EEE443 Digital Signal Processing
Implementation of Discrete-Time Systems

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Introduction

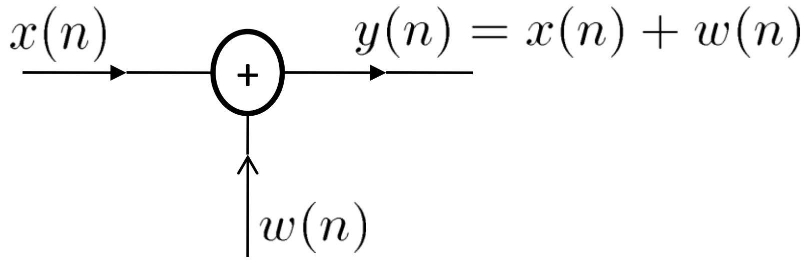
- A linear-time invariant system (LTI) is described by linear constant coefficient difference equation (LCCDE)
  - If we are given a rational transfer function having the input $x(n)$ and output $y(n)$, the LTI is described as

$$y(n) = \sum_{k=0}^{q} b(k)x(n-k) - \sum_{k=1}^{p} a(k)y(n-k)$$

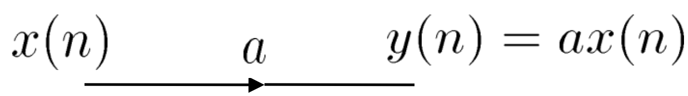
  - What we understand from this equation?
    - Recurvise system – IIR (which have the terms of $a(k)$)
    - Non-recursive system – FIR (which its all $a(k) = 0$)

Block Diagram for Illustration Purpose

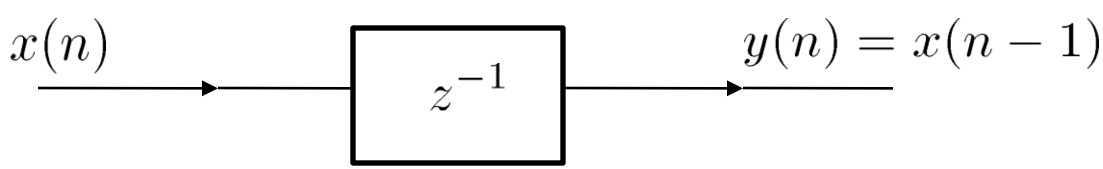
- There are three basic computational elements:
  - Adders



  - Multipliers



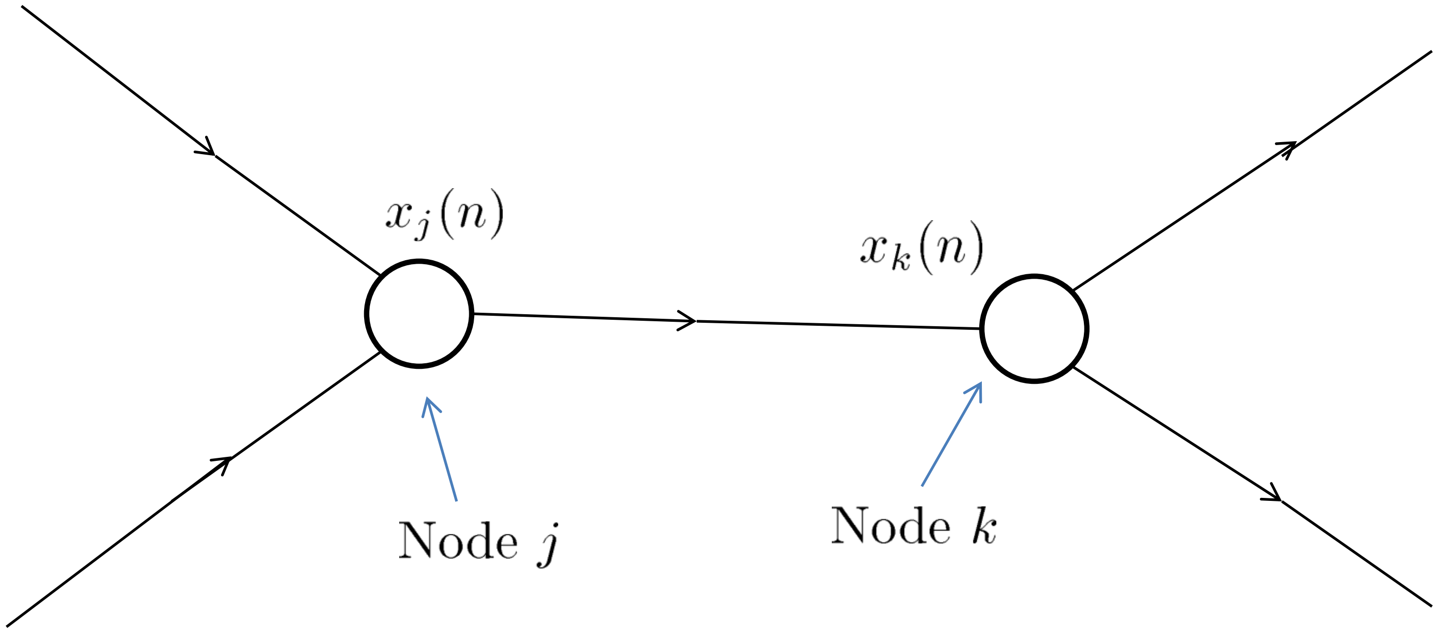
  - Delays



Signal Flow Graph

- Originated from the basic computational elements
- Signal flow graph is drawn using nodes to show the network of directed branches
- The nodes correspond to two type of points
  - Adders – more than one incoming branches
  - Branch points – more than one outcoming branches

