Discrete-Time Systems

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Lecture #3

Definition of Discrete-Time Systems

- Transformation or operator that maps an input sequence with values $x[n]$ into an output sequence with values $y[n]$.
  - The transformation is expressed with $T\{\cdot\}$

Example 2.3

- The example is an ideal delay system:

$$y[n] = x[n - n_d]$$

- n_d is a fixed positive integer.
- What can you conclude from the equation above?? What system is it??

The output *y[n]* is equivalent to the input *x[n]* that has been delayed by n_d.

The system is delayed.

- What operation involved??

Shifting

Discrete-Time Signals Manipulation
- transformation of independent variable -

- Shifting
  - Advanced(+) and delayed(-)
  - $y[n] = x[n+k]$ and $y[n] = x[n-k]$
- Revearsal
  - Folding or reflection
  - $y[n] = x[-n]$ which is $[n] \rightarrow [-n]$
- Time Scaling
  - Replacing [n] by [μn]
  - $y[n] = x[\mu n]$

Transformation of independent variable
- delayed and advanced examples -

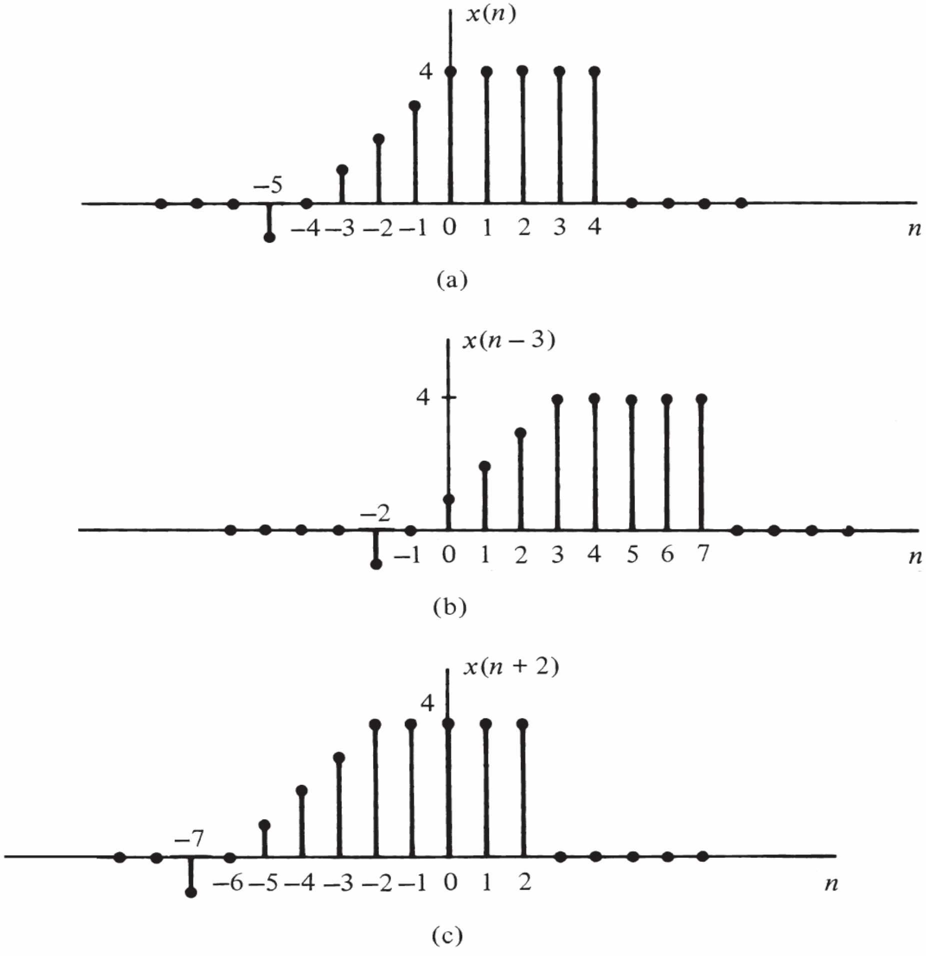

Figure 2.1.9 Graphical representation of a signal, and its delayed and advanced versions.

