CHAPTER 6: FIRST-ORDER CIRCUITS

6.1 Introduction

- This chapter considers RL and RC circuits.
- Applying the Kirshoff's law to RC and RL circuits produces differential equations.
- The differential equations resulting from analyzing the RC and RL circuits are of the first order.
- Hence, the circuits are known as *first-order* circuits.

A first-order circuit is characterized by a first-order differential equation.

- Two ways to excite the first-order circuit:

  (i) *source-free circuit*

  The energy is initially stored in the capacitive of inductive elements. The energy couses the current to flow in the circuit and gradually dissipated in the resistors.

  (ii) *Exciting by independent sources*

6.2 The Source-Free RC Circuit

- A source-free RC circuit occurs when its dc source is suddenly disconnected.

- The energy already stored in the capacitor is released to the resistors.
- Consider the circuit in Figure 6.1:

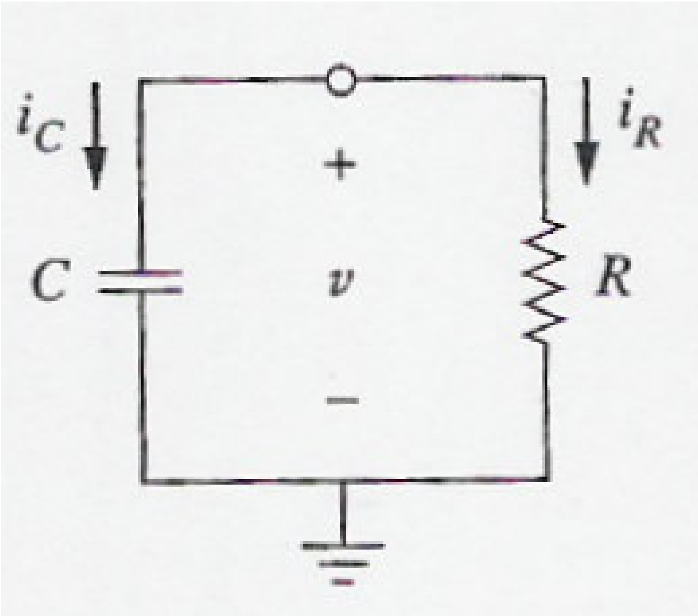

Figure 6.1

Assume voltage $v(t)$ across the capacitor.

Since the capacitor is initially charged, at time $t=0$, the initial voltage is

$$v(0)=V_0$$

with the corresponding of the energy stored as

$$w(0)=\frac{1}{2}CV_0^2$$

applying KCL at the top node of the circuit,

$$i_C+i_R=0$$

By definition, $i_C=C\dfrac{dv}{dt}$ and $i_R=\dfrac{v}{R}$.

Thus,

$$C\frac{dv}{dt}+\frac{v}{R}=0$$

or

$$\frac{dv}{dt}+\frac{v}{CR}=0$$

This is the first-order differential equation.

Rearrange the equation,

$$\frac{dv}{v}=-\frac{1}{RC}dt$$

Intergrating both sides,

$$\ln v=-\frac{t}{RC}+\ln A$$

where ln A is the integration constant.

Thus,

$$\ln\frac{v}{A}=-\frac{t}{RC}$$

Taking powers of *e* produces,

$$v(t)=Ae^{-t/RC}$$

But from the initial condition, $v(0)=A=V_0$

Thus,

$$v(t)=V_0e^{-t/RC} \qquad (6.1)$$

This shows that the voltage response of the RC circuit is an exponential decay of the initial voltage.

Since the response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external volatge or current source – it is called the *natural response* of the circuit.

The voltage response of the RC circuit:

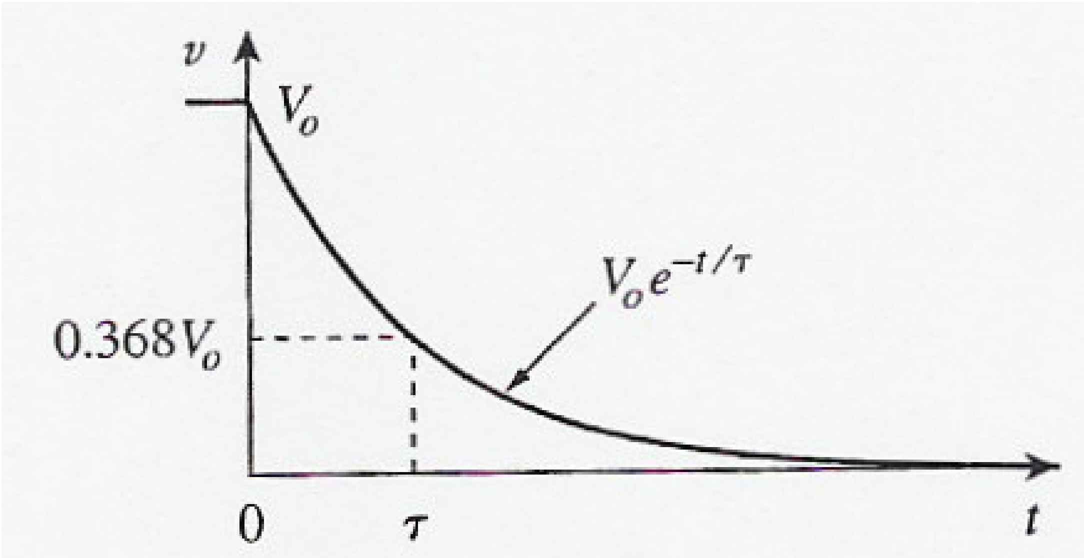

Figure 6.2

As t increases, the voltage decreases toward zero.

The rapidity with which the voltage decreases is expressed in terms of the *time constant*, τ.

> The **time constant** of a circuit is the time required for the response to decay by a factor of 1/e or 36.8% of its initial value.

This implies at $t = \tau$,

$$v(t) = V_0 e^{-\tau/RC} = V_0 e^{-1} = 0.368V_0$$

or

$$\tau = RC \tag{6.2}$$

The voltage is less than 1% after 5 time constant – the circuit reaches its final state or staedy state.

- The current $i_R(t)$ is given by

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau} \tag{6.3}$$

- The power dissipated in the resistor is

$$p(t) = vi_R = \frac{V_0^2}{R} e^{-2t/\tau} \tag{6.4}$$

- The energy absorbed by the resistor up to time t is

$$w_R(t) = \int_0^t p\,dt = \int_0^t \frac{V_0^2}{R} e^{-2t/\tau}$$

$$= -\frac{\tau V_0^2}{2R} e^{-2t/\tau} \Big|_0^t$$

$$\therefore w_R(t) = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau}) \tag{6.5}$$

Note: as $t \to \infty, w_R(t) \to \frac{1}{2}CV_0^2$, which is the same as $w_C(0)$, the energy initially stored in the capacitor.

- The key to working with a source-free RC circuit:
  (i) Find the intial voltage $v(0) = V_0$ across the capacitor.
  (ii) Find the time constant τ.
  (iii) Obtain the capacitor voltage $v_C(t) = v(t) = v(0)e^{-t/\tau}$

  Note: In finding the time constant $\tau = RC$, R is often the Thevenin equivalent resistance at the terminals of the capacitor (take out the capacitor and find $R = R_{TH}$ at its terminal).
- Example 1:
  In Figure 6.3, let $v_C(0) = 15$V. Find v_c, v_x and i_x for $t > 0$.

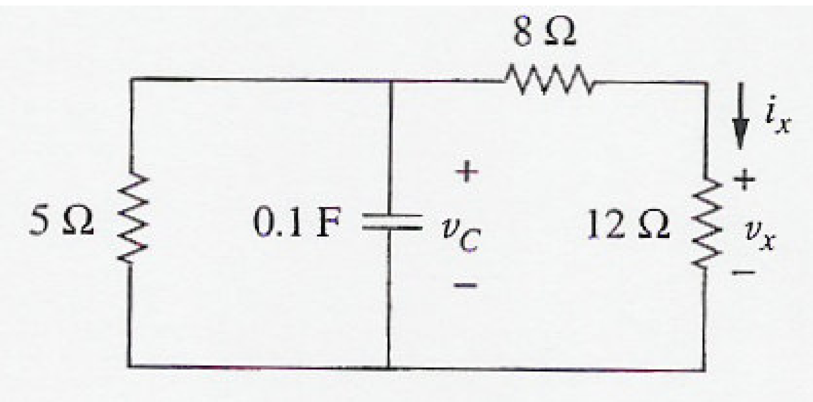

Figure 6.3

Change the circuit to the standard RC circuit as shown in Figure 6.4

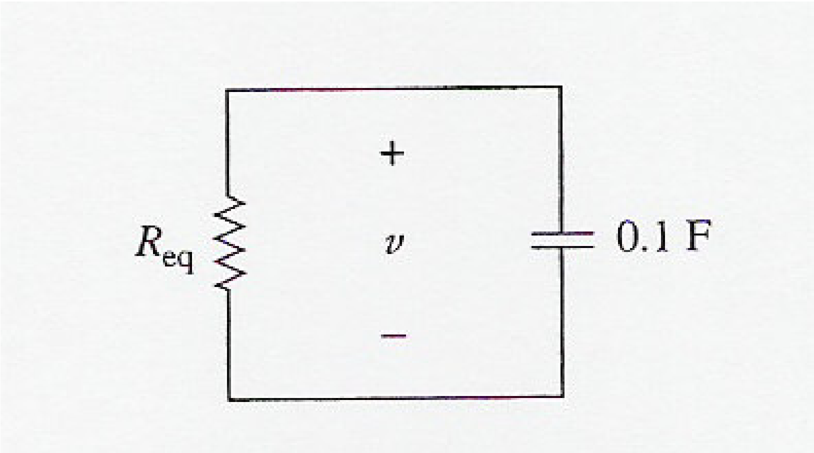

Figure 6.4

$$R_{eq} = (8+12)\|5 = 4\Omega$$

$$\tau = R_{eq}C = (4)(0.1) = 0.4\,\text{s}$$

Thus,

$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\,\text{V}$$

$$v_C = v = 15e^{-2.5t}\,\text{V}$$

Using voltage division,

$$v_x = \frac{12}{12+8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\,\text{V}$$

Finally,

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\,\text{A}$$

- Example 2:

The switch in the circuit in Figure 6.5 has been closed for a long time and it is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.