Signal Flow Graph Example



Facts About Signal Flow Graph

- Output of each branch is a linear transformation of the branch input and the linear operator is indicated next to the arrow
  - Notice that for these linear operators, multipliers and delays will be included
- Two types of special nodes:
  - Source nodes – these are nodes that have no incoming branches and are used for sequences that input to the filter
  - Sink nodes – these are nodes that have only entering branches and are used to represent output sequences

Structures of FIR Systems

- FIR – finite impulse response
- What we know about FIR?
  - All its $a(k)$ terms will be zero
  - Non-recursive
- A causal FIR filter/system has a transfer function:

$$H(z) = \sum_{n=0}^{M-1} h(n) z^{-n}$$

  - Notice that this transfer function has no poles (non-recursive)
- An FIR system is described by the difference equation

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k) \quad \xrightarrow{\text{System Function}} \quad H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$$

- There are several different realization block diagrams for this system:
  - Direct form, cascade form, frequency-sampling, and lattice

Structures of FIR Systems (cnt'd)

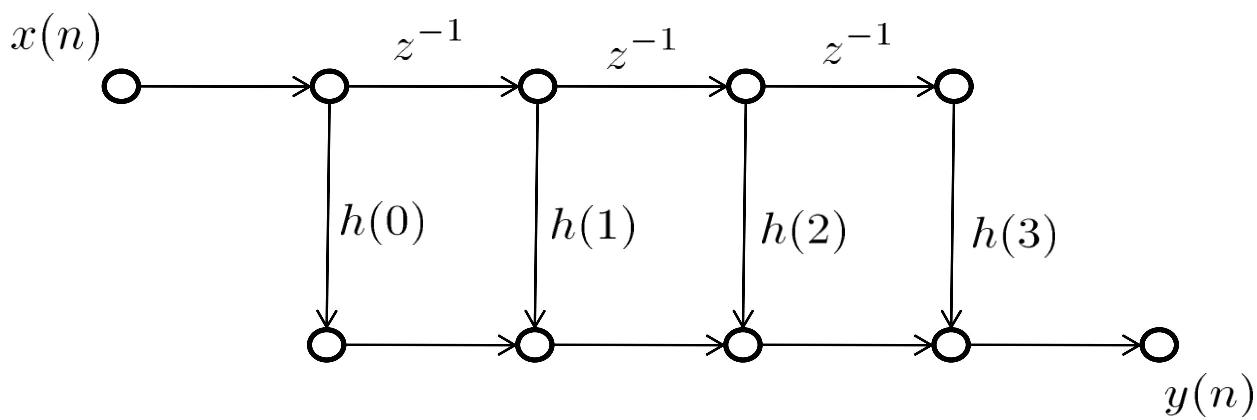
- Unit sample response of the FIR system is identical to the coefficient $\{b_k\}$, ie.

$$h(n) = \begin{cases} b_n, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

- M is the length of the FIR filter

Direct Form (1)

- The most common way to implement an FIR
- Called a tapped delay line or a transversal system (below is shown the signal flow graph)



Direct Form (2)

$$y(n) = \sum_{n=0}^{M-1} h(k)x(n-k)$$

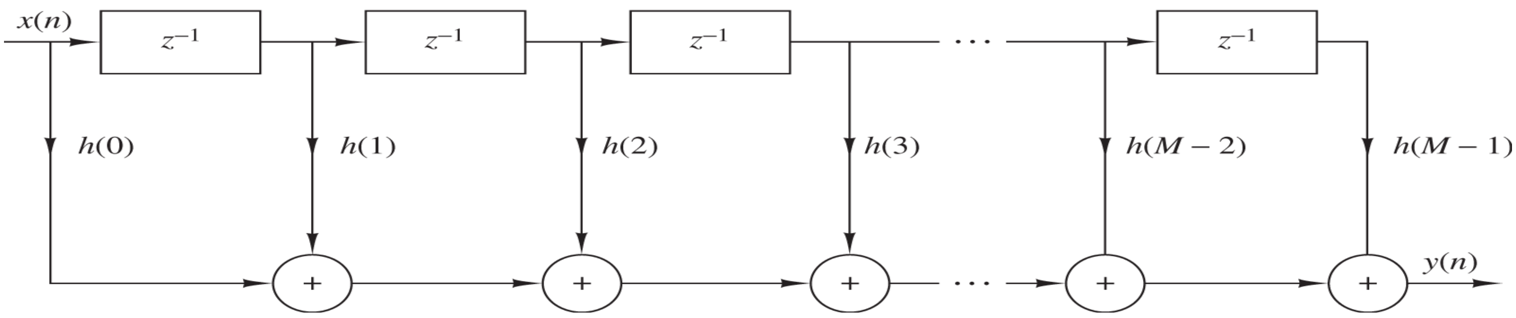


Figure 9.2.1 Direct-form realization of FIR system.