Transformation of independent variable - folding and shifting examples -

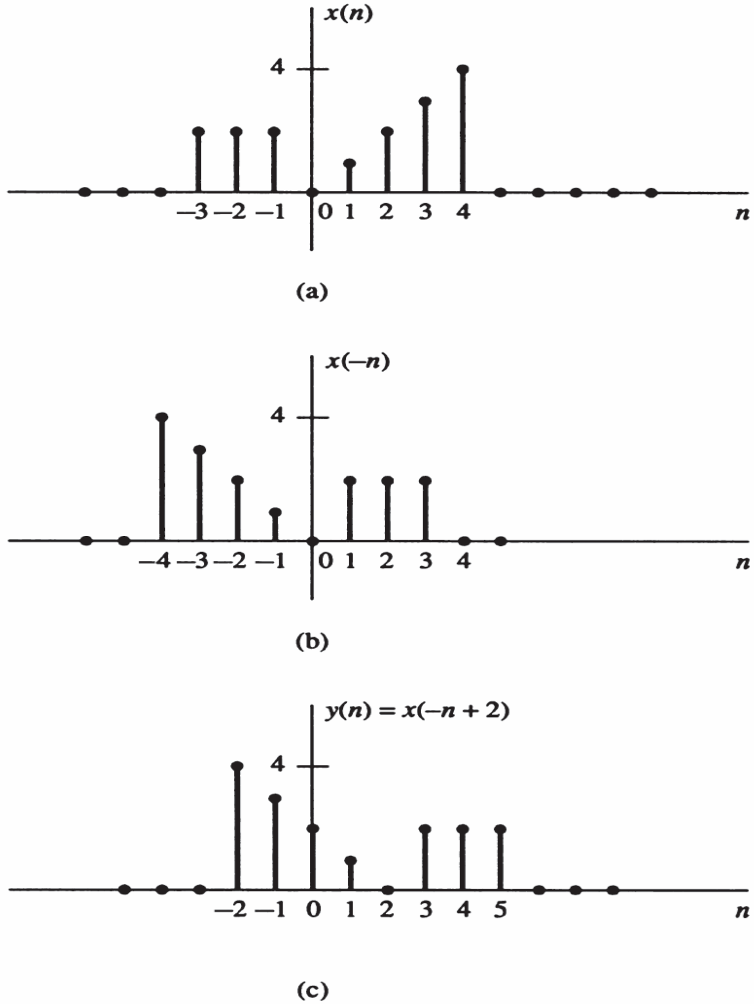

Figure 2.1.10 Graphical illustration of the folding and shifting operations.

Discrete-Time Signals Manipulation
– addition, multiplication and scaling -

- Addition
  - $y[n] = x_1[n] + x_2[n] \text{ where } -\infty < n < \infty$
- Multiplication
  - $y[n] = x_1[n]x_2[n] \text{ where } -\infty < n < \infty$
- Scaling
  - $y[n] = Ax[n] \text{ where } -\infty < n < \infty$

Classification of Discrete-Time Signals

- Energy signals and power signals
- Periodic signals and aperiodic signals
- Symmetric (even) and antisymmetric (odd) signals

Energy signals and power signals

- The energy is defined with

$$E \equiv \sum_{n=-\infty}^{\infty} |x[n]|^2$$

- E can be finite or infinite
  - If finite, then it is called ***<u>energy signal</u>***
- Average power of discrete-time signal

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Energy signals and power signals

- The signal energy of $x[n]$ over the infinite interval of $-N \leq n \leq N$ is

$$E_N \equiv \sum_{n=-N}^{N} |x[n]|^2$$

- Therefore,

$$E \equiv \lim_{N \to \infty} E_N$$

- Average power of the signal

$$P = \lim_{N \to \infty} \frac{1}{2N+1} E_N$$

Periodic signals and aperiodic signals

- A signal is said to be ***periodic*** with period N (N>0) if and only if

$$x[n+N] = x[n], \quad \forall n$$

- The smallest N will give the fundamental period
- Power for a periodic signal with fundamental period N and takes on finite values is