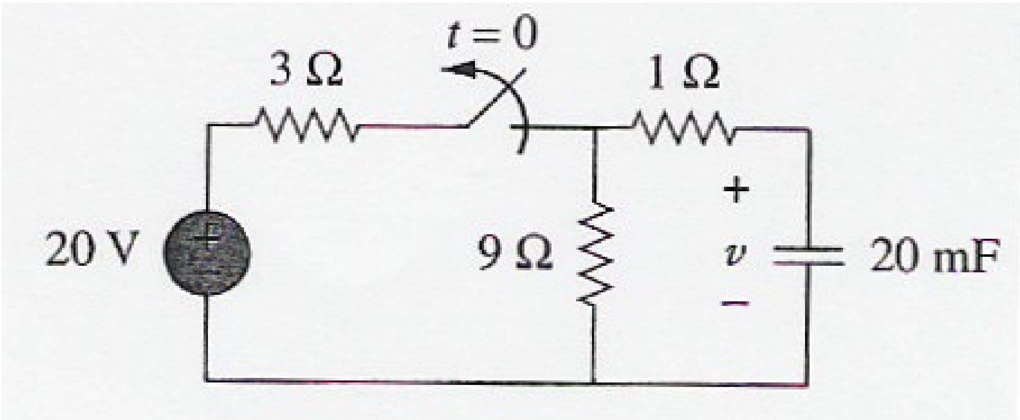

Figure 6.5

For $t < 0$, the switch is closed; the capacitor is an open circuit to dc.

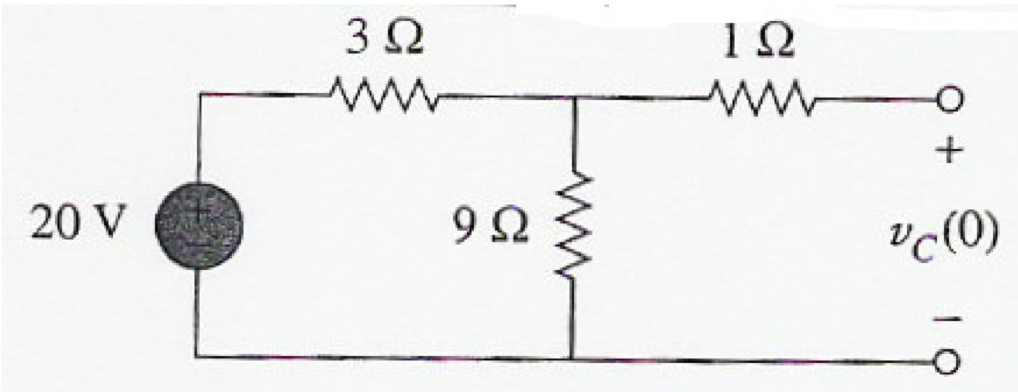

Figure 6.6

Using voltage division,

$$v_C(t) = \frac{9}{9+3}(20) = 15 \text{ V}, \qquad t < 0.$$

Since the voltage across a capacitor acnnot change instantaneously, the voltage across the capacitor at $t = 0^-$ is the same at $t = 0$, or

$$v_C(0) = V_0 = 15\text{V}$$

For t > 0,

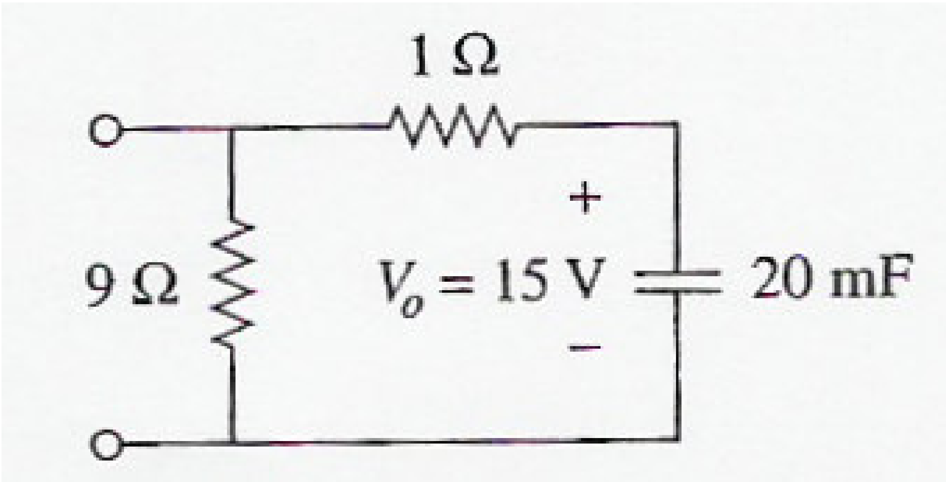

Figure 6.7

$$R_{eq} = 1 + 9 = 10\Omega$$

$$\tau = R_{eq}C = (10)(20\times10^{-3}) = 0.2\text{s}$$

Thus,

$$v(t) = v_C(0)e^{-t/\tau} = 15e^{-t/0.2}\text{V}$$

The initial energy stored in the capacitor is

$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2}(20)(10^{-3})(15^2) = 2.25\text{J}$$

6.3 The Source-Free RL Circuit

- Consider the circuit in Figure 6.8:

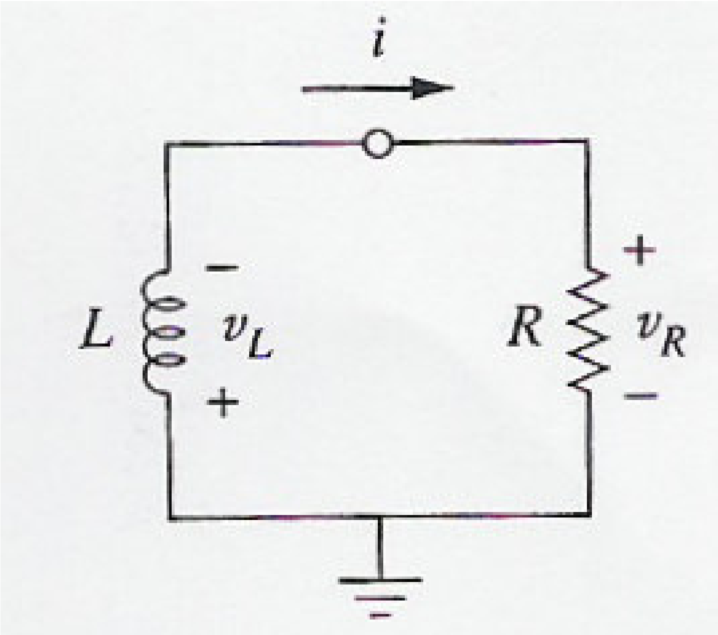

Figure 6.8

- Goal – to determine the current $i(t)$ through the inductor.
- Why we select the inductor current as the response? The inductor current cannot change instantaneously.
- At $t = 0$, we assume that the inductor has an initial current I_0 or $i(0) = I_0$.

  Energy stored in the inductor,

$$w(0) = \frac{1}{2} L I_0^2$$

Applying KVL,

$$v_L + v_R = 0$$

But,

$$v_L = L\frac{di}{dt} \text{ and } v_R = iR$$

Thus,

$$L\frac{di}{dt} + Ri = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

Rearranging terms and intergrating gives,

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L}dt = \ln i\Big|_{I_0}^{i(t)} = -\frac{Rt}{L}\Bigg|_0^t$$

$$\therefore \ln i(t) - \ln I_0 = -\frac{Rt}{L} + 0$$

$$\ln\frac{i(t)}{I_0} = -\frac{Rt}{L}$$

Taking the powers of *e*,

$$i(t) = I_0 e^{-Rt/L} \qquad (6.6)$$

Thus, the natural (current) response of the RL circuit is as shown:

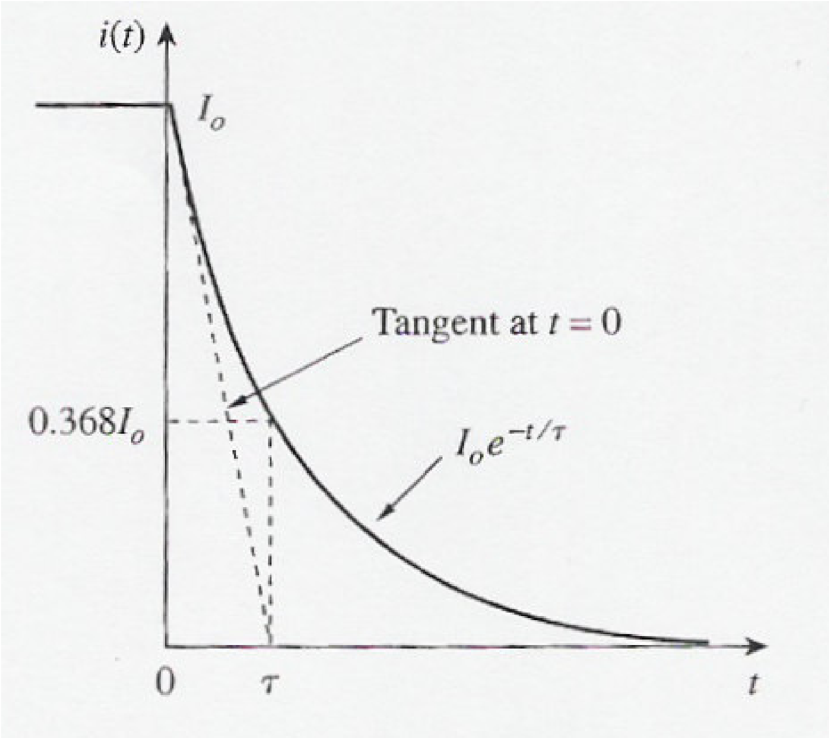

Figure 6.9

- From Equation 6.6, the time constant is

$$\tau = \frac{L}{R} \qquad (6.7)$$

- The voltage across the resistor,

$$v_R(t) = iR = I_0 R\ e^{-t/\tau} \qquad (6.8)$$

- The power dissipated in the resistor is,

$$p = v_R i = I_0^2 R\ e^{-2t/\tau} \qquad (6.9)$$

- The energy absorbed by the resistor is,

$$w_R(t) = \int_0^t p dt = \int_0^t I_0^2 R \ e^{-2t/\tau} dt$$

$$= -\frac{1}{2}\tau I_0^2 R \ e^{-2t/\tau}\Big|_0^t$$

$$\therefore w_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau}) \qquad (6.10)$$

Note: as $t \to \infty, w_R(t) \to \frac{1}{2} L I_0^2$, which is the same as $w_L(0)$, the energy initially stored in the inductor.

- The key to working with a source-free RC circuit:

(iv) Find the intial current $i(0) = I_0$ across the capacitor.

(v) Find the time constant τ.

(vi) Obtain the capacitor voltage $i_L(t) = i(t) = i(0)e^{-t/\tau}$

Note: *R* is often the Thevenin equivalent resistance at the terminals of the inductor (take out the inductor and find $R = R_{TH}$ at its terminal).

- Example 1:

  Assuming that $i(0) = 10$A, calculate $i(t)$ and $i_x(t)$ in the circuit in Figure 6.10.

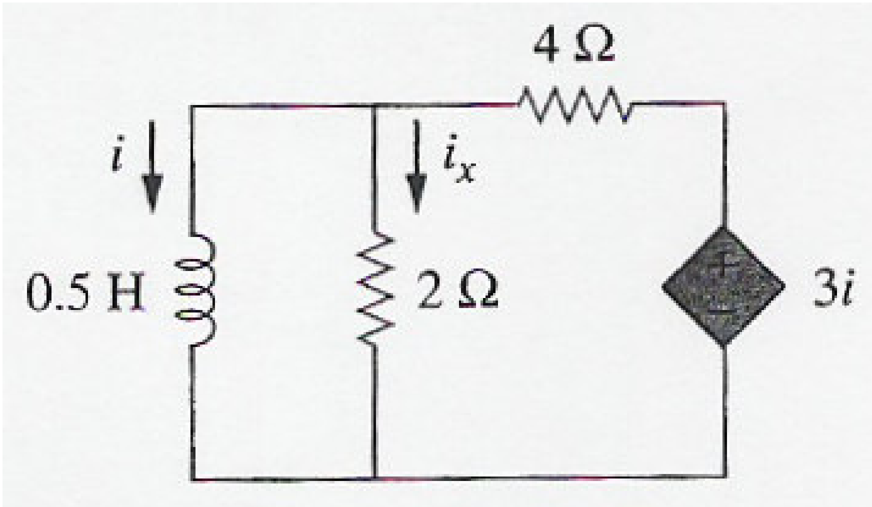

Figure 6.10

Two Methods to find the $i(t)$,

Method 1

The equivalent resistance is the same as the Thevenin resistance at the inductor terminals.

Existance of dependent source – insert a voltage source with $v_0 = 1$V at the inductor terminals.

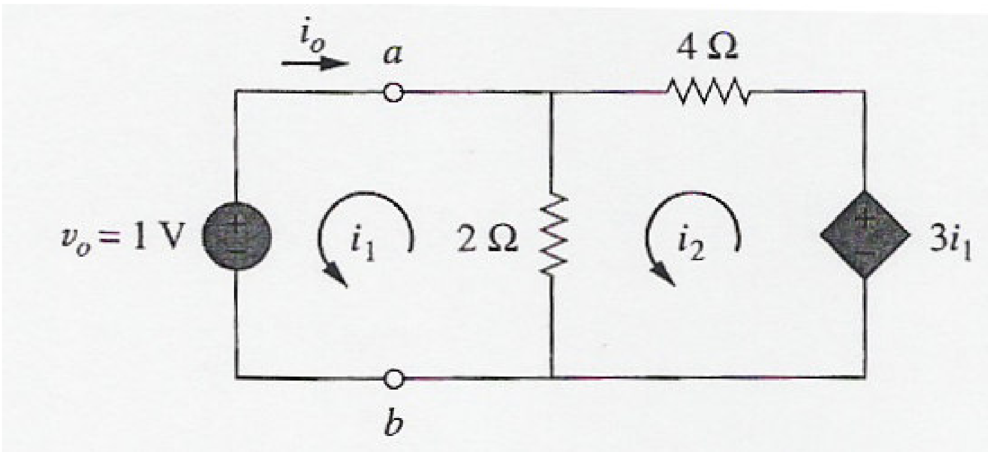

Figure 6.11

Applying KVL,

Loop 1:

$$2(i_1 - i_2) + 1 = 0$$

$$i_1 - i_2 = -\frac{1}{2}$$

Loop 2:

$$6i_2 - 2i_1 - 3i_1 = 0$$

$$i_2 = \frac{5}{6} i_1$$

Thus,

$$i_1 = -3\text{A and } i_0 = -i_1 = 3\text{A}$$

Hence,

$$R_{eq} = R_{TH} = \frac{v_0}{i_0} = \frac{1}{3}\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{3}{2}\text{s}$$

Thus,

$$i(t) = i(0)e^{-t/\tau} = 10e^{-t/\tau}\text{A}$$

Method 2:

Consider the foloowing circuit:

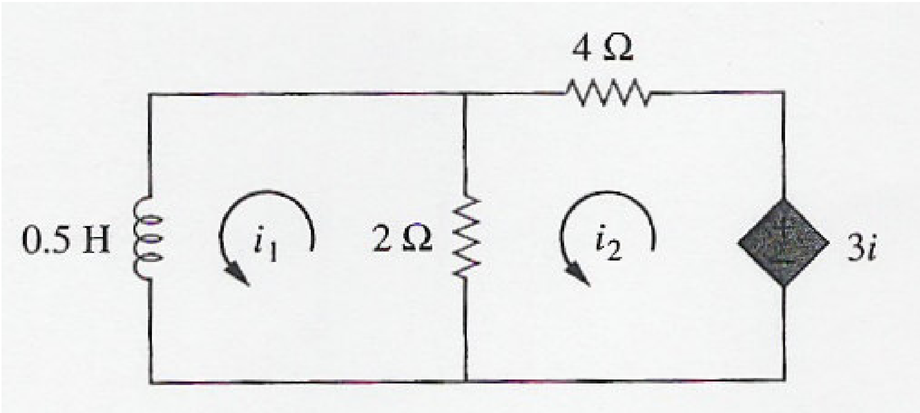

Figure 6.12

Applying KVL,

Loop 1:

$$\frac{1}{2}\frac{di_1}{dt}+2(i_1-i_2)=0$$

$$\frac{di_1}{dt}+4(i_1-i_2)=0$$

Loop 2:

$$6i_2-2i_1-3i_1=0$$

$$i_2=\frac{5}{6}i_1$$

Thus,

$$\frac{di_1}{dt}+\frac{2}{3}i_1=0$$

$$\frac{di_1}{i_1}=-\frac{2}{3}dt$$

Since $i_1 = 1$,

$$\frac{di}{i} = -\frac{2}{3}dt$$

Integrating gives,

$$\ln i\Big|_{i(0)}^{i(t)} = -\frac{2}{3}t\Big|_0^t$$

$$\ln\frac{i(t)}{i(0)} = -\frac{2}{3}t$$

Taking the power of *e*,

$$i(t) = i(0)e^{-(2/3)t} = 10e^{-(2/3)t}\text{ A}$$

which is the same as Method 1

The voltage across the inductor is,

$$v = L\frac{di}{dt} = (0.5)(10)\left(-\frac{2}{3}\right)e^{-(2/3)t}\text{ V}$$

$$\therefore v = -\frac{10}{3}e^{-(2/3)t}\text{ V}$$

Thus,

$$i_x(t) = \frac{v}{2} = -1.667e^{-(2/3)t}\text{ A for } t > 0$$

- Example 2:

  In the circuit shown in Figure 6.13, find i_0, v_0 and i for all time, assuming that the switch was open for a long time.