- This structure requires
  - M-1 memory locations for storing the M-1 previous inputs
  - M multiplications
  - M-1 additions per output point
- When the FIR system has linear phase, the unit sample response of the system satisfies either the symmetry or asymmetry condition

$$h(n) = \pm h(M-1-n)$$

  - The advantage → number of multiplications is reduced:
    - From M to M/2 for M even
    - From M to (M-1)/2 for M odd

Cascade Form (1)

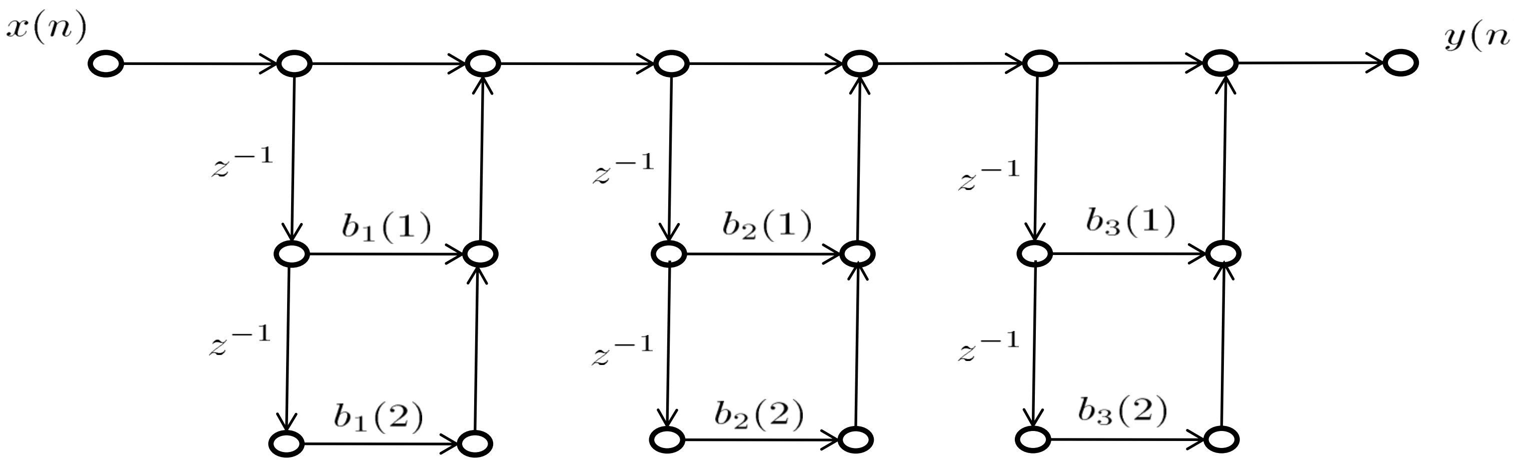
- The transfer function is factored into a product of first order factors

$$H(z) = \sum_{n=0}^{N} h(n) z^{-n} = A \prod_{k=1}^{N} (a - \alpha_k z^{-1}) \quad \text{where, for } \alpha_k, k = 1, 2, \ldots$$

- If $h(n)$ is real, the complex roots of $H(z)$ occur in complex conjugate pairs, and therefore these conjugate pairs may be combined to form second order factors with real coefficients

$$H(z) = A \prod_{k=1}^{N} \left[1 + b_k(1) z^{-1} + b_k(2) z^{-2}\right]$$

Cascade Form (2)



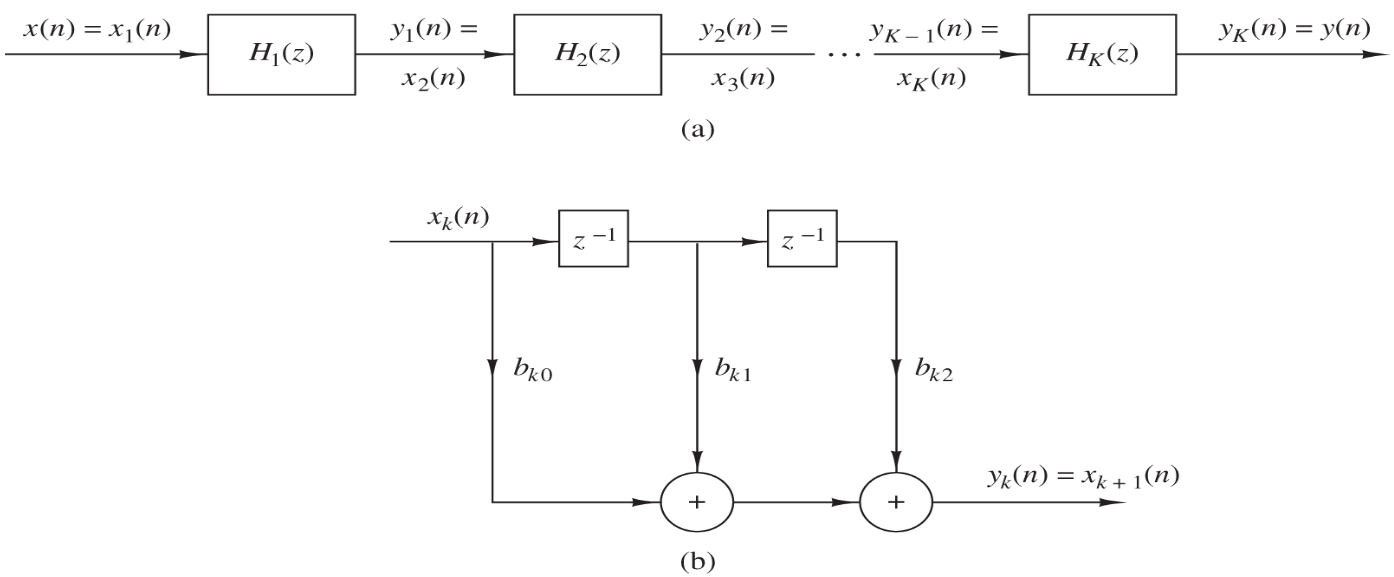


Figure 9.2.3 Cascade realization of an FIR system.