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2$$

Periodic signals are power signals

Symmetric (even) and antisymmetric (odd) signals

- Symmetric (even) if

$$x[-n] = x[n]$$

- Antisymmetric (odd) if

$$x[-n] = -x[n]$$

Discrete-Time Systems

Continued…

Block Diagram Representation

- Adder and multiplier

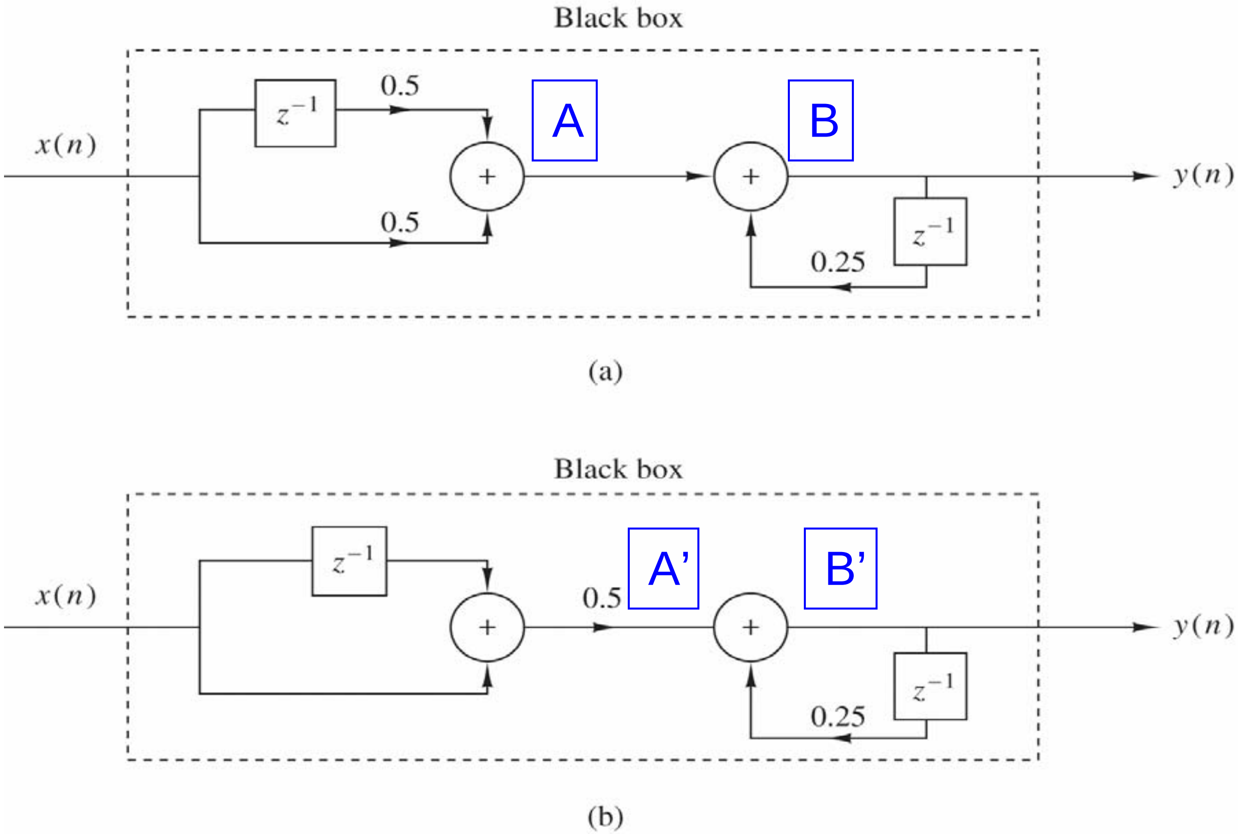

Figure 2.2.7 Block diagram realizations of the system $y(n) = 0.25y(n-1) + 0.5x(n) + 0.5x(n-1)$.

How to Determine y[n] From a Block Diagram? (1)

Referring to Figure 2.2.7 (Previous slide) ...

At A: Top: $0.5x[n-1]$, Bottom:$0.5x[n]$, so adding them become $0.5x[n-1]+0.5x[n]$.

At B: $y[n]$ is delayed and multiplied by 0.25, therefore, the mathematical expression for this is $0.25y[n-1]$. Combining both of this becomes:

$$y[n] = 0.5x[n-1] + 0.5x[n] + 0.25y[n-1] \tag{1}$$

Factorizing this, it becomes

$$y[n] = 0.5(x[n-1] + x[n]) + 0.25y[n-1] \tag{2}$$

Rearrange

$$y[n] = 0.25y[n-1] + 0.5(x[n] + x[n-1]) \tag{3}$$

How to Determine y[n] From a Block Diagram? (2)

Similarly, we apply this knowledge to the A' and B'.

At A': Top: $0.5x[n-1]$, Bottom:$0.5x[n]$, both of these have to multiplied with 0.5, therefore, at A' we yield $0.5(x[n-1]+x[n])$.