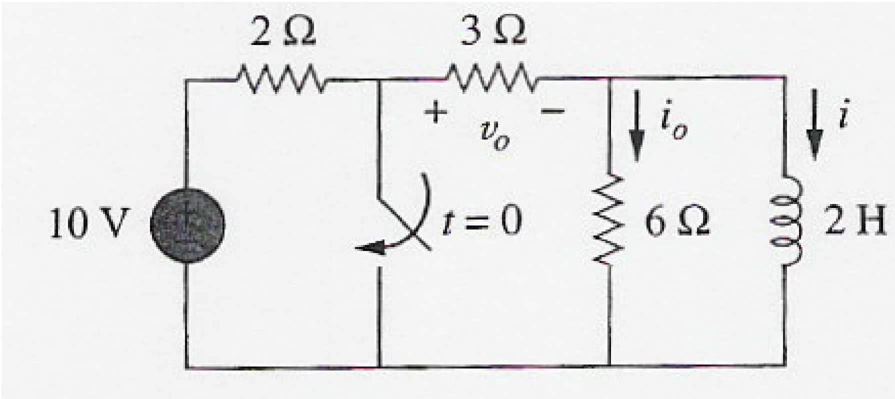

Figure 6.13

For $t < 0$, the switch is opened – the inductor acts like a short circuit to dc,

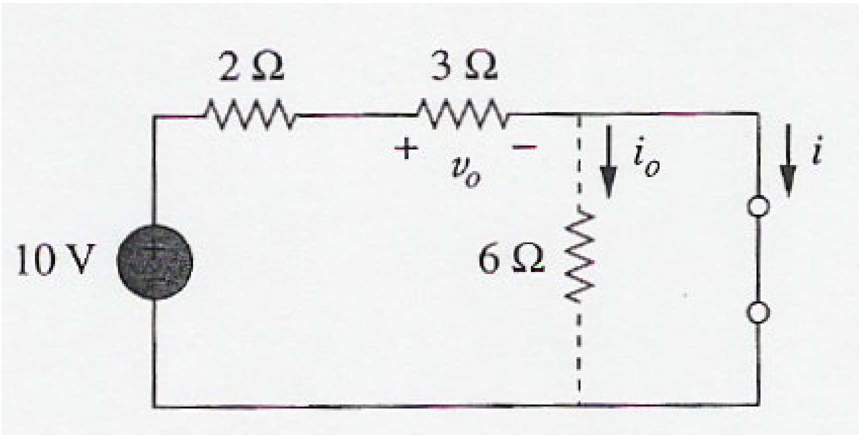

Figure 6.14

From Figure 6.14,

$$i_0 = 0\,\text{A}.$$

$$i(t) = \frac{10}{2+3} = 2 \text{ A for } t < 0$$

$$v_0(t) = 3i(t) = 6 \text{ V for } t < 0$$

Thus, $i(0) = 2\text{A}$

For $t > 0$, the switch is closed – the voltage source is short-circuited.

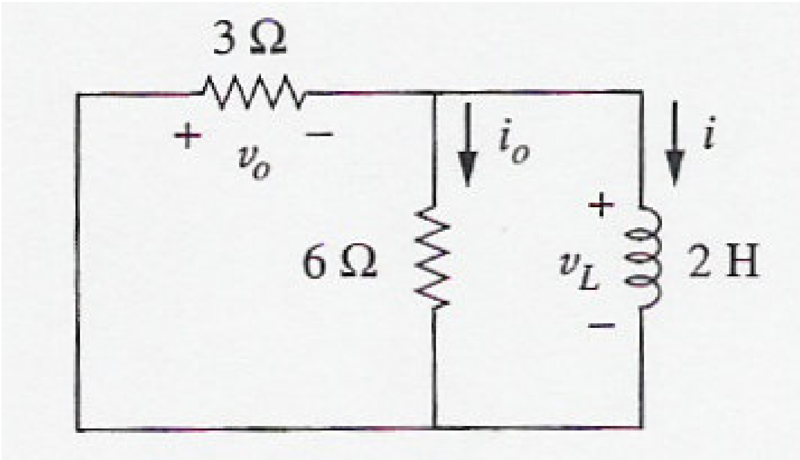

Figure 6.15

At the inductor terminal,

$$R_{TH} = 3 \| 6 = 2\Omega$$

$$\tau = \frac{L}{R_{eq}} = 1\text{s}$$

Hence,

$$i(t) = i(0)e^{-t/\tau} = 2e^{-t}\text{A for } t > 0$$

$$v_0(t) = -v_L = -L\frac{di}{dt} \qquad \text{for } t > 0$$

$$= -2(-2e^{-t}) = 4e^{-t}V$$

$$i_0(t) = \frac{v_L}{6} = -\frac{2}{3}e^{-t}\,\text{A for } t > 0$$

Thus, for all time:

$$i_0(t) = \begin{cases} 0A & t < 0 \\ -\frac{2}{3}e^{-t}A & t > 0 \end{cases}$$

$$v_0(t) = \begin{cases} 6V & t < 0 \\ 4e^{-t}V & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2A & t < 0 \\ 2e^{-t}A & t \geq 0 \end{cases}$$

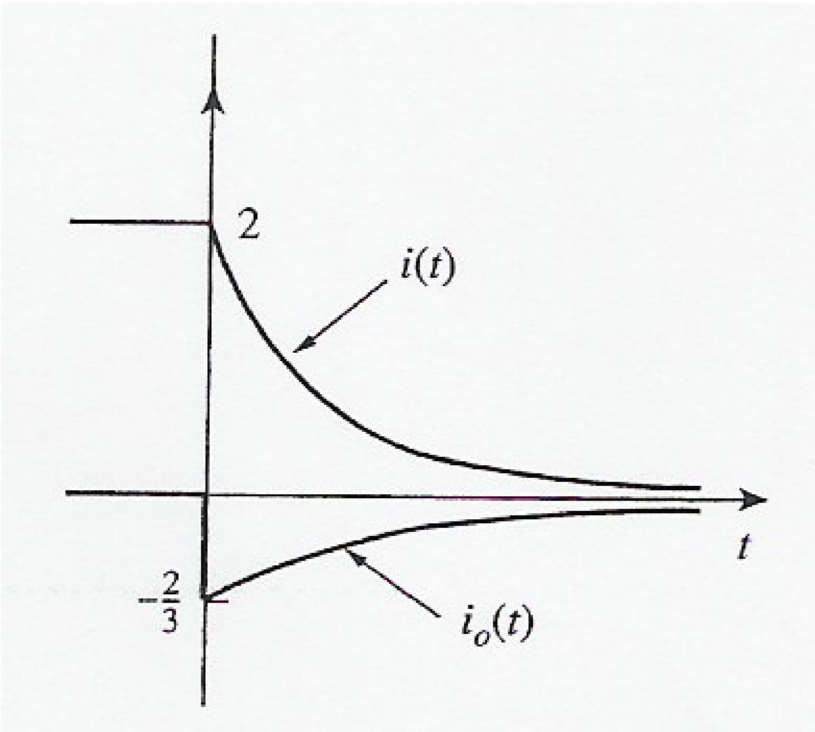

Figure 6.16

6.4 Singularity Functions

- Definition:

Singularity functions are functions that either are dicontinuous or have discontinuous derivatives.

- Unit step function:

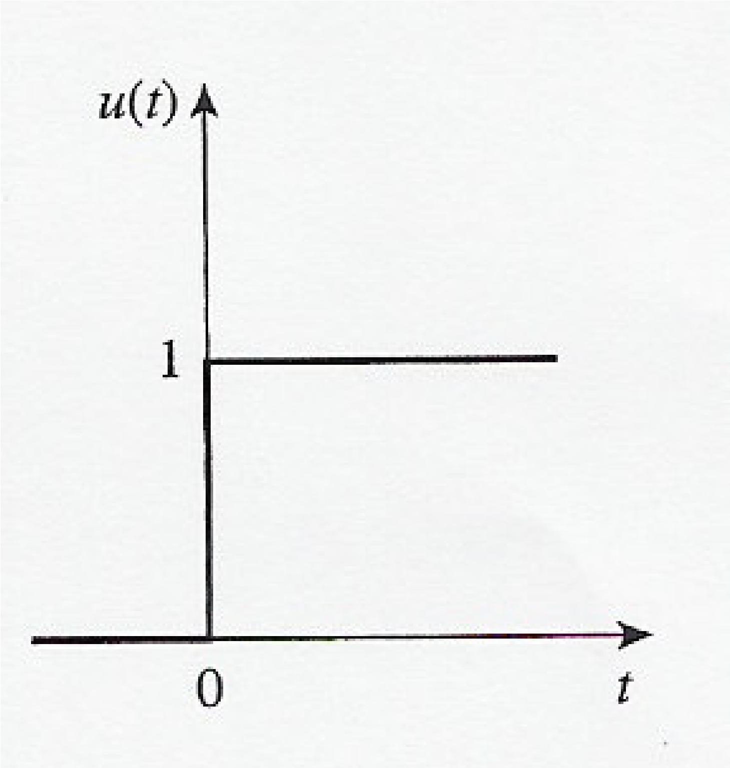

Figure 6.17

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases} \qquad (6.11)$$

If $u(t)$ is delayed by t_0 seconds:

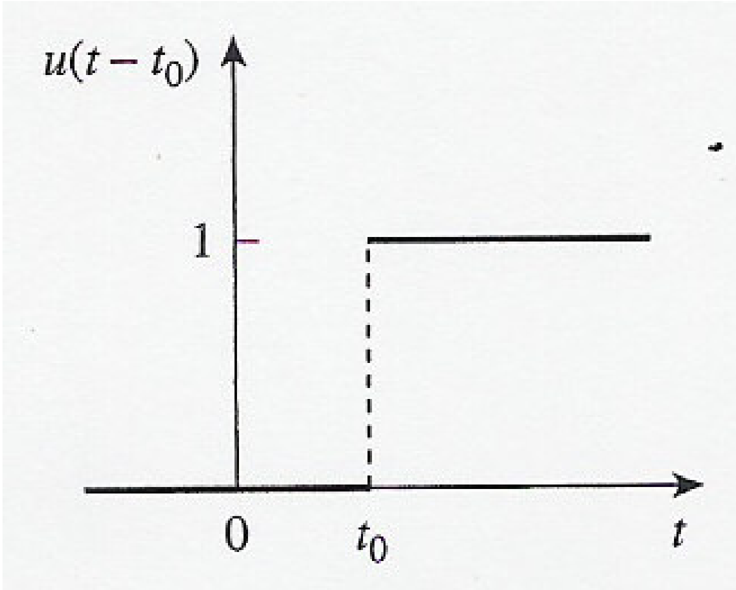

Figure 6.18

$$u(t-t_0)=\begin{cases}0 & t<t_0\\ 1 & t>t_0\end{cases}$$

If $u(t)$ is advanced by t_0 seconds:

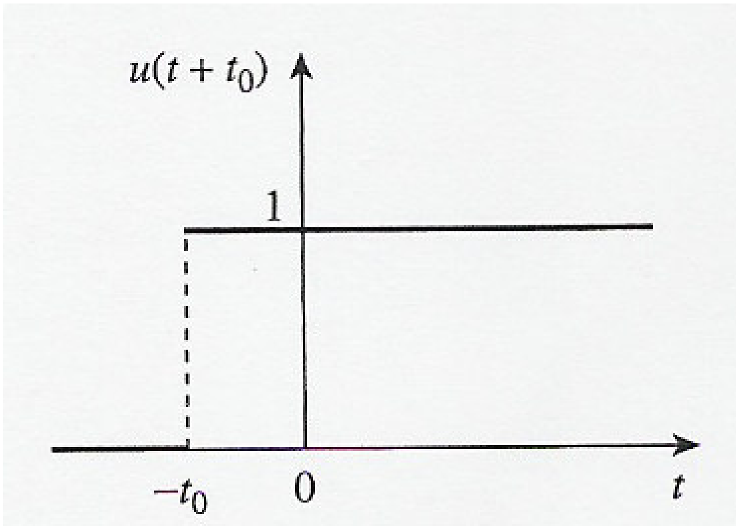

Figure 6.19

$$u(t+t_0)=\begin{cases}0 & t<-t_0\\ 1 & t>-t_0\end{cases}$$

The step function can be used to represent an abrupt change in voltage or current.
For example, if the voltage is represented by,

$$v(t) = \begin{cases} 0 & t < t_0 \\ V_0 & t > t_0 \end{cases}$$

can be expressed as

$$v(t) = V_0 u(t - t_0)$$

The voltage source of $V_0 u(t)$ and its equivalent circuit:

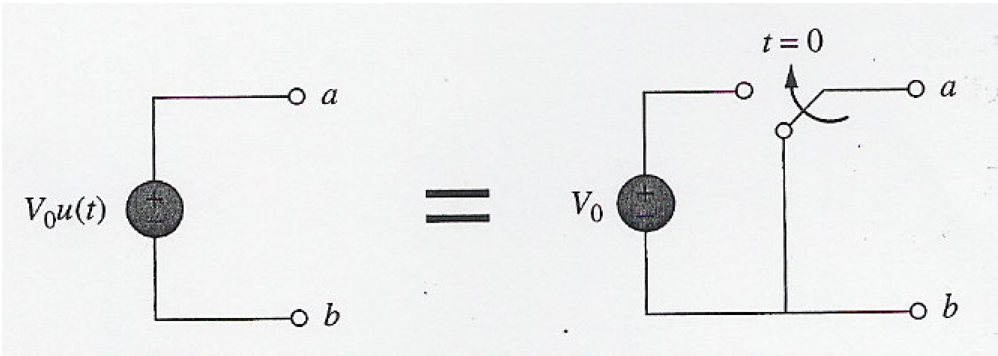

Figure 6.20

The current source of $I_0 u(t)$ and its equivalent circuit:

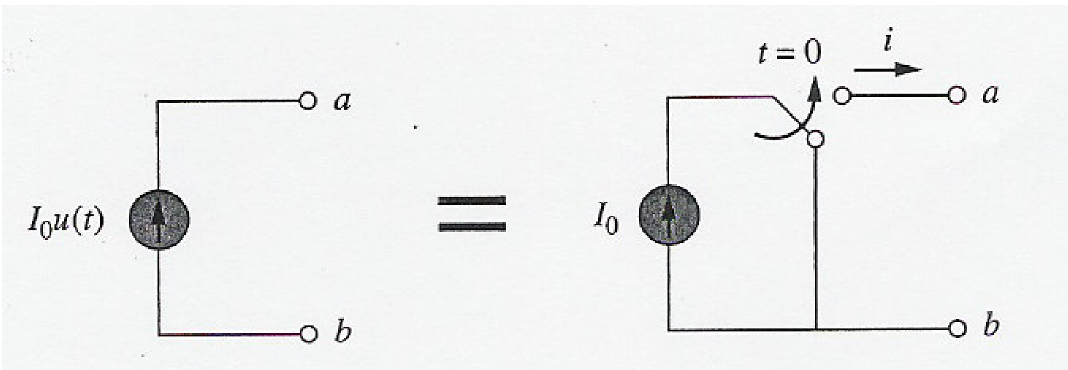

Figure 6.21

- Unit impulse function:

The derivative of the unit step function $u(t)$ - the *unit impulse function* $\delta(t)$.

$$\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0 & t < 0 \\ Undefined & t = 0 \\ 0 & t > 0 \end{cases} \quad (6.12)$$

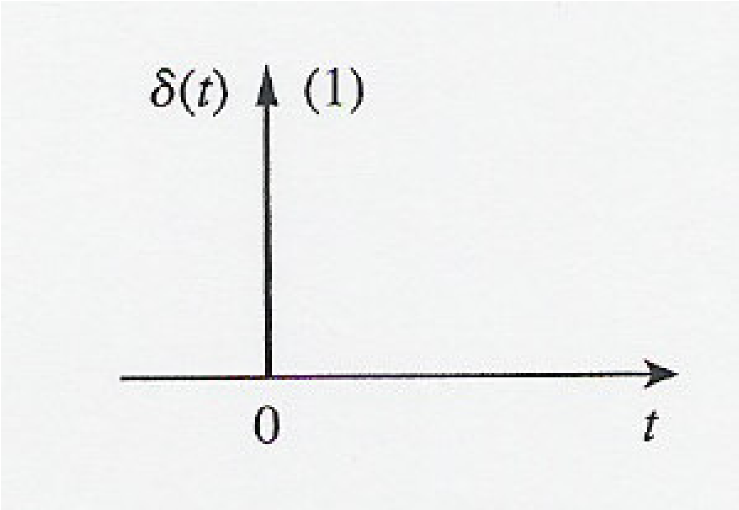

Figure 6.22

Also known as *delta function*.

The *unit impulse function* $\delta(t)$ is zero everywhere except at t = 0, where it is undefined.

Impulsive currents or voltages occur in electric circuits as a result of switching operations or impulsive sources.

It may be visualized as a very short duration pulse of unit area.

In mathematical form:

$$\int_{0^-}^{0^+} \delta(t)dt = 1$$

where $t = 0^-$ denotes the time just before $t = 0$, $t = 0^+$ denotes the time just after $t = 0$ and 1 (refer to unity) denotes the unit area.

The unit area is the strength of the impulse function.

The effect of the impulse function to other functions:

Let us evaluate the integral

$$\int_a^b f(t)\delta(t - t_0)dt$$

where $a < t_0 < b$.

Since $\delta(t - t_0) = 0$ except $t = t_0$, the integrand is zero except at t_0.

Thus,

$$\int_a^b f(t)\delta(t - t_0)dt = \int_a^b f(t_0)\delta(t - t_0)dt$$

$$= f(t_0)\int_a^b \delta(t - t_0)dt = f(t_0)$$

*when a function is integrated with the impulse function, we obtain the value of the function at the point where the impulse occur.

