# EEE443 Digital Signal Processing

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Lecture #5

Chapter 2 (2.5, 2.6)

# What we will learn today?

- 2.5 Implementation of Discrete-Time Systems
  - 2.5.1 Structures for the Realization of Linear Time-Invariant Systems
  - 2.5.2 Recursive and Nonrecursive Realizations of FIR Systems
- 2.6 Correlation of Discrete-Time Signals
  - 2.6.1 Crosscorrelation and Autocorrelation Sequences
  - 2.6.2 Properties of the Autocorrelation and Crosscorrelation Sequences
  - 2.6.3 Correlation of Periodic Sequences
  - 2.6.4 Input-Output Correlation Sequences

# 2.5 Implementation of Discrete-Time Systems

- Starting from this section, we will characterize and analyze LTI systems in the *frequency domain* – so far, we have done this in time-domain.
- Other topics to be discusses are:
  - Design, *and*
  - Implementation
    - To this extent, we have
      - considered this as one issue at a time, rather than separate issues, *and*
      - described this by linear constant-coefficient difference equations

# 2.5.1 Structures for the Realization of Linear Time-Invariant Systems

- Let's consider the first-order system shown in (a)

$$y(n) = -a_1 y(n-1) + b_0 x(n) + b_1 x(n-1)$$

- This system can be rewritten in recursive and nonrecursive manners

$$v(n) = b_0 x(n) + b_1 x(n-1)$$

$$y(n) = -a_1 y(n-1) + v(n)$$

- As interchanging the order of cascaded LTI systems will yield the same response, we obtain an alternative structure of such system shown in (b)

$$w(n) = -a_1 w(n-1) + x(n)$$

$$y(n) = b_0 w(n) + b_1 w(n-1)$$

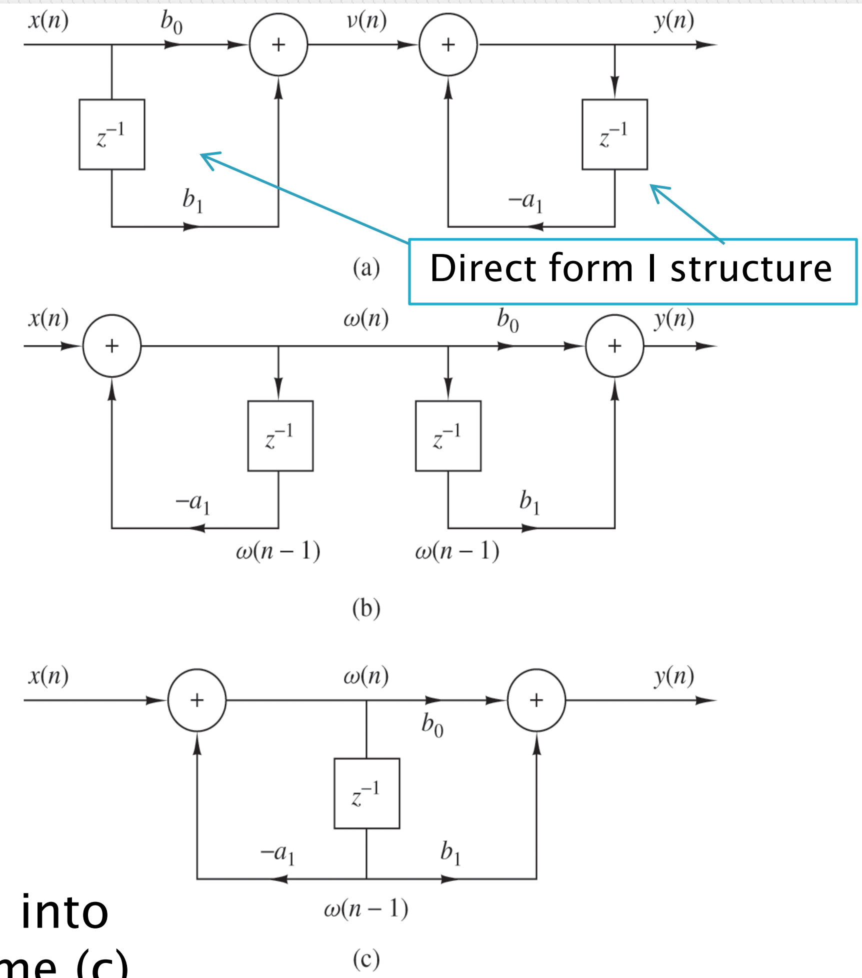


Figure 2.5.1 Steps in converting from the direct form I realization in (a) to the direct form II realization in (c).

