = -\frac{X}{R^2 + B^2}$$

Summary of Today's Lecture

- We have learnt about the application of voltage and current in the phasor(frequency) domain.
- From the relationship, we have come across terms like *impedance, resistance, reactance, admittance, conductance and susceptance.* These are expressed as follows:
  - impedance – $\mathbf{Z}$
  - resistance – R
  - reactance – X
  - admittance – $\mathbf{Y}$
  - conductance – G
  - susceptance – B