- Unit ramp function:
  Integrating the unit step function $u(t)$ results in the unit ramp function $r(t)$.
  In mathematical form:

$$r(t) = \int_{-\infty}^{t} u(t)dt = tu(t)$$

or

$$r(t) = \begin{cases} 0, & t \le 0 \\ t, & t \ge 0 \end{cases} \qquad (6.13)$$

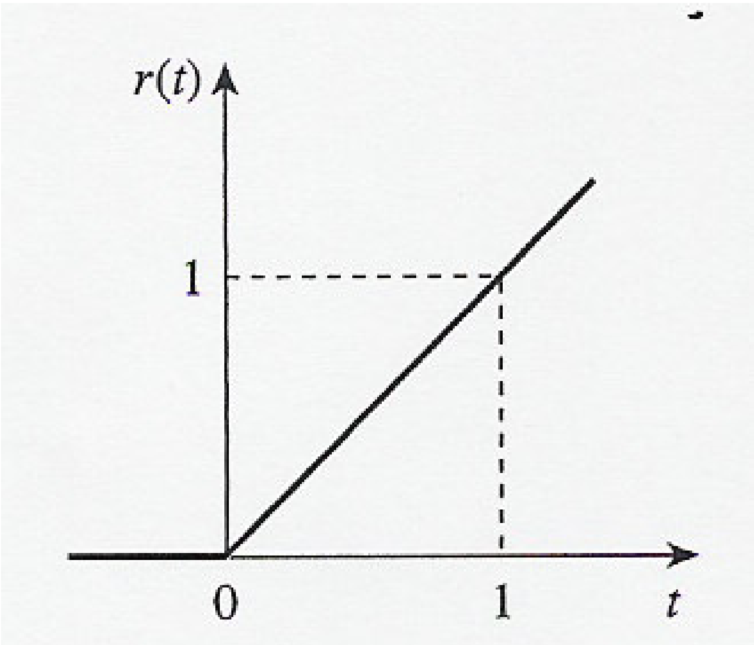

Figure 6.23

If $r(t)$ is delayed by t_0 seconds:

$$r(t - t_0) = \begin{cases} 0 & t < t_0 \\ t - t_0 & t > t_0 \end{cases}$$

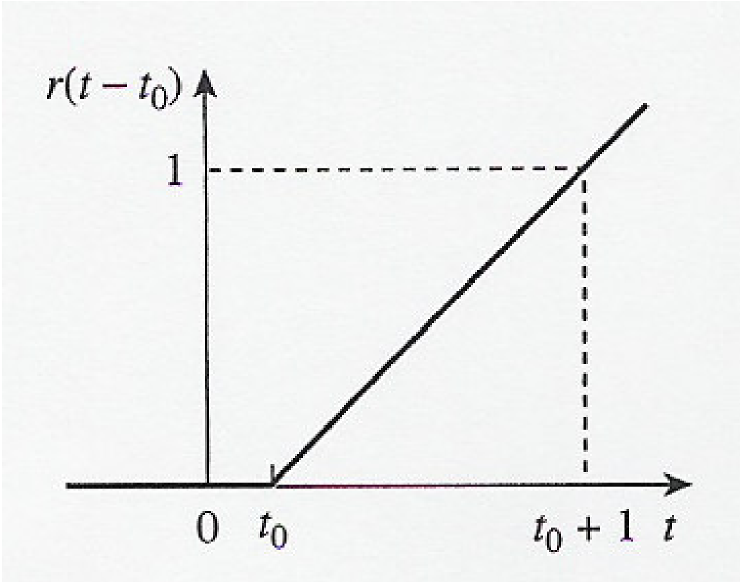

Figure 6.24

If $r(t)$ is advanced by t_0 seconds:

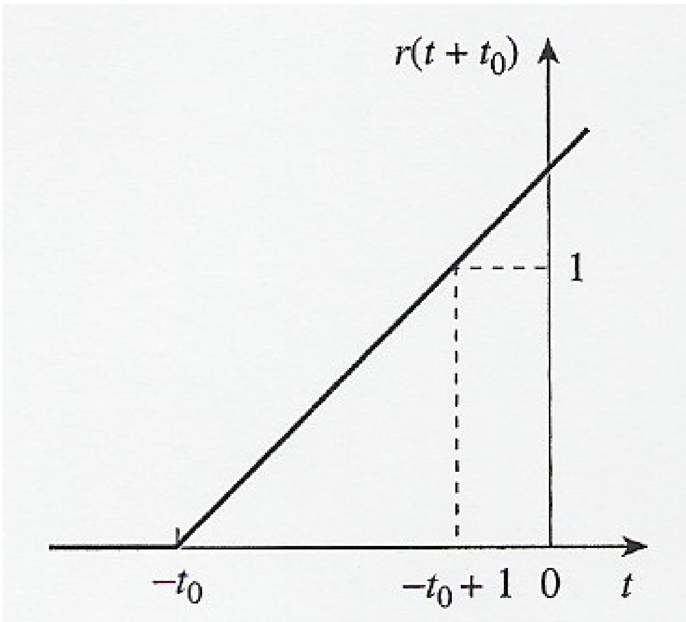

Figure 6.25

$$r(t+t_0)=\begin{cases} 0 & t<t_0 \\ t+t_0 & t>t_0 \end{cases}$$

- Example:

Express the voltage pulse in Figure 6.26 in terms of the unit step. Calculate its derivative and sketch it.

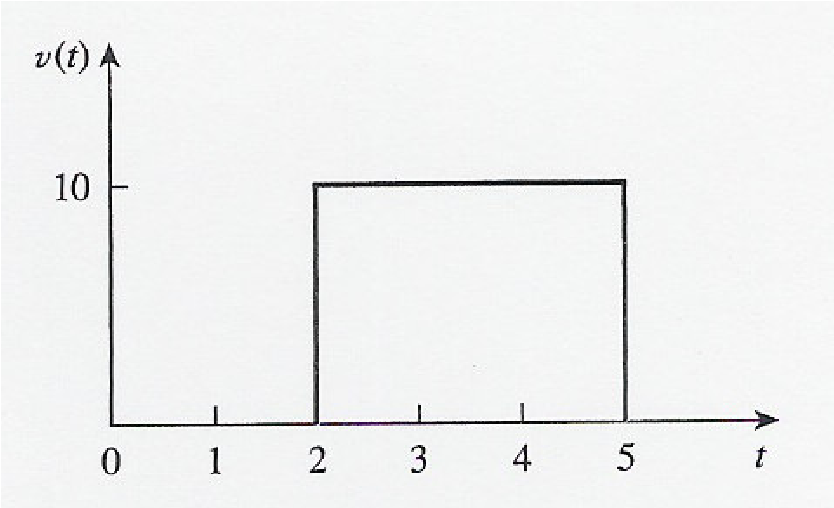

Figure 6.26

The pulse consists of the sum of two unit step functions

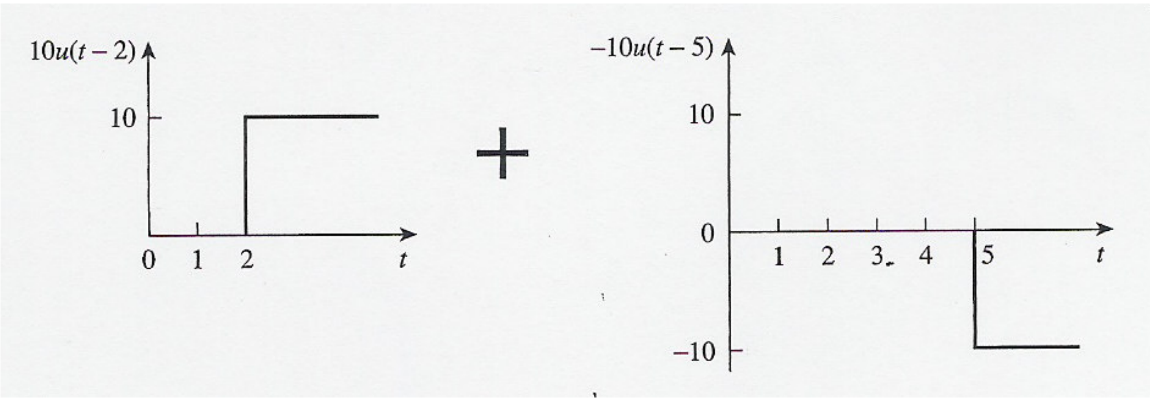

Figure 6.27

Thus,

$$v(t) = 10u(t-2) - 10u(t-5)$$
$$= 10[u(t-2) - u(t-5)]$$

Its derivative:

$$\frac{dv}{dt} = 10[\delta(t-2) - \delta(t-5)]$$

The pulse:

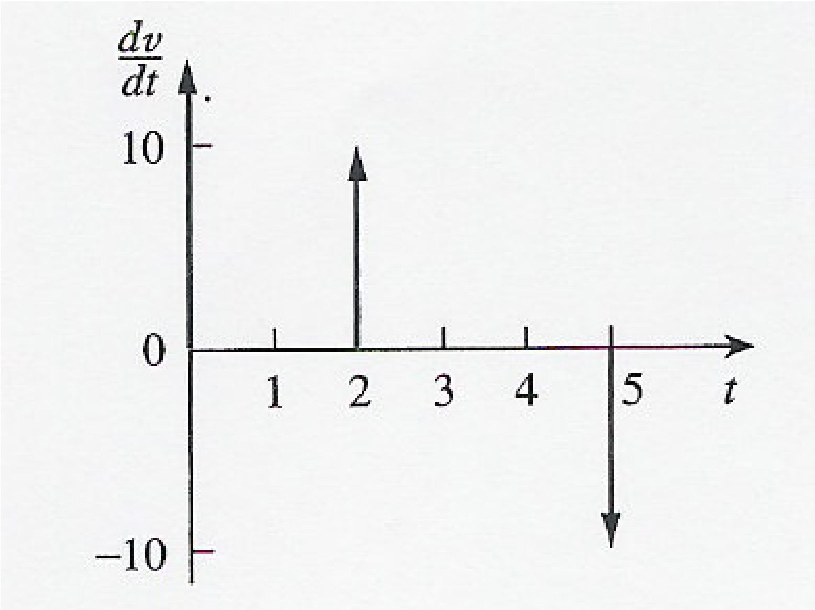

Figure 6.28

6.5 Step Response of an RC Circuit

- When the dc source is suddenly applied, the voltage or current can be modeled as a step function.
- Known as a step response

> The **step response** of a circuit is its behavior when the excitation is the step function, which may be a voltage or a current source.