Since the two delays in (b) can be merged into one, we may rewrite the diagram to become (c), i.e. using one delay for more efficient in terms of memory requirement and more practical

# General LTI Recursive System(1)

- Describing this using the difference equation

$$y(n) = -\sum_{k=1}^{N} a_k y(n-1) + \sum_{k=0}^{M} b_k x(n-k) \qquad (2.5.6)$$

  - Requires $M+N$ delays and $M+N+1$ multiplications
- Can be viewed as the cascade of
  - A nonrecursive system

$$v(n) = \sum_{k=0}^{M} b_k x(n-k)$$

  - A recursive system

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + v(n)$$

*This explanation uses form I structure*

# General LTI Recursive System(2)

- By reversing the order of these two systems, we obtain the form II structure
- Using this structure is more convenient as it may suggest the memory required to realize the system, ie.
  - If $N \geq M$, delays = order of $N$
  - If $M > N$ , the required memory is specified by $M$
- This structure is the cascade of
  - Recursive system

$$w(n) = -\sum_{k=1}^{N} a_k w(n-1) + x(n)$$

  - Nonrecursive system

$$y(n) = \sum_{k=0}^{M} b_k w(n-k)$$

# General LTI Recursive System(3)

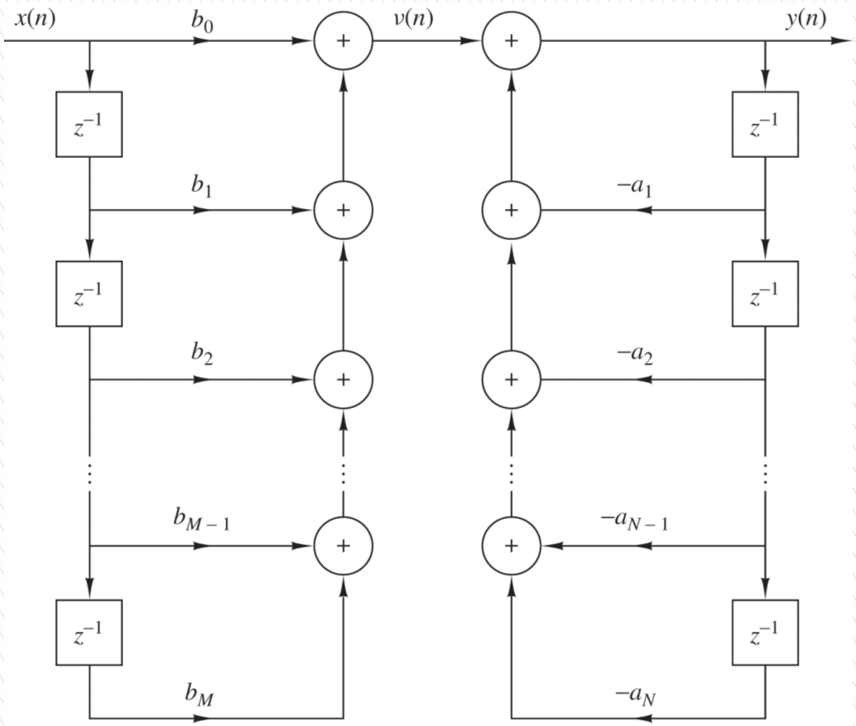


Figure 2.5.2 Direct form I structure of the system described by (2.5.6).