Linear Phase Filter/Systems

- A linear phase filter/system is said to have linear phase if the frequency response has the form
$$H(e^{j\omega}) = |H(e^{j\omega})|e^{-j\alpha\omega}$$
- The constant group delay $\tau_h(\omega) = \alpha$
- The unit impulse response of a linear phase system is either (taking into account $N = M - 1$)
  - Symmetric $h(n) = h(N - n)$
  - Non-symmetric $h(n) = -h(N - n)$

Type of FIR Filter with Linear Phase

- The symmetry can be exploited to simplify the system/filter structure
- Therefore, there are four types of FIR filter
  - Type I : N is even and h(n) is symmetric
  - Type II : N is odd and h(n) is symmetric
  - Type III: N is even and h(n) is antisymmetric
  - Type IV: N is odd and h(n) is antisymmetric

Structures of IIR Filters

- As being described before, a causal IIR filter with rational system or transfer function is expressed as

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^{q} b(k) z^{-k}}{1 + \sum_{k=1}^{p} a(k) z^{-k}}$$

- Describing it in LCCDE

$$y(n) = \sum_{k=0}^{q} b(k) x(n-k) - \sum_{k=1}^{p} a(k) y(n-k)$$

- Similar to FIR, there are four types of IIR structures or realizations
  - Direct form, cascade form, lattice, and lattice-ladder

Direct-Form (1)

- The rational system function that characterizes an IIR system can be viewed as two systems in cascade

$$H(z) = H_1(z)H_2(z)$$

$$H_1(z) = \sum_{k=0}^{M} b_k z^{-k} \qquad H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

- In Section 2.5, we have come across this where the cases were $H_1(z)$ precedes $H_2(z)$, or vice versa
- Since the top equation is an FIR system, we know how write the diagram for this (Figure 9.2.1)
- To finalize the structure for an IIR, we attach the all-pole system in cascade with all-zero system (FIR system) as shown in Figure 9.3.1

Direct-Form (2)