- Consider the circuit in Figure 6.29 (a) and (b):

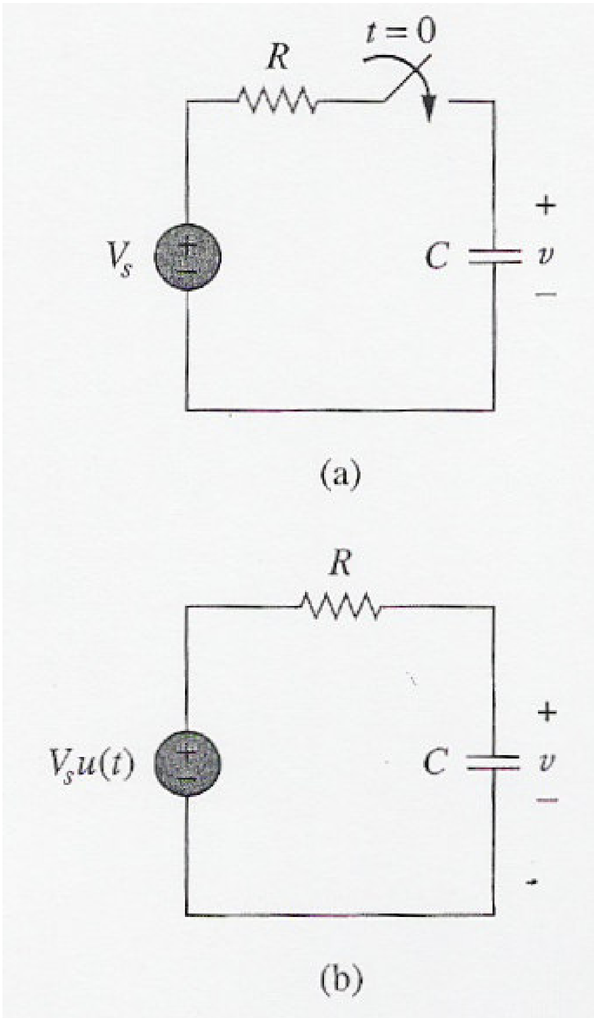

Figure 6.29

- Assume the initial voltage voltage V_0 at the capacitor.
- Since the voltage of a capacitor cannot change instantenously,

$$v(0^-) = v(0^+) = V_0$$

- Applying KVL,

$$C\frac{dv}{dt} + \frac{v - V_S u(t)}{R} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_S}{RC}u(t)$$

where v is the voltage across the capacitor.

- For $t > 0$,

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_S}{RC}$$

$$\frac{dv}{dt} = -\frac{v - V_S}{RC}$$

$$\frac{dv}{v - V_S} = -\frac{dt}{RC}$$

- Integrating both sides and introducing the initial conditions,

$$\ln(v - V_S)\Big|_{V_0}^{v(t)} = -\frac{t}{RC}\Big|_0^t$$

$$\ln\left(v(t)-V_S\right)-\ln\left(V_0-V_S\right)=-\frac{t}{RC}+0$$

$$\ln\frac{v-V_S}{V_0-V_S}=-\frac{t}{RC}$$

$$v(t)=V_S+(V_0-V_S)e^{-t/\tau}, \qquad t>0$$

Thus,

$$v(t)=\begin{cases} V_0, & t<0 \\ V_S+(V_0-V_S)e^{-t/\tau}, & t>0 \end{cases} \qquad (6.14)$$

- Equation 6.14 is known as the complete response (or total response) of the RC circuit to a sudden application of a dc voltage source, assuming the capacitor is initially charged.
- Assuming that $V_S > V_0$, a plot of $v(t)$ is,

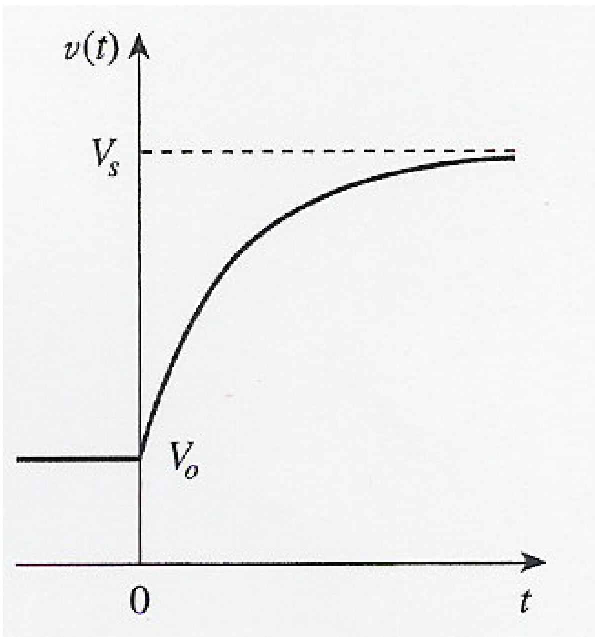

Figure 6.30

- If the capacitor is uncharged initially,

$$V_0 = 0$$

$$\therefore v(t) = \begin{cases} 0, & t < 0 \\ V_S(1 - e^{-t/\tau}), & t > 0 \end{cases}$$

or

$$v(t) = V_S(1 - e^{-t/\tau})u(t)$$

⇒ the complete step response of the RC circuit when the capcitor is initially uncharged.

The current is obtained using $i(t) = C\dfrac{dv}{dt}$ from $v(t)$ equation.

- $v(t)$ has two components – two ways to decompose the components.
- First way – natural response and forced response.

Complete response = Natural Response (stored energy) + Forced Response (independent source)

or

$$v = v_n + v_f$$

where

$$v_n = V_0 e^{-t/\tau}$$

and

$$v_f = V_S(1 - e^{-t/\tau})$$

v_n is as discussed in Section 6.2.

v_f is known as the forced response because it is produced by the circuit when an external 'force' (a voltage or current source) is applied.

- Second way –transient response and steady-state response.

Complete Response = Transient Response (temporary part) + Steady-state Response (permanent part)

or

$$v = v_t + v_{ss}$$

where

$$v_t = (V_0 - V_S)e^{-t/\tau}$$

and

$$v_{ss} = V_S$$

The transient response v_t is temporary – the portion of the complete response that decays to zero as time approaches infinity.

The **transient response** is the circuit's temporary response that will die out with time.

The steady-state response v_{ss} is the portion of the complete response that remains after the transient response has died out.

The **steady-state response** is the behavior of the circuit a long time after an external excitation is applied.

- The complete response may be written as

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \quad (6.15)$$

where $v(0)$ is the initial voltage at $t = 0^+$ and $v(\infty)$ is the final or steady-state value.

- To find the step response of an RC circuit
  (i) Find the initial capacitor voltage, $v(0)$.
  - obtain from the given circuit for $t < 0$.
  (ii) Find the final capacitor voltage, $v(\infty)$.
  - obtain from the given circuit for $t > 0$.
  (iii) The time constant, τ.
  - obtain from the given circuit for $t > 0$.

- Example 1:

The switch in Figure 6.31 has been in position A for a long time. At $t=0$, the switch moves to B. determine $v(t)$ for $t>0$.

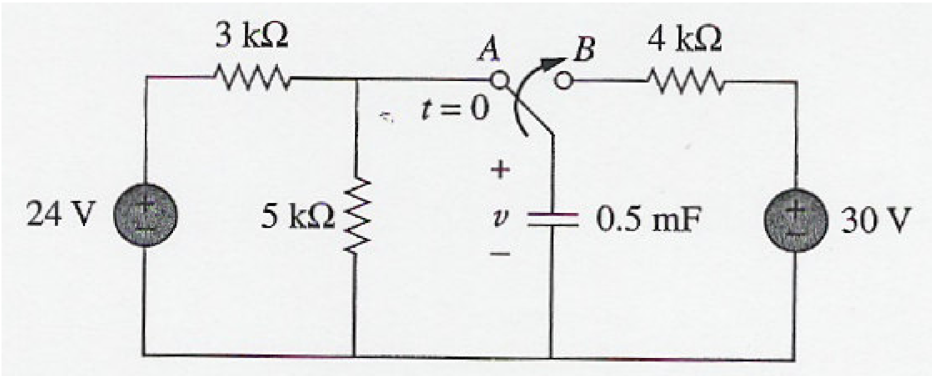

Figure 6.31

For $t<0$,

The capacitor acts like an open circuit to dc, but v is the same as the voltage across the $5k\Omega$ resistor.

Hence, the voltage across the capacitor just before $t=0$ is obtained by volatge division as

$$v(0^-)=\frac{5}{5+3}(24)=15\text{V}$$

Since the capacitor voltage cannot change instantaneously,

$$v(0)=v(0^-)=v(0^+)=15\text{V}$$

For $t > 0$,

$$R_{TH} = 4k\Omega$$

$$\tau = R_{TH}C = 4\times10^{3}\times0.5\times10^{-3} = 2\text{s}$$

Since the capacitor acts like an open circuit to dc at steady state, $v(\infty) = 30\text{V}$.

Thus,

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$= 30 + (15 - 30)e^{-t/\tau}$$

$$v(t) = 30 - 15e^{-0.5t}\text{ V}$$

- Example 2:

  The switch in Figure 6.32 is closed at $t = 0$. Find $i(t)$ and $v(t)$ for all time.