At B': Similar to (a), $y[n]$ is delayed and multiplied by 0.25, therefore, the mathematical expression for this is $0.25y[n-1]$. Combining both of this becomes:

$$y[n] = 0.5(x[n-1]+x[n]) + 0.25y[n-1] \tag{1}$$

Rearrange this,

$$y[n] = 0.25y[n-1] + 0.5(x[n]+x[n-1]) \tag{2}$$

Therefore, both diagram are *actually the same*.

Classification of Discrete-Time Systems

- Memoryless systems
- Linear systems
- Time-Invariant systems
- Causality
- Stability

Memoryless Systems

- Output at every value of n depends only on the input at the same value of n
  - Example systems: adder, constant multiplier and signal multiplier
  - Example of non-memoryless (need memory) systems: delayed and advanced

Linear Systems

- Defined by the principle of *superposition*

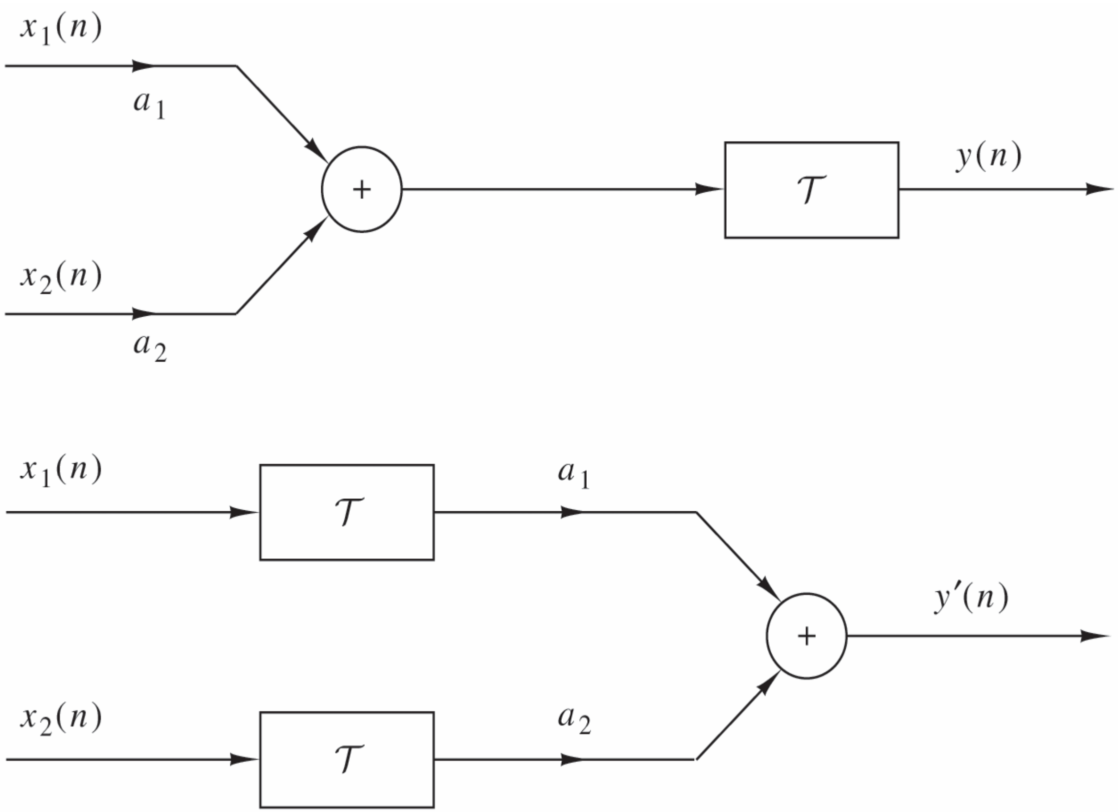

Figure 2.2.9 Graphical representation of the superposition principle. $\mathcal{T}$ is linear if and only if $y(n) = y'(n)$.

Time-Invariant Systems

- System for which a *time shift* or *delay* of the input sequence causes a *corresponding shift* in the output sequence
- Also referred to as a *shift-invariant system*

Causality

- A system is causal if, for every choice of n_0, the output sequence value at the index $n = n_0$ depends only on the input sequence values for $n \leq n_0$
- The system is nonanticipative

Stability

- A system is stable in the bounded-input, bounded output (BIBO) sense if and only if every bounded input sequence produces a bounded output sequence.
- The input $x[n]$ is bounded if

$$|x[n]| \leq B_x < \infty, \qquad \forall n$$

Note that:

Stability requires that, for every bounded input, there exist a fixed positive finite value

Fixed positive finite value