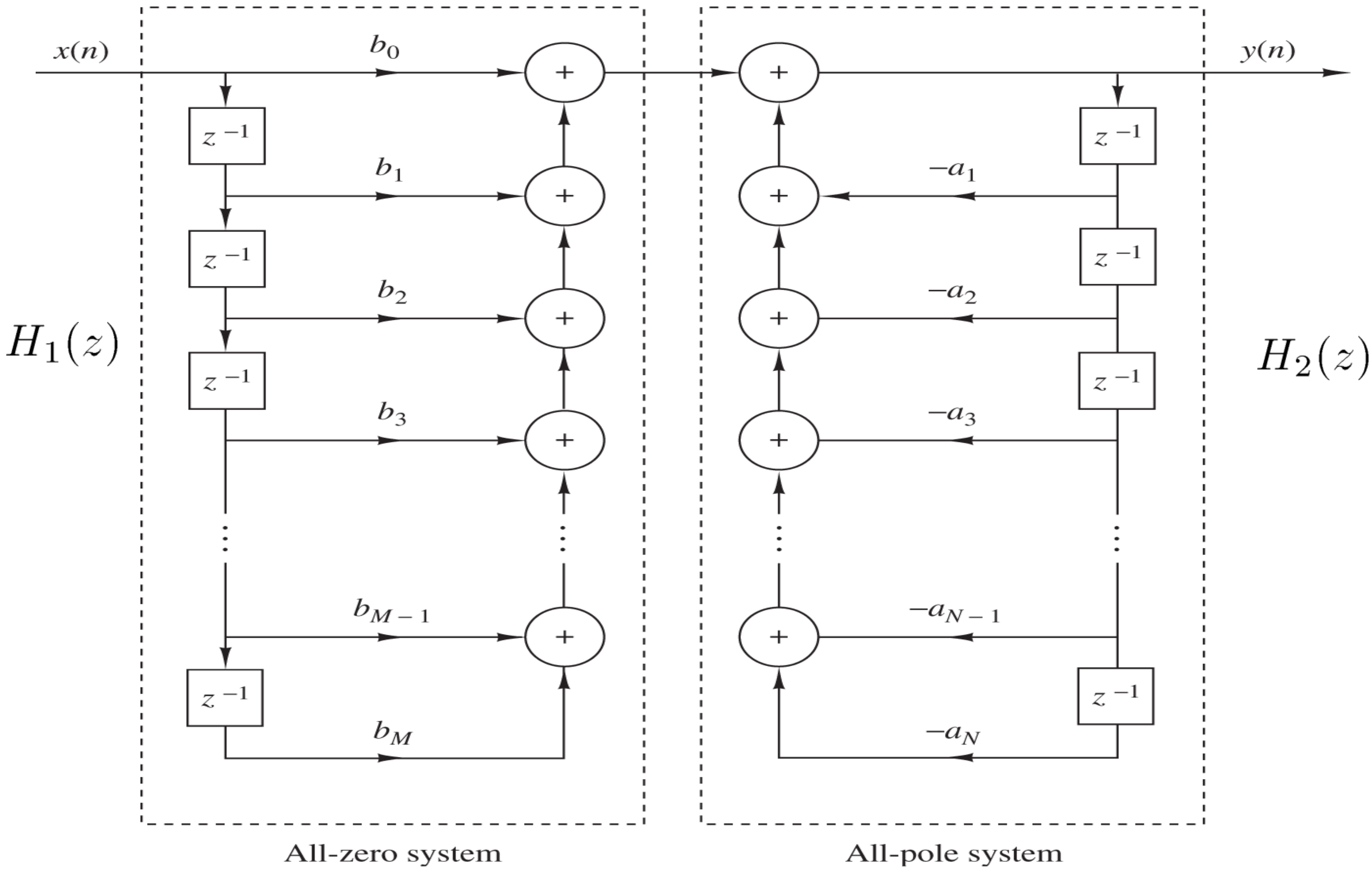


Figure 9.3.1 Direct form I realization.

Direct-Form (3)

- Figure 9.3.1 can be redrawn as shown in Figure 9.3.2 by considering linear equation for all-pole filter

$$w(n) = -\sum_{k=1}^{N} a_k w(n-k) + x(n)$$

- Since $w(n)$ is the input to the all-zero system, its output is

$$y(n) = \sum_{k=0}^{M} b_k w(n-k)$$

- Looking at both equations above, we may note that both involve delayed versions of the sequence $\{w(n)\}$
  - Resulting only a single delay line or single set of memory locations is required for storing the past values of $\{w(n)\}$
  - This structure type is called a direct form II
  - Requires M+N+1 multiplications, M+N additions and max of {M,N} memory locations
  - Is said to be *canonic* → as it minimizes the number of memory locations
- "Direct-form" is given due to it can be obtained directly from the rational function or system function
  - BUT SENSITIVE TO PARAMETER QUANTIZATION → NOT RECOMMENDED IN PRACTICAL APPLICATIONS

Direct-Form (4)

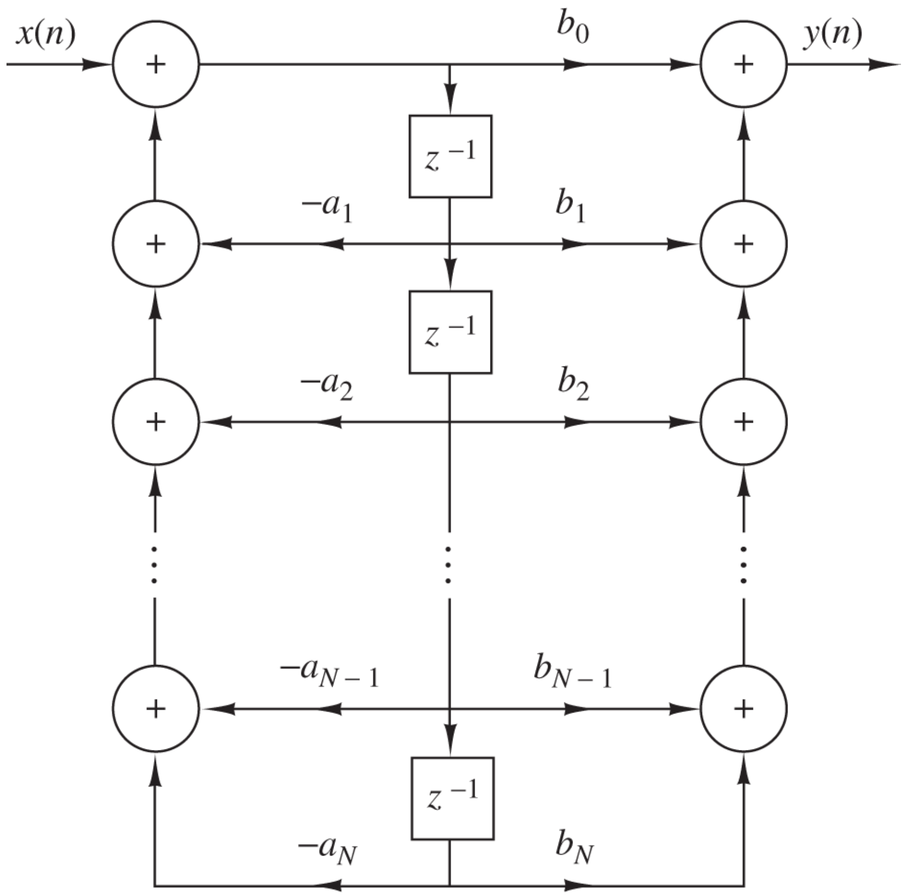


Figure 9.3.2 Direct form II realization $(N = M)$.