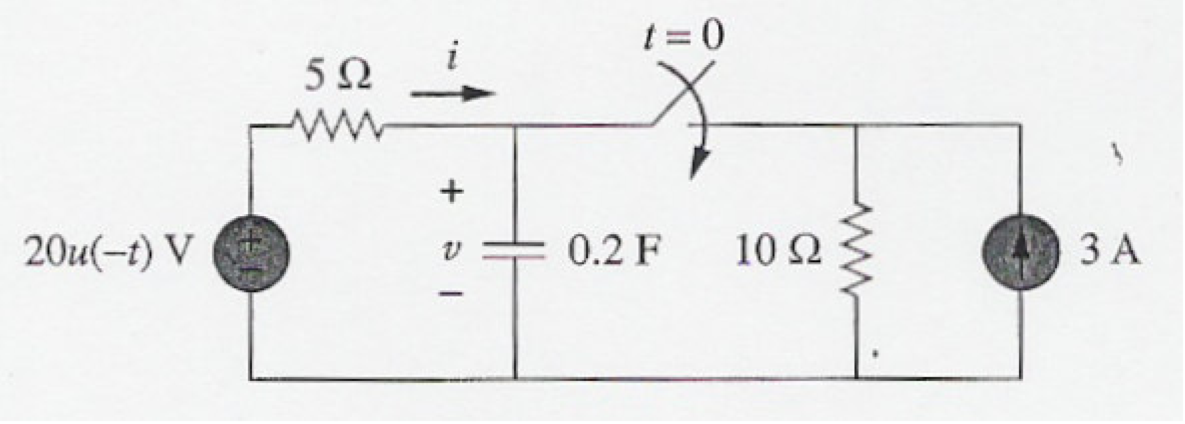

Figure 6.32

$$u(-t) = \begin{cases} 1, & t < 0 \\ 0, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 0, & t < 0 \\ -2(1+e^{-1.5t})A, & t > 0 \end{cases}$$

$$v = \begin{cases} 20V, & t < 0 \\ 10(1+e^{-1.5t})V, & t > 0 \end{cases}$$

6.6 Step Response of an RL Circuit

- Consider the circuit in Figure 6.33,

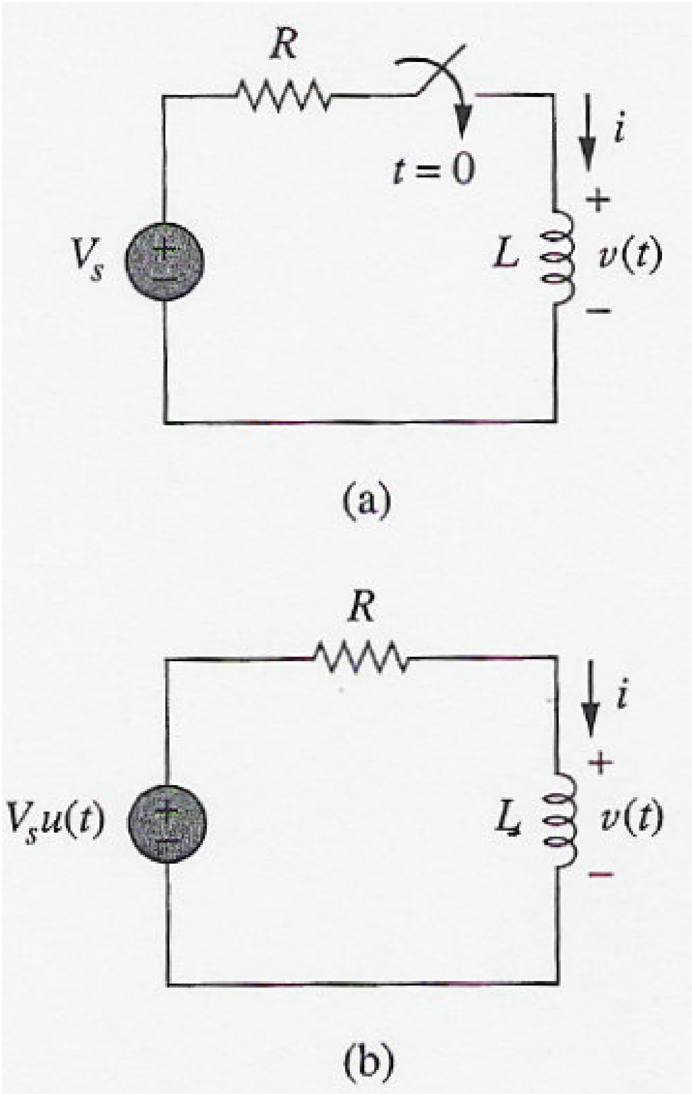

Figure 6.33

- Goal – find the inductor current i as the circuit response.
- Decompose the response into natural and forced current:

$$i = i_n + i_f$$

where

$$i_n = Ae^{-t/\tau}, \qquad \tau = \frac{L}{R}$$

A is the constant to be determined

- The natural response dies out after five time constants – the inductor becomes a short circuit and the voltage across it is zero.
- The entire source voltage V_S appears across R.
- Thus, the forced response is

$$i_f = \frac{V_S}{R}$$

$$\therefore i = Ae^{-t/\tau} + \frac{V_S}{R}$$

- To find A, let I_0 be the initial current through the inductor.
- Since the current through cannot change instantaneously,

$$i(0^+) = i(0^-) = I_0$$

- Thus, at $t = 0$,

$$I_0 = A + \frac{V_S}{R}$$

$$A = I_0 - \frac{V_S}{R}$$

Thus,

$$i(t) = \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R} \right) e^{-t/\tau}$$

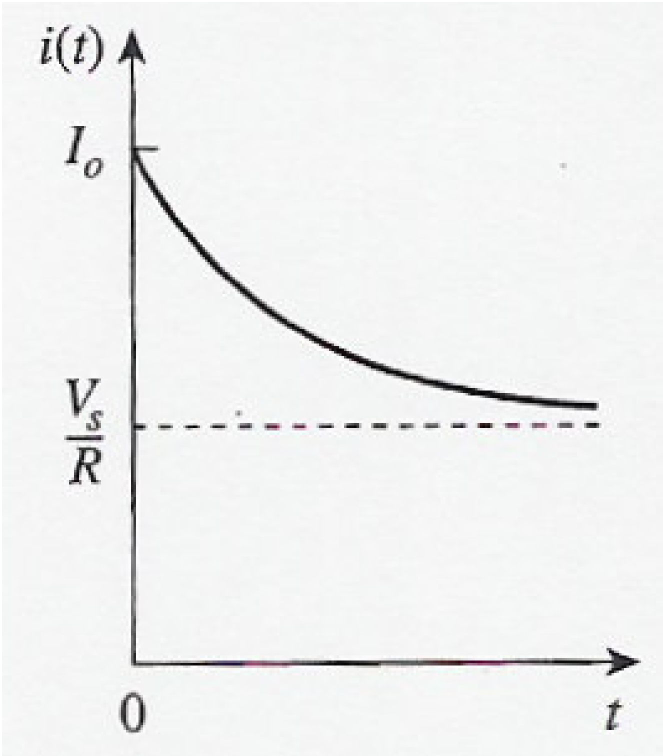

Figure 6.34

- Or, the response can be written as,

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} \quad (6.16)$$

- To find the step response of an RL circuit

(iv) Find the initial inductor current, $i(0)$.

- obtain from the given circuit for $t < 0$.

(v) Find the final inductor current, $i(\infty)$.

- obtain from the given circuit for $t > 0$.

(vi) The time constant, τ.

- obtain from the given circuit for $t > 0$.

- Example 1:

Find $i(t)$ in the circuit in Figure 6.35 for $t > 0$. Assume the circuit has been closed for a long time.

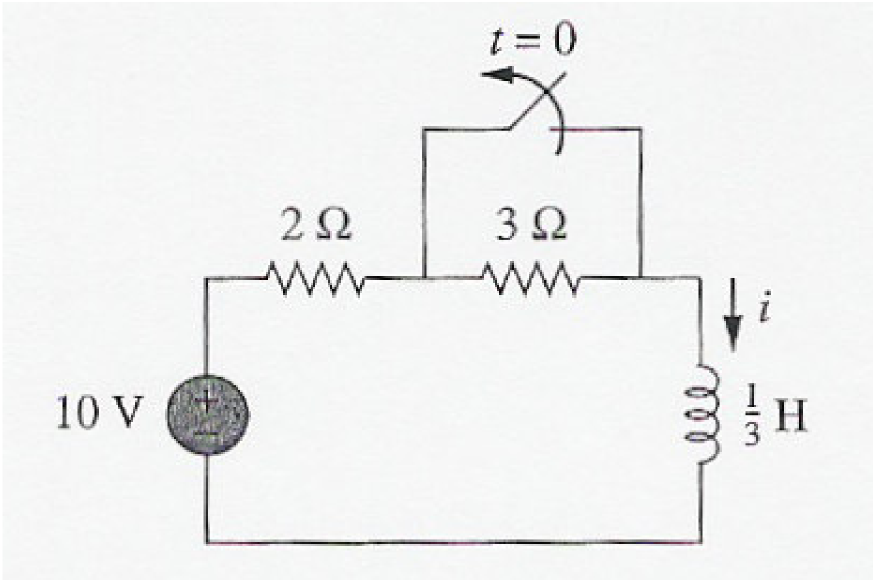

Figure 6.35

When $t < 0$,

The 3Ω is short-circuited.

The inductor acts like a short circuit.

$$i(0^-) = \frac{10}{2} = 5\text{A}$$

Since the inductor current cannot change instantaneously,

$$i(0) = i(0^+) = i(0^-) = 5\text{A}$$

When $t > 0$,

The switch is open.

$$i(\infty) = \frac{10}{2+3} = 2\text{A}$$

$$R_{TH} = 2+3 = 5\Omega$$

$$\tau = \frac{L}{R_{TH}} = \frac{\frac{1}{3}}{5} = \frac{1}{15}\text{s}$$

Thus,

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$= 2 + (5-2)e^{-15t} = 2 + 3e^{-15t}A$$