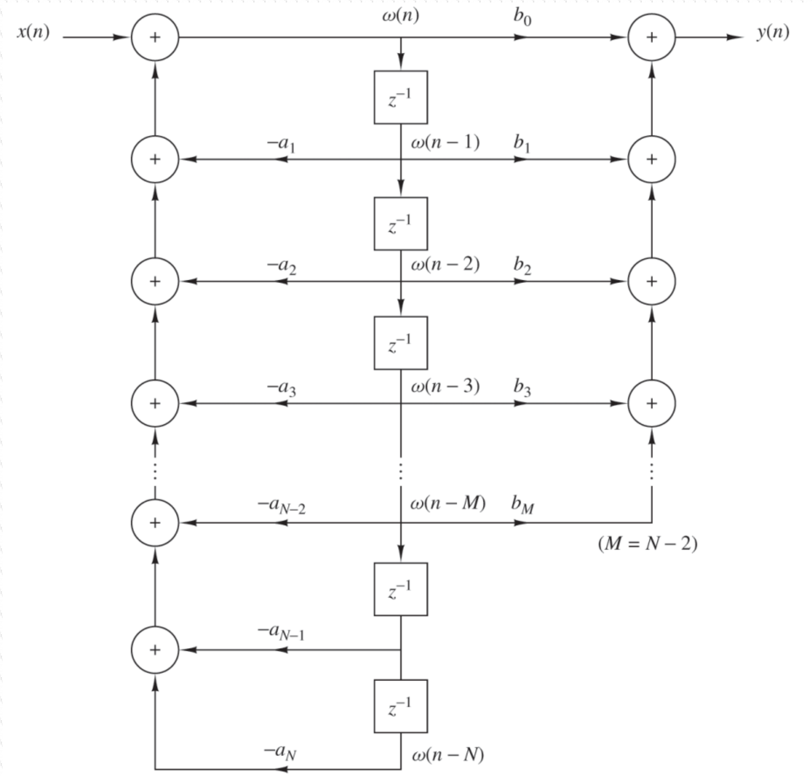


Figure 2.5.3 Direct form II structure for the system described by (2.5.6).

Form I Structure

Form II Structure
– Sometimes called "canonic form"

# Cases where only nonrecursive LTI system exists

- If some parameters like $a_k$ is set to 0, only nonrecursive LTI system will exist

$$y(n) = \sum_{k=0}^{M} b_k x(n-k)$$

- The response of this system is said to be a *weighted moving average of the input signal*
  - Sometimes called a *moving average (MA) system*

FIR (Finite Impulse Response) system with an impulse response

$$h(k) = \begin{cases} b_k, & 0 \le k \le M \\ 0, & \text{otherwise} \end{cases}$$

# 2.5.2 Recursive and Nonrecursive Realizations of FIR Systems

- What we should have known at this stage?
  - *FIR and IIR systems* based on how the impulse response $h(k)$ is set
  - *Causal recursive system* – can be described by an input–output equation (Eq. (2.5.17))
  - *Linear time-invariant system* – can be described by the difference equation (Eq. (2.5.18))
  - *Causal nonrecursive system* – as mentioned before, can be described by an input–output equation but do not depend on past values (Eq. (2.5.19))

# 2.5.2 Recursive and Nonrecursive Realizations of FIR Systems

- FIR systems can be realized from recursive and nonrecursive systems
  - Every FIR system can be realized nonrecursively
    - Usual case
  - FIR system can also be realized from recursive system
    - Check moving average system …
- Summary:
  - *FIR and IIR systems* are two different LTI systems
  - The terms *recursive* and *nonrecursive* are used to define the structures for realizing or implementing the system

# 2.6 Correlation of Discrete-Time Signals

- Similar to convolution in the sense that two signal sequences are involved
- Important to measure the degree of similarity between two signals
- Widely used in radar, sonar, digital communications, geology, and other areas in science and technology
- Example:
  - Radar target detection
  - Digital communication transmission

# 2.6.1 Crosscorrelation and Autocorrelation Sequences

- Suppose we have two real signal sequences

$$x(n) \text{ and } y(n)$$

- The crosscorrelation of these two signals is

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l), \quad l = 0, \pm 1, \pm 2, \ldots$$

  - Or

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l)y(n), \quad l = 0, \pm 1, \pm 2, \ldots$$