About Signal Flow Graphs and Transposed Structures

Signal Flow Graph

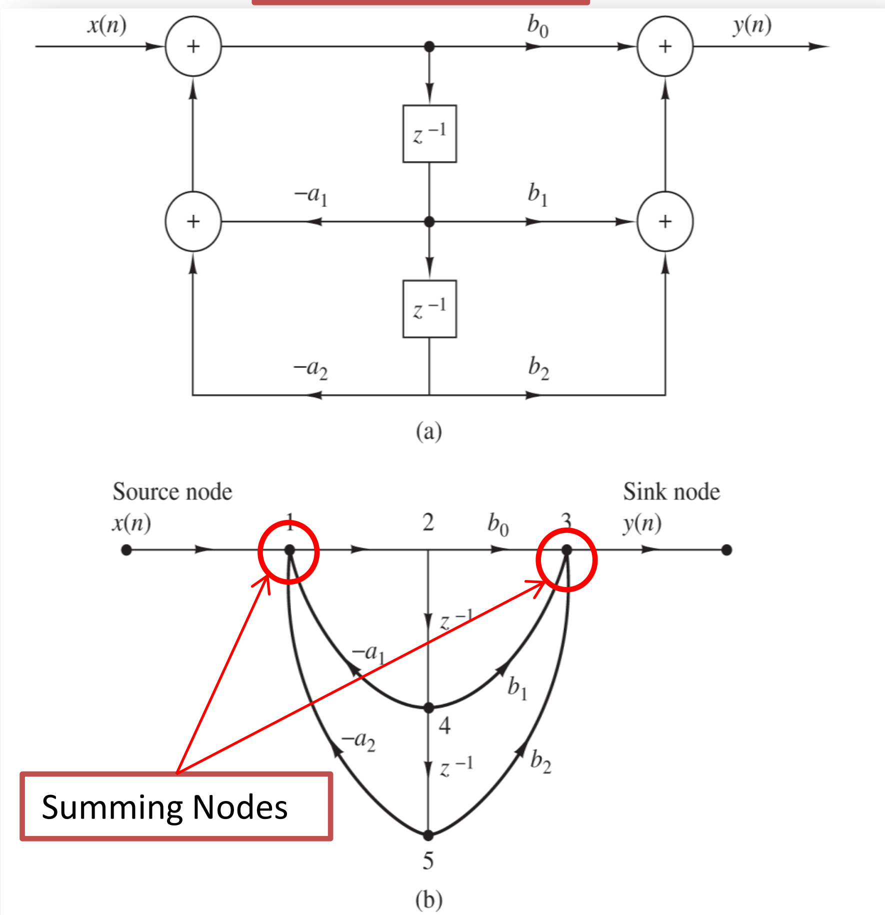


Figure 9.3.3 Second-order filter structure (a) and its signal flow graph (b).

Transposed Structures

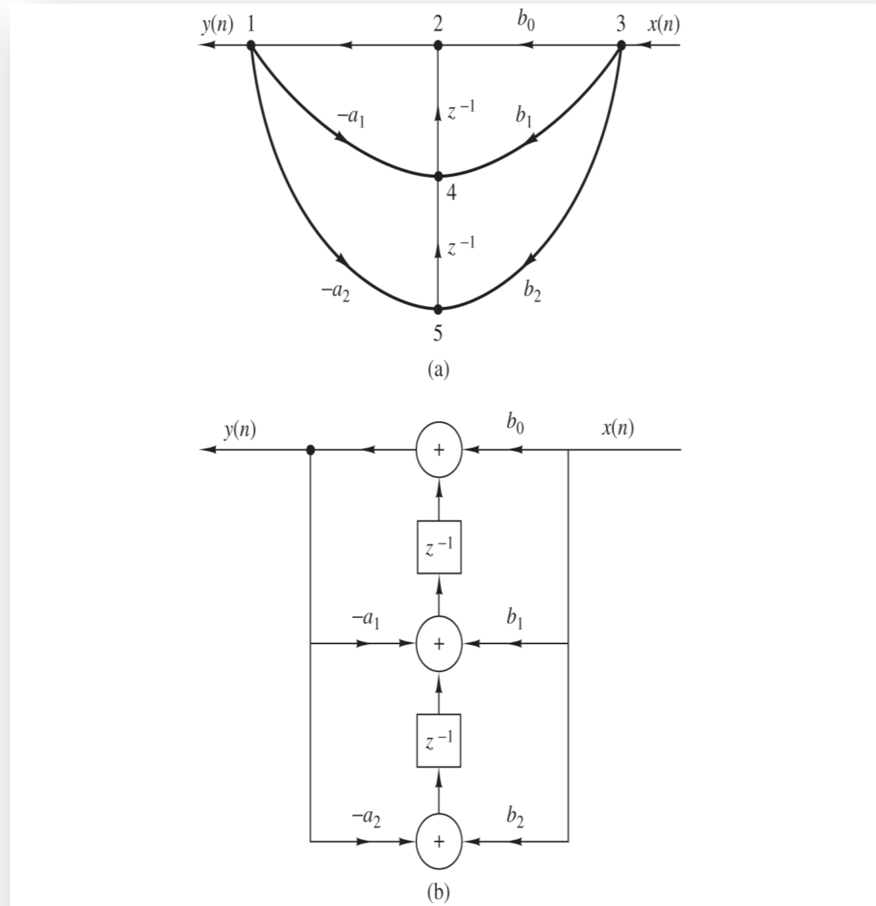


Figure 9.3.4 Signal flow graph of transposed structure (a) and its realization (b).

Let’s try using the transposition theorem to the direct form II

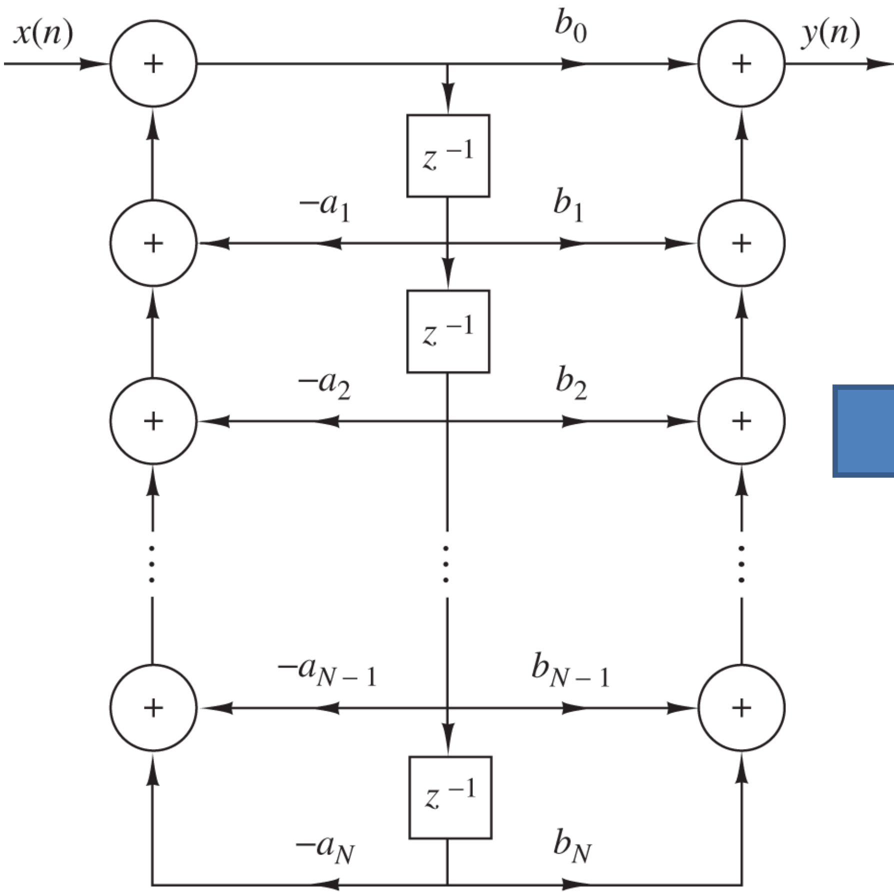


Figure 9.3.2 Direct form II realization $(N = M)$.

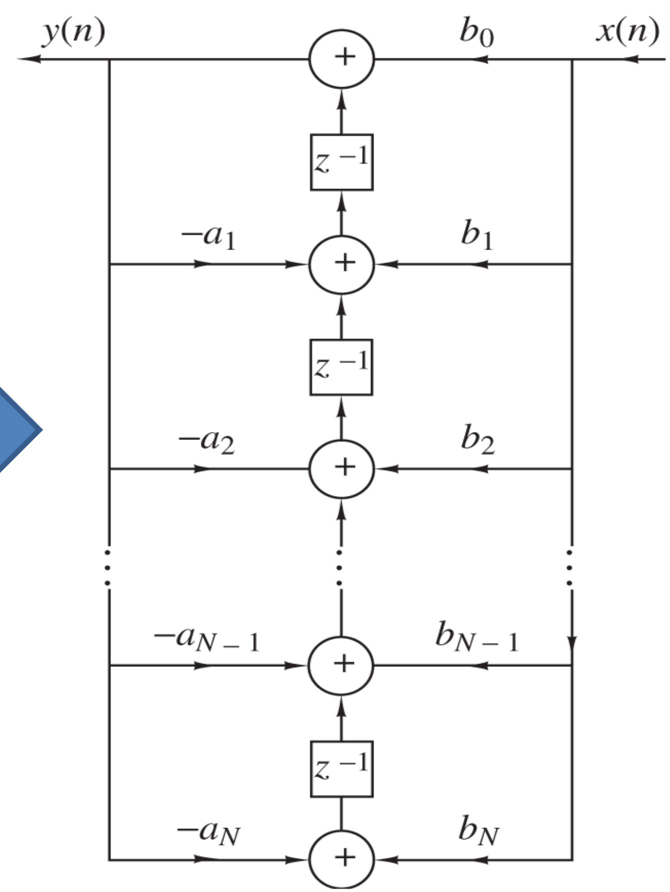


Figure 9.3.5 Transposed direct form II structure.

Cascade-Form

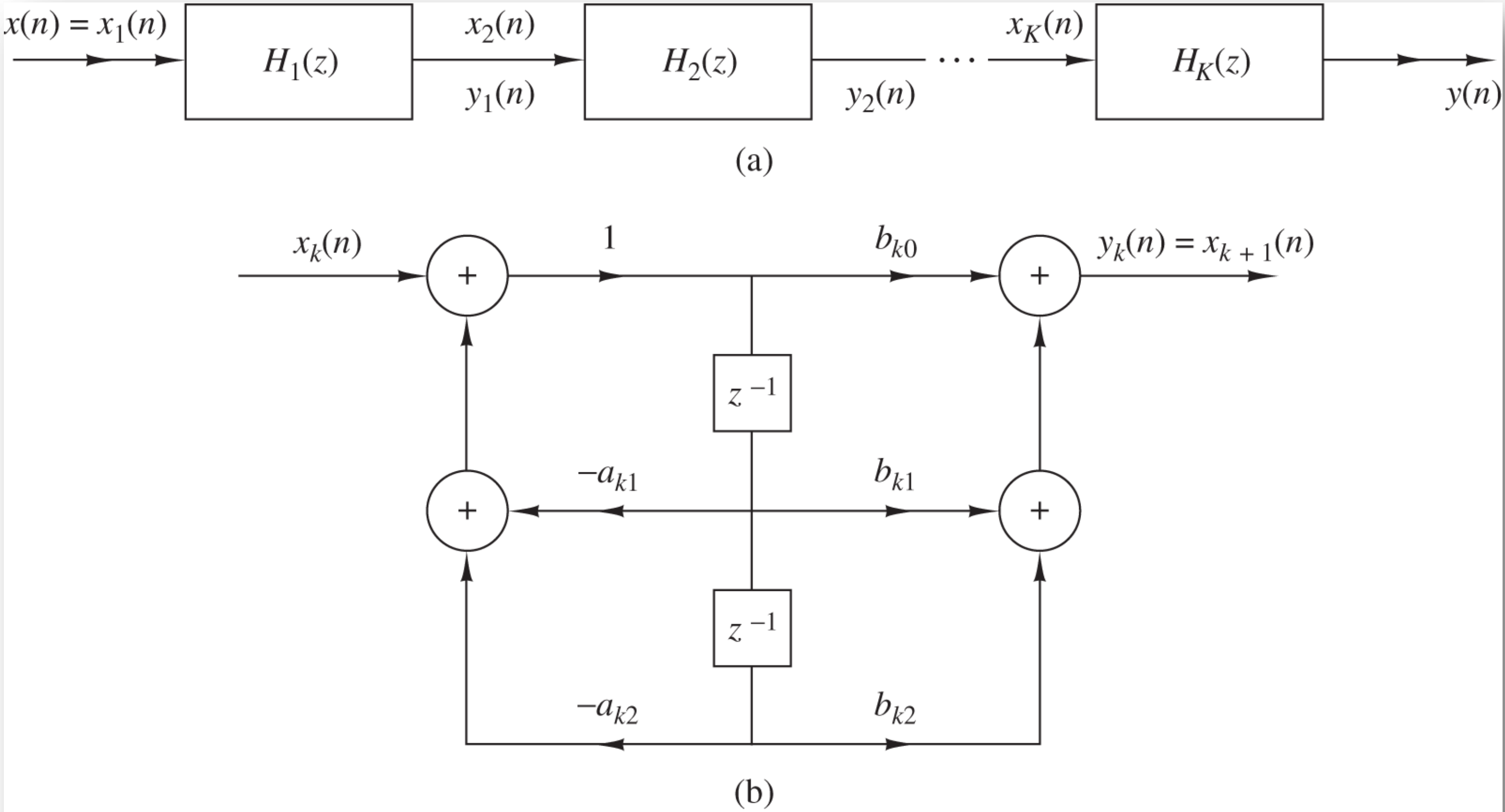


Figure 9.3.8 Cascade structure of second-order systems and a realization of each second-order section.

Parallel-Form

Performing partial fraction expansion of $H(z)$, we obtain

$$H(z) = C + \sum_{k=1}^{N} \frac{A_k}{a - p_k z^{-1}}$$

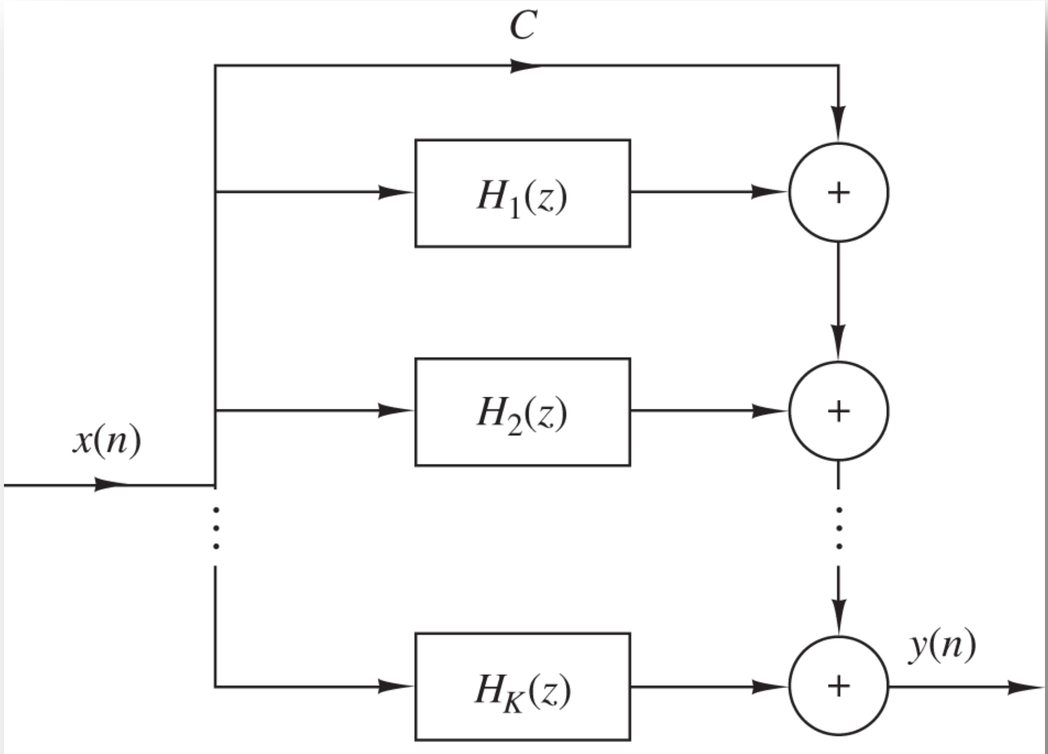


Figure 9.3.9 Parallel structure of IIR system.